



Hybrid Decision-Making Method within Dempster-Shafer Framework

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Abstract

Modern multi-criteria decision-making (MCDM) approaches must handle ambiguous and imprecise initial information about decision-making phenomena in an efficient way. Application of the Dempster-Shafer theory (DST) provides a strong mathematical basis for processing uncertain information. Due to the close relationship between fuzzy sets and DST, various fuzzy set environments are applied to model the DST system to solve multi-criteria decision-making problems. To work on this challenge, a picture fuzzy set (PFS) can be applied to model the vague information. This paper presents a novel MCDM approach, namely the hybrid SWARA DS and CoCoSo DS method, developed within Dempster-Shafer's framework. The benefits of the new proposed approach in handling MCDM problems involving uncertain criteria and expert weightings are demonstrated through its application in selecting the optimal roof shape for a renovated single-family house. Sensitivity and comparative analyses validate the method's reliability and effectiveness.

Keywords: Multi-Criteria Decision Making; SWARA DS; CoCoSo DS; Dempster-Shafer; Picture Fuzzy Set theory.

1 Introduction

Despite significant progress and the development of more than 200 methods, the field of MCDM remains an active and vital area of research and practical application. Its technical aspects are still being refined. Recent developments in MCDM include the integration of fuzzy logic, data-driven models,

and different forms of the collaboration in the decision support systems [35]. The new fuzzy extensions of the most popular MCDM approaches have been proposed in recent years. These advancements have enhanced the robustness and adaptability of MCDM methods, making them more effective in handling complex decision-making scenarios involving multiple objectives and diverse criteria.

MCDM has put a lot of effort into considering more realistic situations in the decision-making field. Therefore, the attribute data cannot be expressed by real crisp numbers. Contemporary business society needs advanced decision-making tools to provide solutions to complicated real-life problems. The alternative details can be modelled in different ways since, in real-life situations, this information always contains inaccurate and wrong reports. Classical fuzzy sets [62] provide an advanced membership (Mem) degree categorization. This Mem degree is usually utilized at the multipurpose unit interval of the Mem function $[0, 1]$. During the last few years, another modelling trend has evolved, where decision-makers have simultaneously obtained information about object membership and non-membership functions. To handle this situation, the intuitionistic fuzzy sets (IFS) [4] have been established. In order to handle special cases, extensions have been proposed, such as cubic IFS [17] and interval-valued IFS [3]. IFS shelters deficient information, applying two functions: membership and non-membership. The complex real-life situations in decision-making are also modelled by soft sets [37], fuzzy soft sets [33], IFSS [34], interval-valued IFS [57].

Decision-makers encounter challenges when managing high levels of vague, imprecise, and uncertain initial information about real-life applications. In this situation, it is not so easy to consider all characteristics of the alternative modes and the evaluation criteria. Despite the great capacities of IFSs to model such MCDM problems, their utilization spectrum still has some limitations in covering more complicated fuzzy information. In other words, IFSs do not consider neutral degree function as an independent component. The extended IFS with a separate function of the degree of neutral membership is named by the picture fuzzy set (PFS) [20]. Recently, Coung [9] proposed operational laws for picture fuzzy sets to model imprecise and ambiguous information about the alternatives in decision-making problems. These progressive fuzzy sets reflect the human thinking phenomena in the closest way.

MCDM-based approaches provide valuable tools for addressing the complex and conflicting situations that arise when managing different energy resources in the buildings' energy consumption sector [51], [46]. Solar energy research focuses on technological improvements, waste management solutions, and analysis of energy savings and environmental benefits [36], [5], [19]. MCDM methods are known to be effective in solving challenges related to the installation of solar arrays, e.g., in evaluating the replacement of fossil fuels with alternative energy [1], in evaluating the installation options of solar photovoltaic (PV) systems at different locations [8], and in ranking and selecting solar panel alternatives based on conflicting criteria [47]. Studies related to single-family homes look at efficiency, cost-effectiveness, durability, and compatibility with the homeowner's energy needs [50]. However, there is no study or proposed method for installing solar panels on the roof of a single-family house, depending on the chosen shape of the house.

Despite advancements, challenges remain in integrating MCDM with emerging technologies and improving its ability to handle uncertainty [15]. This ongoing research ensures that MCDM remains a dynamic and evolving field, continually adapting to new real-world applications and decision-making needs. Currently, several contributions are directed towards DM methods that effectively model imprecise and uncertain initial information [7], [30], [42]. The Dempster-Shafer theory (DST) [11], [48] provides a strong mathematical basis for processing uncertain information. Due to the close relationship between fuzzy sets and DST, various fuzzy set environments are applied to model the DST system to solve multi-criteria decision-making problems [14], [21], [44], [16], [45]. Therefore, the main contributions of the present paper consist of:

- Applying PFS operational algebra in the DST framework for the CoCoSo method enables the avoidance of drawbacks of operational laws established by employing Archimedean t-norm and t-conorm.
- Combining the proposed novel SWARA DS and CoCoSo DS methods, relevant results appraise the vagueness exhibited in the decision-making process.

- A novel hybrid decision-making approach combining SWARA and CoCoSo within the Dempster-Shafer framework appraises the vagueness exhibited in the decision-making process. Including in the analysis the vagueness of the expert's opinions expressed through the criterion ratings and obtaining adequate rankings of the alternatives of solar panel installation for renovated houses.

The remainder of this article is structured as follows: Section 2 presents a literature review of the CoCoSo and PFS-based methods for decision-making, along with their applications. Section 3 provides detailed explanations of the Dempster-Shafer CoCoSo method within the context of a Picture Fuzzy Set environment and the SWARA DS method for addressing multi-criteria decision-making problems. Section 4 demonstrates the application of our proposed method through real-life examples. Discussion and conclusions finalize the paper.

2 Literature Review

The literature review is divided into two subsections: the WASPAS based methods and PFS based methods for decision-making. As the field is relevant and popular, more recent scientific advances in the field are presented.

2.1 CoCoSo based Methods and their Applications

Recent applications of CoCoSo in the development of MCDM methods and their research focus are presented in Table 1.

Table 1: CoCoSo recent results

| Author(s) | Research focus | Application type) | Parameter type |
|-----------------------------|---|----------------------|---|
| Turskis et al. [55] | Evaluation of building retrofit strategies | Real-life | m-generalized q-neutrosophic numbers |
| Demir et al. [10] | Sustainable urban mobility planning | Real-life | Fuzzy numbers |
| Pankaj et al. [39] | Selection of optimal sustainable materials | Real-life | Shannon's entropy |
| Ghoushchi et al. [18] | Identification of wind farm failures | Real-life | Spherical Fuzzy Numbers |
| Sumerli Sarıgül et al. [52] | Evaluation of airport hotel service quality | Real-life | Quantitative criteria (deterministic data) |
| Ortíz-Barrios et al. [38] | Identification of weak points in food industry supply chains | Real-life | Intuitionistic Fuzzy AHP (IF-AHP) and Fuzzy DEMATEL (IF-DEMATEL), CoCoSo |
| Karami et al. [24] | Selection of contractors in construction projects | Real-life | Interval-valued fuzzy sets |
| Parsa Rad et al. [40] | Optimization of supplier selection according to real conditions | Illustrative example | Fuzzy best-worst method (FBWM), gray CoCoSo |
| Popović [43] | Ranking of candidate selection | Illustrative example | Deterministic crisp parameter |
| Torkayesh et al. [54] | Assessment of the level of social sustainability | Illustrative example | Deterministic crisp parameter |
| Joshi [23] | Evaluation of user interface (UI) model | Illustrative example | MARCOS (Measurement of Alternatives and Ranking according to Compromise Solution), CoCoSo |
| Chatterjee et al. [6] | Optimization of drilling parameters | Fuzzy parameters | |
| Peng et al. [41] | Optimization of tank layout | Real-life | Intuitionistic Fuzzy Soft Set, IFS |
| Xie et al. [60] | Evaluation of English language quality | Illustrative example | Intuitionistic Fuzzy Sets, IFS |
| Zheng et al. [66] | Decision-making in the case of a diagnosis of sepsis | Illustrative example | Interval-Valued q-Rung Orthopair Fuzzy Sets, IVq-ROFS |
| Our | Assessment of solar panel installation for the renovated single-family houses | Real-life | Picture Fuzzy Dempster-Shafer (PFDS) theory |

Turskis et al. [55] proposed a new CoCoSo-based MCDM methodology. This methodology uses m-generalized q-neutrosophic sets (CoCoSomGqNN) to better handle uncertainty in evaluating modernization strategies in civil engineering. Demir et al. [10] applied a fuzzy MCDM-based framework that combines F-FUCOM and F-CoCoSo methods. This framework is used to prioritize sustainable urban mobility measures. Pankaj et al. [39] presented an integrated MCDM approach using Shannon entropy and CoCoSo. This method is designed to select the most sustainable engineering material by evaluating seven alternatives according to six criteria for optimal performance and cost. Ghoushchi et al. [18] presented a modified FMEA method using spherical fuzzy SWARA and CoCoSo methods to more accurately prioritize failure modes under uncertainty. This method was applied in a case study of the Manjil wind farm in Iran. Sumerli Sarigül et al. [52] evaluated the quality of service of 17 five-star airports using 11 criteria. They used the MEREC method to determine the importance of the criterion and the MARCOS and CoCoSo methods for ranking the alternatives. Ortíz-Barrios et al. [38] developed an integrated IF-AHP, IF-DEMATEL and CoCoSo approach for improving sustainability management in digital food supply chains. They identified “location” as the main criterion and “manufacturing capacity” as the main influencing factor. Karami et al. [24] introduced an interval-valued fuzzy MCDM framework based on IVF-SWARA and IVF-CoCoSo for improving contractor selection under uncertainty, demonstrating enhanced decision-making accuracy, flexibility, and reliability through a real-life application and sensitivity analysis.

Parsa Rad et al. [40] presented a fuzzy Best-worst method and grey CoCoSo-based approach to improve supplier selection under uncertainty in the downstream oil, gas, and petrochemical industry. Popović [43] developed a personnel selection framework that combines the SWARA method for determining criteria weights and the CoCoSo method for ranking candidates. The system was designed to increase the efficiency of decision-making in recruiting the most suitable individuals for a given position. Torkayesh et al. [54] evaluated the social sustainability performance of G7 countries using OECD data and a new integrated weighting system combining CRITIC, Shannon’s Entropy, and CoCoSo methods. Joshi [23] proposed a novel evaluation framework using MARCOS, CoCoSo, and entropy-based weighting to optimize journal website UI designs by systematically assessing key criteria, including content accessibility and readability.

Chatterjee et al. [6] applied five fuzzy versions of the CoCoSo method for the optimization of drilling parameters for aluminium metal matrix composites. Peng et al. [41] proposed an IFS-based CoCoSo decision-making method that integrates subjective and objective weights via a new score function and the CRITIC approach, effectively evaluating CCN cache placement strategies under uncertainty. Xie et al. [60] developed the IFN-CoCoSo approach, which integrates intuitionistic fuzzy sets and multiple distance measures to effectively evaluate and rank the quality of blended English teaching in vocational colleges within a MAGDM framework. Zheng et al. [66] introduced a novel CoCoSo method based on interval-valued q-rung ortho-pair fuzzy sets (IVq-ROFS), incorporating a new fuzzy entropy and enhanced compromise scores to improve decision-making accuracy and robustness, validated through a sepsis diagnosis case.

2.2 Picture Fuzzy Sets based Methods and their Practical Examples

Recent results of Picture Fuzzy Sets based methods, and their research focus are presented. Recent results of Picture Fuzzy Sets based methods, and their research focus are presented in Table 2.

Liu et al. [29] introduced the Linguistic Picture Fuzzy Set (LPFS) to address multi-criteria decision-making challenges in supplier selection. The authors proposed linguistic picture fuzzy weighted averaging and geometric operators and generalized weighted distance measures between LPFSs. They also extended the Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) method and the TODIM method (which stands for interactive and multi-criteria decision-making in Portuguese) to incorporate the proposed distance measures. Kumar and Bisht [28] developed an effective entropy measure designed for picture fuzzy sets. A practical example was demonstrated for the selection of an optimal air-conditioning system in a superspecialty hospital. Wei [59] expanded the TODIM model using picture fuzzy numbers (PFNs) to solve the commercialization problem of emerging technologies.

Zhang et al. [64] used the traditional EDAS (Evaluation based on Distance from Average Solution)

Table 2: Picture Fuzzy Sets recent results

| Author(s) | Research focus | Method) | Application type |
|----------------------|---|--|----------------------|
| Liu et al [29] | Supplier selection | Linguistic PF TOPSIS+Todim | Illustrative example |
| Kumar and Bisht [28] | Air-conditioning system for a super-specialty hospital | PF Entropy | Illustrative example |
| Wei [59] | Emerging technology commercialization | PF TODIM | Illustrative example |
| Zhang et al. [64] | Green supplier selection | PF EDAS | Illustrative example |
| Shi and Peng [49] | Service quality evaluation of online medical services | Normal PF COPRAS | Illustrative example |
| Duong and Thao [13] | Market segment evaluation | PF Dissimilarity measure | Illustrative example |
| Tian and Peng [53] | Tourism attraction recommendation evaluation | PF ANP-TODIM | Illustrative example |
| Jang et al. [22] | Enterprise resource planning system evaluation | PF Probabilistic hesitant aggregation | Illustrative example |
| Zhao et al. [65] | Evaluation of the university faculty for tenure and promotion | PF parametric similarity measure | Illustrative example |
| Lu et al. [31] | Investment company evaluation | PF generalized soft aggregation | Illustrative example |
| Luo and Long [32] | Investment evaluation | PF geometric aggregation Trapezoidal number | Illustrative example |
| Ali et al. [2] | Car purchase evaluation | PF complex Aczel-Alsina aggregation | Illustrative example |
| Zhang et al. [63] | Energetic security evaluation | PF ELECTRE TRI | Illustrative example |
| Khan et al. [27] | Circulation center evaluation | PF logarithmic aggregation | Illustrative example |
| Khalil et al. [26] | Human resource evaluation | PF interval-valued | Illustrative example |
| Wang et al. [56] | Patient disease evaluation | PF MABAC | Illustrative example |
| Dhumras et al. [12] | Electronic Marketing Strategic Plan evaluation | PF q-Rung TOPSIS/VIKOR | Illustrative example |
| Our | Assessment of solar panel installation for the renovated single-family houses | PFDS theory | Real-life |

model to construct a picture fuzzy EDAS model. This model was verified by selecting a green supplier. Shi and Peng [49] proposed an extended normal picture fuzzy complex proportional assessment (COPRAS) method to evaluate the quality of online medical services. Duong and Thao [13] developed the picture fuzzy set based dissimilarity measure and applied it to evaluate market segments and select the optimal one. Tian and Peng [53] integrated the PFS, analytic network process (ANP), and TODIM to develop the MCDM method for evaluating the tourism attraction recommendation. The ANP approach was used to decide the criteria weights. Jang et al. [22] proposed the MCDM methods based on the picture hesitant fuzzy sets (PHFSs) called probabilistic picture hesitant fuzzy sets (P-PHFSs) and verified them by evaluating the enterprise resource planning system. Zhao et al. [65] created a parametric similarity measure between picture fuzzy sets and applied it to assess the university faculty for promotion and tenure. Lu et al. [31] integrated the picture fuzzy soft set, and the fuzzy parameter set to create the generalized picture fuzzy soft set. The authors presented a numerical example for the evaluation of an investment company. Luo and Long [32] developed picture fuzzy geometric aggregation operators that are based on trapezoidal fuzzy numbers to evaluate the investment. Ali et al. [2] derived power aggregation operators based on the Aczel–Alsina operational laws for managing complex picture fuzzy (CPF) data. They presented an illustrative example with an evaluation of car purchases. Zhang et al. [63] developed an ELECTRE TRI based group decision-making with picture fuzzy sets and a new method they applied to assess energy security. Khan et al. [27] proposed picture fuzzy logarithmic aggregation operators for fuzzy decision-making problems of circular center evaluation. Khalil et al. [26] constructed an interval-valued picture fuzzy set (IVPFS) to evaluate human resources. Wang et al. [56] developed a theory based multi-attributive border approximation area comparison (MABAC) method under a picture fuzzy environment and applied this method to assess patient disease. Dhumras et al. [12] incorporated the bi-parametric discriminant information measure and q-rung picture fuzzy set in the TOPSIS/VIKOR decision-making models for electronic marketing strategic plan evaluation.

3 Methods for Multi-Criteria Decision-Making

In this section, a detailed explanation of two methods is provided: CoCoSo-DS and SWARA-DS, designed to address multi-criteria decision-making challenges. A framework has been developed to facilitate a coherent and efficient decision-making process that involves stakeholders and promotes informed choices. Figure 1 illustrates the overall methodology used for selecting the optimal roof shape when renovating a single-family house.

3.1 Dempster Shafer CoCoSo under Picture Fuzzy Set Environment

3.1.1 Preliminaries of the Picture Fuzzy Sets

The Picture Fuzzy Set (PFS) is essentially an extension of the classical fuzzy set [62] and intuitionistic fuzzy sets [4]. It can be characterized as:

Definition 1. [20] If X = finite universal set, picture fuzzy set can be expressed by $P = \{\langle x_j, \mu_p(x_j), \eta_p(x_j), \nu_p(x_j) | x_j \in X\}$, where $\mu_p(x_j), \eta_p(x_j), \nu_p(x_j) \in [0, 1]$, $\mu_p(x_j) + \eta_p(x_j) + \nu_p(x_j) \leq 1$, and $\pi_P(x_j) = 1 - (\mu_p(x_j) + \eta_p(x_j) + \nu_p(x_j))$ for any $x_j \in X$. The functions $\mu_p(x_j), \eta_p(x_j), \nu_p(x_j)$ represent the degrees of the positive, neutral, and negative membership, respectively. $\pi_P(x_j)$ signifies the degree of refusal membership of x in the set P . In discrete cases, a triplet $\langle\mu, \eta, \nu\rangle$ is designated as a picture fuzzy number (PFN) if $\mu, \eta, \nu \in [0, 1]$, and $\mu + \eta + \nu \leq 1$.

Definition 2. [58] Let $a = \langle\mu, \eta, \nu\rangle$ be a PFN. Subsequently, its score and accuracy values are represented by (Eq. (1), (2))

$$S(\alpha) = \mu - \eta - \nu \quad (1)$$

$$A(\alpha) = \mu + \eta + \nu \quad (2)$$

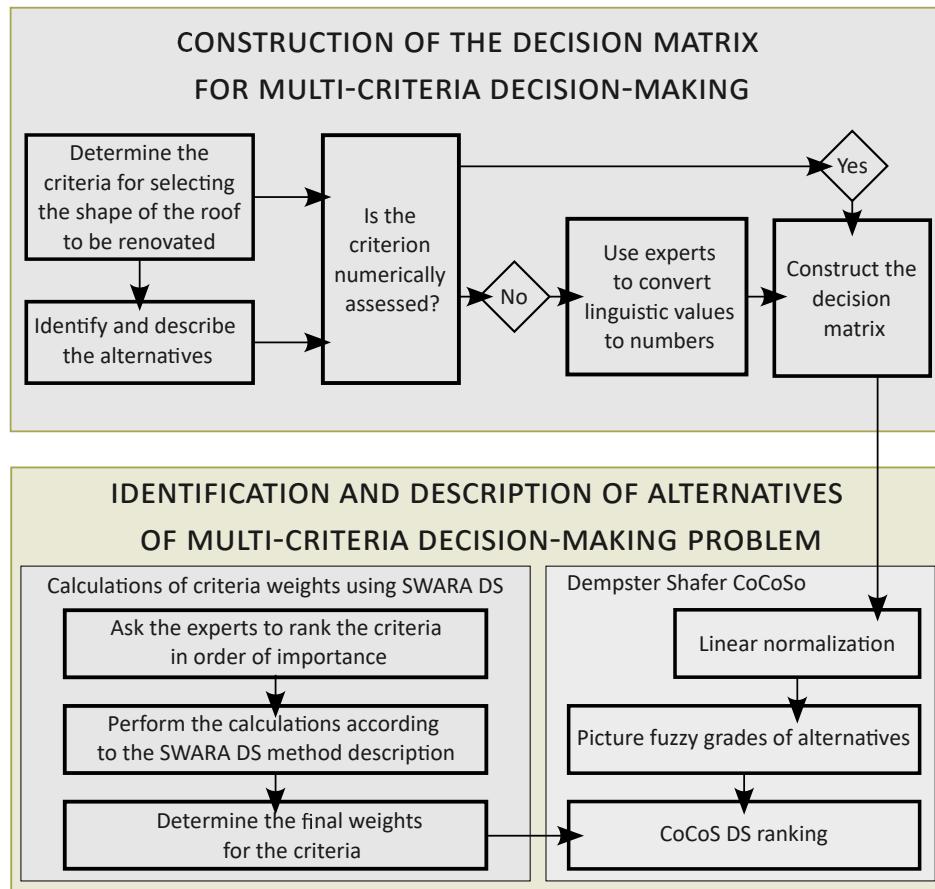


Figure 1: Framework of the proposed Multi-Criteria Decision-Making methodology

Definition 3. [58] When examining two PFNs represented as $a_1 = \langle \mu_1, \eta_1, \nu_1 \rangle$ and $a_2 = \langle \mu_2, \eta_2, \nu_2 \rangle$, following cases can be taken into account.

1. If $S(a_1) < S(a_2)$, then $a_1 < a_2$
2. If $S(a_1) = S(a_2)$ and $A(a_1) < A(a_2)$, then $a_1 < a_2$
3. If $S(a_1) = S(a_2)$ and $A(a_1) = A(a_2)$, then $a_1 = a_2$

3.1.2 Picture Fuzzy Sets in the Environment of Dempster-Shafer Theory

Definition 4. [11], [48] Consider a space of objects, denoted as X , characterized by mutually exclusive and exhaustive hypotheses. In this context, a Basic Probability Assignment (BPA) on X can be conceptualized as a mapping from the set X to the unit interval $[0, 1]$, illustrated as follows:

$$F : P(X) \rightarrow [0, 1] \quad , \text{ which satisfies } F(\emptyset) = 0 \quad \text{and} \quad \sum_{Y \subseteq P(X)} F(Y) = 1$$

Definition 5. [11], [48] If F is BPA on set X such as $F(X) > 0$, then BF can be constructed for F using $Bel(X) = \sum_{Y \subseteq X} F(Y)$.

Definition 6. [11], [48] If F is BPA on set X such as $F(X) > 0$, then PF can be formulated as follows: $Pl(X) = \sum_{Y \cap X \neq \emptyset} F(Y)$.

Definition 7. [11], [48] The belief interval BI on X is defined as $BI(X) = \{Bel(X), Pl(X)\}$ and this interval can be considered as the „true probability“ of X .

A strong connection exists between fuzzy set models and the practical use of DST. Unlike the limitations associated with operational laws governed by Archimedean t-norm and t-conorm for fuzzy

quantities, the operation laws within the DST framework do not encounter the same constraints. Consequently, aggregation methods developed using these operational laws often generate more rational and persuasive outcomes for Multiple Criteria Decision Making (MCDM) problems.

When addressing decision-making problems in a picture fuzzy set environment, three mutually exclusive hypotheses can be identified:

- H1: $x \in C$ with a positive degree μ
- H2: $x \in C$ with a neutral degree η
- H3: $x \in C$ with a negative degree ν

In the context of the DST environment, $\mu_C(x)$ can be interpreted as the probability or evidence of $x \in C$, representing the BPA function $F(\{H_1\}) = \mu_C(x)$. Similarly, $F(\{H_2\}) = \eta_C(x)$ and $F(\{H_3\}) = \nu_C(x)$. With this notation, we can formulate $F(\{H_1, H_2\}) + F(\{H_1, H_3\}) + F(\{H_2, H_3\}) + F(\{H_1, H_2, H_3\})$ as $\pi_C(x) = 1 - (\mu_C(x) + \eta_C(x) + \nu_C(x))$. Since $\mu_C(x) + \eta_C(x) + \nu_C(x) + \pi_C(x) = 1$, it can be concluded that this set of the functions $\mu_C(x)$, $\eta_C(x)$, $\nu_C(x)$, $\pi_C(x)$ represent a correct BPA function. By definitions 6 and 7,

$$Bel_C(x) = F(\{H_1\}) = \mu_C(x) \quad (3)$$

$$\begin{aligned} Pl_C(x) &= F(\{H_1\}) + F(\{H_1, H_2\}) + F(\{H_1, H_3\}) + F(\{H_2, H_3\}) + F(\{H_1, H_2, H_3\}) = \\ &= \mu_C(x) + \pi_C(x) = 1 - (\eta_C(x) + \nu_C(x)) \end{aligned} \quad (4)$$

Using this notation, the definition of the PFS can be restated as follows:

Definition 8. [45] *PFS Cover a finite universe of discourse X within DST environment can be defined as $C = \{\langle, BI_C(x)\rangle | X \in Y$, here $BI_C = [Bel_C(x), Pl_C(x)] = [\mu_C(x), 1 - (\eta_C(x) + \nu_C(x))]$. The functions $\mu_C(x), \eta_C(x), \nu_C(x) : X \rightarrow [0, 1]$ are positive, neutral, and negative membership degrees of $x \in X$ to C and satisfy $0 \leq \mu_C(x) \leq 1 - (\eta_C(x) + \nu_C(x)) \leq 1$.*

Definition 9. [45] *In the DST framework, a PFN can be expressed as $\alpha = BI = [Bel, Pl] = [\mu, 1 - \eta - \nu]$. The assessment of two PFNs in the DST environment involves the evaluation of score and accuracy values (Eq. (5), (6)):*

$$S_{DST}(\alpha) = \frac{(Bel + Pl)}{2} = \frac{(1 + \mu - \eta - \nu)}{2} \quad (5)$$

$$A_{DST}(\alpha) = (Pl - Bel) = (1 - \mu - \eta - \nu) \quad (6)$$

Definition 10. [45] *If two PFN are considered in the DST framework as $a_1 = [Bel_1, Pl_1] = [\mu_1, 1 - \eta_1 - \nu_1]$ and $a_2 = [Bel_2, Pl_2] = [\mu_2, 1 - \eta_2 - \nu_2]$, then the following cases can be considered*

1. If $S_{DNT}(\alpha_1) < S_{DNT}(\alpha_2)$, then $\alpha_1 < \alpha_2$
2. If $S_{DNT}(\alpha_1) = S_{DNT}(\alpha_2)$ and $A_{DST}(\alpha_1) < A_{DST}(\alpha_2)$, then $\alpha_1 < \alpha_2$
3. If $S_{DNT}(\alpha_1) = S_{DNT}(\alpha_2)$ and $A_{DST}(\alpha_1) = A_{DST}(\alpha_2)$, then $\alpha_1 = \alpha_2$

Definition 11. [45] *Consider two PFNs in the DST environment, $\alpha_1 = [Bel_1, Pl_1] = [\mu_1, 1 - \eta_1 - \nu_1]$, and $\alpha_2 = [Bel_2, Pl_2] = [\mu_2, 1 - \eta_2 - \nu_2]$. Let λ - be a real number in the range $[0, 1]$. Then, operations related to PFNs in the DST environment can be carried out as follows (Eq. (7) - (10)):*

$$\alpha_1 \oplus \alpha_2 = [\frac{1}{2}(Bel_1 + Bel_2), \frac{1}{2}(Pl_1 + Pl_2)] = [\frac{1}{2}(\mu_1 + \mu_2), \frac{1}{2}(1 - \mu_1 - \nu_1 - \mu_2 - \nu_2)] \quad (7)$$

$$\alpha_1 \otimes \alpha_2 = [(Bel_1 \cdot Bel_2), (Pl_1 \cdot Pl_2)] = [(\mu_1 \cdot \mu_2), (1 - \mu_1 - \nu_1) \cdot (1 - \mu_2 - \nu_2)] \quad (8)$$

$$\lambda \cdot \alpha_1 = [\lambda \cdot Bel_1, (\lambda \cdot Pl_1)] = [(\lambda \cdot \mu_1), \lambda \cdot (1 - \mu_1 - \nu_1)] \quad (9)$$

$$\alpha_1^\lambda = [Bel_1^\lambda, Pl_1^\lambda] = [\mu_1^\lambda, (1 - \mu_1 - \nu_1)^\lambda] \quad (10)$$

3.1.3 Dempster Shafer CoCoSo under Picture Fuzzy Set Environment

The CoCoSo method is a multi-criteria evaluation method that combines various aggregation principles (sum, product, trade-off) to determine which alternative is closest to the ideal, taking into account all evaluation criteria [61]. A new version of this method, called CoCoSo DS, has been developed. This new method is based on the Dempster-Shafer theory and is designed for use in the picture fuzzy number environment. The primary purpose of the method is to provide a solution for the MCDM (multi-criteria decision-making) problem. The main concepts behind CoCoSo DS can be summarized in the following way.

Step 1. *First, decision-making X is constructed. Each element in this matrix represents the rating of a particular alternative based on the options considered. These elements can be denoted by $x_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n$, which is the rating of the criterion i concerning the alternative j . Finally, the aggregated decision matrix is represented by Eq. (11) as a criterion*

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (11)$$

Step 2. *The process of normalizing the decision matrix X is done using the linear normalization method. This method can be expressed by Eq. (12) as follows for the benefit criteria:*

$$\tilde{x}_{ij} = \frac{x_{ij}}{\Delta + \max_{1 \leq i \leq m} x_{ij}} \quad (12)$$

And for the cost criteria (Eq. (13)):

$$\tilde{x}_{ij} = \frac{-\Delta + \min_{1 \leq i \leq m} x_{ij}}{x_{ij}} \quad (13)$$

In the above equations, the variable Δ is a non-zero value introduced to ensure numerical stability.

Step 3. *The normalized aggregated decision matrix \tilde{X} in the crisp form obtained in the second step is converted to the picture fuzzy numbers. This conversion involves applying the relationships between normalized terms of the alternatives and picture fuzzy numbers. The conversion grades are defined by using linguistic expressions.*

Step 4. *In this step, weighted comparability and power-weighted comparability series are constructed for each alternative. They are denoted as R_j and P_j , respectively (Eq. (14)-(15)):*

$$R_j = \sum_{i=1}^m \tilde{x}_{ij}^p \cdot w_i \quad (14)$$

$$P_j = \sum_{i=1}^m (\tilde{x}_{ij}^p)^{w_i} \quad (15)$$

Step 5. *The basis for the calculations of the relative weights is three score assessment strategies, which are expressed by the following Eq. (16)-(18):*

$$k_{ja} = \frac{S(P_j) + S(R_j)}{\sum_{j=1}^n (S(P_j) + S(R_j))}, \quad (16)$$

$$k_{jb} = \frac{S(R_j)}{\min_{1 \leq j \leq n} S(R_j)} + \frac{S(P_j)}{\min_{1 \leq j \leq n} S(P_j)}, \quad (17)$$

$$k_{jc} = \frac{\lambda S(R_j) + (1 - \lambda)S(P_j)}{\lambda \max_{1 \leq j \leq n} S(R_j) + (1 - \lambda) \max_{1 \leq j \leq n} S(P_j)}; 0 \leq \lambda \leq 1. \quad (18)$$

The arithmetic mean values of the weighted sum model (WSM) and the weighted product model (WPM) are obtained by the Eq. (16). By Eq. (17), relative scores of WSM and WPM are calculated, which is determined with respect to minimum values of $S(P_j)$ and $S(R_j)$. The balanced compromise scores of the WSM and WPM models are calculated using Eq. (18). In our case, the value for $\lambda = 0.5$ is chosen.

Step 6. The final ranking function is expressed by Eq. (19):

$$k_j = (k_{ja}k_{jb}k_{jc})^{\frac{1}{3}} + \frac{1}{3}(k_{ja} + k_{jb} + k_{jc}). \quad (19)$$

3.2 Determination of Criteria Weights using SWARA DS

The stepwise weighted assessment relative analysis (SWARA) method can be applied to developing specialized decision support systems for rational dispute resolution [25]. When ranking the criteria, the most important criterion was prioritized, and then the levels of importance among the other criteria were determined in the picture fuzzy set environment. The final level of importance among the criteria is calculated by applying the proposed operational laws in the Dempster-Shafer framework, and the score function is used to execute reverse conversion to define the criteria weights:

$$\tilde{S}_{i \leftrightarrow i+1} = \left[\frac{1}{l} \sum_{k=1}^l \mu_{k,i \leftrightarrow i+1} \cdot \tilde{\omega}_k, \frac{1}{l} \sum_{k=1}^l (1 - \eta_{k,i \leftrightarrow i+1} - \nu_{k,i \leftrightarrow i+1}) \cdot \tilde{\omega}_k \right] \quad (20)$$

$$S_{i \leftrightarrow i+1} = S_{DST}(\tilde{S}_{i \leftrightarrow i+1}) \quad (21)$$

Where the relative importance of criterion $\langle \mu_{k,i \leftrightarrow i+1}, \eta_{k,i \leftrightarrow i+1}, \nu_{k,i \leftrightarrow i+1} \rangle$ is expressed by k expert and presented in a picture fuzzy set environment, $\tilde{\omega}_k$ is the weight of the particular expert.

Appropriate calculations (Eq. (22) - (24)) are used for the representation of the criteria weights:

- cumulative values of comparative importance $s_{i \leftrightarrow i+1}$;
- defined importance coefficient

$$k_i = s_{i \leftrightarrow i+1} + 1; i = 2 \dots n \quad (22)$$

- recalculated average criteria weights

$$q_i = \frac{q_{i-1}}{k_i} \quad (23)$$

- final weights of the criteria set

$$w_i = \frac{q_i}{\sum_{i=1}^n q_i} \quad (24)$$

here n is a count of criteria.

4 A Case Study Model of Installing Solar Panels on the Roofs of Renovated Houses

A house renovation involves a wide range of works, including upgrading the heating system, replacing windows, roofing, etc. This study focuses on selecting the right roof shape, so the various works mentioned above were not analyzed. It is appropriate to use MCDM methods to select the optimal roof shape for installing solar panels on a single-family house under renovation. This requires using experts to carefully evaluate a set of criteria [51], [46]. When assessing the roof of a renovated house, it is essential to select experts from different fields, as each expert brings unique knowledge and perspectives to the project. Eleven experts with specific expertise were selected to participate in the

study. The team consists of three architects, one structural engineer, one fire protection specialist, two solar installers, one electrical engineer, one energy efficiency expert, one cost estimator, and one building inspector.

4.1 Alternatives for Selecting the Roof Shape

In preparation for the case study, the experts were presented with the most popular single-pitched, double-pitched and four-pitched roof projects. After consultation with experts/specialists from different fields (architects, engineers, etc.), seven roof-shape projects (alternatives) acceptable for a specific case were agreed upon. The possibility that the house could have additional single-story spaces or verandas was also considered. The slope angle values for all sides of the roof are given in Figure 2.

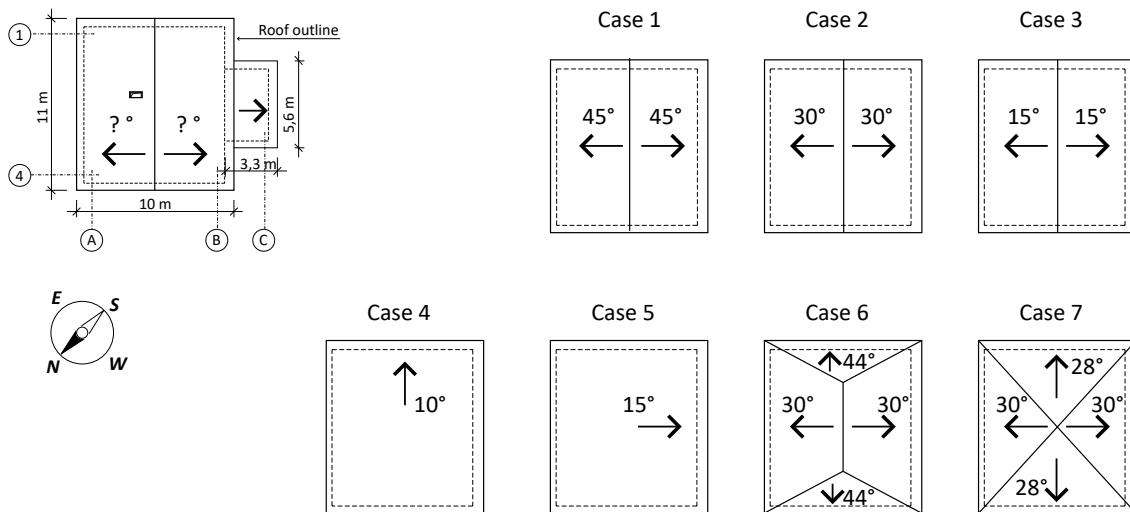


Figure 2: Plot plan and orientation of the typical house and the most popular roof shape and slope projects

Specific aspects of the renovated house correspond to a typical project:

- The house is planned to have a floor area of 156.0 m² after renovation and is located at 700.00 m²; it is a single-story house with an attic and porch,
- Before the renovation, the roof was gable, and the attic had two windows in the walls between the A-B and B-A axes,
- The courtyard is oriented north-east,
- One corner of the walls of the house is oriented precisely to the south, which means that for the calculation of the actual output of the solar PV panels, the angle will be chosen according to the azimuth 45°,
- The experts from a company specializing in the installation of solar panels found that the electrical connection capacity is sufficient and that the external and internal supporting walls are strong enough to accommodate different roof shapes and slopes.

To ensure that the second-floor walls had enough windows, roof cornices of varying heights were included in all alternatives. The chimney flue was not relocated and remained in the same position as before the renovation. In all cases (seven alternatives), the entire roof plan is covered with solar panels. A comparison of the two types of solar panels is made. For this purpose, the coverage of the roof planes of all the alternatives with both type 1 and type 2 solar panels was simulated. For example, when modelling solar panels on a roof with a pitch of 15° (Case 3, Figure 2), two types of solar panels can be installed. In the first case, type 1 (2384×1303×35 mm, 120 EUR) solar panels are modelled; in the second case, type 2 (1755×1038×35 mm, 230 EUR) solar panels are modelled.

Table 3: Transform linguistic evaluations into picture fuzzy numbers

| Evaluations for Alternatives | Abb. | PFN |
|------------------------------|------|------------------|
| Absolutely High | AH | (0.85,0.05,0.05) |
| Very High | VH | (0.75,0.05,0.15) |
| High | H | (0.65,0.05,0.25) |
| Slightly High | SH | (0.55,0.05,0.35) |
| Moderate | M | (0.45,0.05,0.45) |
| Slightly Low | SL | (0.35,0.05,0.55) |
| Low | L | (0.25,0.05,0.65) |
| Very Low | VL | (0.15,0.05,0.75) |
| Absolutely Low | AL | (0.05,0.05,0.85) |

Despite the possibility of installing fewer type 2 solar panels (50 units of type 1, 36 units of type 2), calculating the total power of all elements reveals that it is even 23 per cent higher. In this case, a higher power option is selected when the power of the sun panels is higher:

- Type 1 – 50 (units) \times 370 (W) = 18.50 (KW),
- Type 2 – 36 (units) \times 670 (W) = 24.12 (KW).

After modelling all seven alternative scenarios, the potential heated volume of the second floor was calculated. The assessment of the aesthetic appearance of the house after the renovation was made, considering contemporary trends and personal expert opinion. Depending on the shape of the roofs in the projects, for various alternatives, one or two skylights are installed for cleaning the solar panels: A1, A2, A6 and A7 – two skylights each, A3 and A4 – one skylight each (Figure 2).

4.2 Construction of Decision Matrix

In this study, the criteria weights were defined using the SWARA DS method. This method involves two steps for ranking the criteria weights. In the first step, experts and specialists use their knowledge, information, and experience to rank the criteria in order of importance. The most important criterion is assigned the rank of 1, while the least important is ranked last. The intermediate criteria are then ranked according to their levels of importance. In the second step, the criteria weights are calculated using the SWARA DS method (Eq. (20–24)).

The first most important criterion identified by the experts was “realistic solar panel capacity”, and the second was “aesthetic appearance of the house”, which were considered very important by all experts. The remaining four criteria were determined by consensus through discussion. In this way, the criteria are ranked as follows: C1 – Power rating of the solar panels, C2 – Aesthetic rating of the house, C3 – Additional technical measures, C4 – The price of the mounted solar panels, C5 – The volume of the heated space on the 2nd floor, C6 – Panels wipe availability, the ratio between cleaned and total area of the surface. Experts assessed the alternatives based on the linguistic terms (Table 3). Table 4 shows the values determined by the experts, indicating how much more important each expert considered one criterion to be than another.

The SWARA DS method is used to calculate the weights of the experts by combining their ratings according to the importance levels and using Dempster-Shafer theory: $w_k = [0.15 \ 0.1 \ 0.08 \ 0.11 \ 0.09 \ 0.15 \ 0.1 \ 0.06 \ 0.06 \ 0.06 \ 0.05 \ 0.05]$. Table 5 shows the mean values of the relative comparative importance of the criteria obtained from Table 4, as well as the relative, recalculated and final criteria weights. The CoCoSo DS decision-making method uses the final criteria weights to select the most rational solution.

The final decision matrix for rank evaluation is shown in Table 6.

The ultimate outcomes of the described CoCoSo DS (steps 4–6) are outlined in Table 7. The intermediate results of CoCoSo DS are also shown in this table.

The results obtained from the CoCoSo DS and SWARA DS methods show that solar panels are best installed on roofs with the steepest slope (alternatives A1 and A6). According to the ranking,

Table 4: Assessment of the relative significance of the criteria in terms of linguistic meaning

| Expert | Pairwise comparison of criteria relative importance | | | | |
|--------|---|---------------------------|---------------------------|---------------------------|---------------------------|
| | $C_{1 \leftrightarrow 2}$ | $C_{2 \leftrightarrow 3}$ | $C_{3 \leftrightarrow 4}$ | $C_{4 \leftrightarrow 5}$ | $C_{5 \leftrightarrow 6}$ |
| 1 | VL | H | VL | SH | AL |
| 2 | AL | H | L | M | VL |
| 3 | VL | VH | AL | L | AL |
| 4 | VL | L | M | AL | AL |
| 5 | L | AL | SL | VL | M |
| 6 | M | VL | AL | AL | AL |
| 7 | H | L | VL | SL | AL |
| 8 | L | SL | SH | L | VL |
| 9 | AL | SH | VH | SH | AL |
| 10 | L | SL | AL | SL | AL |
| 11 | AL | SL | M | M | AL |

Table 5: Evaluating criteria weights by SWARA DS method

| C1 - C6 | Average values of comparative importance indicators, $s_{j \leftrightarrow j+1}$ | Coefficients of comparative importance indicators, k_j | Recalculated (intermediate) indicators weights, w_j | Final indicators weights, q_j |
|---------|--|--|---|---------------------------------|
| C_1 | – | 1,0000 | 1.0000 | 0,3030 |
| C_2 | 0,2455 | 1,2455 | 0,8030 | 0,2430 |
| C_3 | 0,4818 | 1,4818 | 0,5420 | 0,1640 |
| C_4 | 0,3364 | 1,3364 | 0,4050 | 0,1230 |
| C_5 | 0,3909 | 1,3909 | 0,2920 | 0,0880 |
| C_6 | 0,1364 | 1,1364 | 0,2570 | 0,0780 |
| | | | 3,298 | |

these two alternatives are the best, regardless of the C_4 and C_5 criteria: the highest price of solar

Table 6: The criteria and alternatives for the weight ratio matrix

| C1 - C6 | Optimum | Weight | Alternatives | | | | | | |
|---------|---------|--------|--------------|---------|---------|---------|---------|---------|---------|
| | | | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
| C_1 | max | 0,3030 | 24,370 | 19,960 | 19,400 | 21,490 | 21,580 | 17,680 | 14,980 |
| C_2 | max | 0,2430 | 9,730 | 4,910 | 2,090 | 2,180 | 3,180 | 8,090 | 7,000 |
| C_3 | min | 0,1640 | 1,180 | 3,910 | 7,820 | 9,820 | 7,910 | 3,000 | 4,270 |
| C_4 | min | 0,1230 | 10270 | 8040 | 7820 | 7440 | 7820 | 7080 | 6240 |
| C_5 | min | 0,0880 | 288,000 | 227,160 | 182,610 | 198,000 | 221,130 | 244,760 | 222,720 |
| C_6 | max | 0,0780 | 0,979 | 0,993 | 0,886 | 0,920 | 0,931 | 0,956 | 0,962 |

Table 7: Intermediate results of the CoCoSo DS and the final ranking of the alternatives

| Alternatives | R | P | k_a | k_b | k_c | k | Rank |
|--------------|------------------|------------------|--------|--------|--------|--------|------|
| A_1 | (0.0787, 0.0934) | (0.9103, 0.9291) | 0.1501 | 2.4784 | 1.0000 | 1.9286 | 1 |
| A_2 | (0.0649, 0.0818) | (0.8797, 0.9067) | 0.1437 | 2.2186 | 0.9578 | 1.7802 | 4 |
| A_3 | (0.0640, 0.0811) | (0.8457, 0.8854) | 0.1378 | 2.0122 | 0.9184 | 1.6567 | 6 |
| A_4 | (0.0691, 0.0854) | (0.8447, 0.8844) | 0.1376 | 2.0000 | 0.9166 | 1.6499 | 7 |
| A_5 | (0.0865, 0.0999) | (0.8599, 0.8934) | 0.1400 | 2.0785 | 0.9331 | 1.6981 | 5 |
| A_6 | (0.0801, 0.0945) | (0.8936, 0.9168) | 0.1466 | 2.3315 | 0.9766 | 1.8452 | 2 |
| A_7 | (0.0787, 0.0934) | (0.8818, 0.9084) | 0.1442 | 2.2388 | 0.9610 | 1.7916 | 3 |

panels (in alternative A1) and the highest heated volume on the 2nd floor (in alternatives A1 and A6). A comparison between alternatives A6 (2nd place) and A4 (7th place) shows that although the criterion with the highest weight (C1) is higher in alternative A4 the scores of the following two most important criteria (C2 and C3) have led to the last place of the latter alternative in the ranking.

4.3 Sensitivity Analysis

A sensitivity analysis is used to assess how changes in critical parameters can affect replacement decisions. Due to varying experiences and specializations, experts assessed the criteria differently. For example, difficulties arise when assessing initial costs (C4) and operating and maintenance costs (C3) due to potential differences in house infrastructure, installation complexity, and additional work, among other factors.

Therefore, to obtain a wider range of opinions and insights, a sensitivity analysis of different expert groups was conducted using the results of an additional survey involving seven final-year students and six potential customers with no prior experience in home design but interested in installing photovoltaic solar panels (Table 8). Based on the opinions of students and customers, we created 13 (Test 2-Test 14) additional criteria weights (Table 8), and the results are shown in Figure 3. In Figure 3, “Test 1” corresponds to the opinion of qualified experts, and “Test 2-Test 14” - the evaluation information of seven final-year students and six potential customers. It is easy to notice that according to this proposed model, experts, students and inexperienced customers provide similar evaluations of alternatives. The sensitivity test performed only confirms the reliability of the proposed model since the ranking of alternatives did not change drastically.

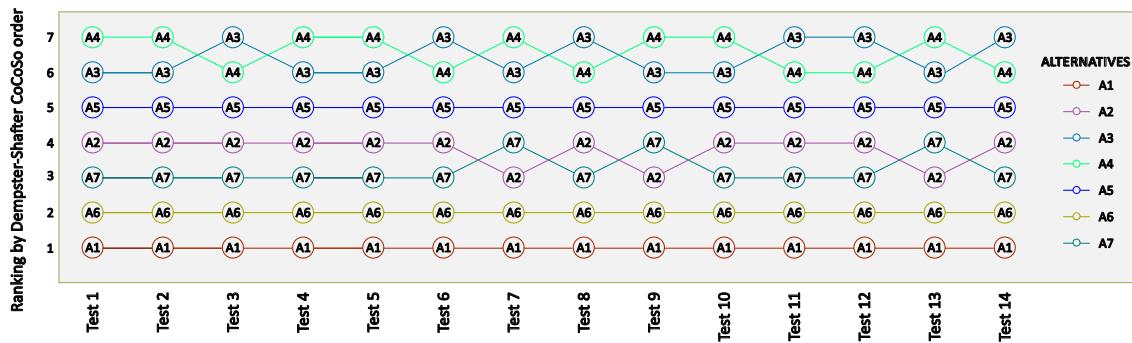


Figure 3: The results of the sensitivity analysis of different expert groups

Table 8: Weight of the tests for sensitivity analysis of different expert groups

| Expert groups | C1 | C2 | C3 | C4 | C5 | C6 |
|--------------------|----|----|----|----|----|----|
| Test 2 (student 1) | L | SH | VL | AL | AL | L |
| Test 3 (student 2) | SL | SL | AL | VL | L | VL |
| Test 4 (student 3) | L | L | L | AL | L | L |
| Test 5 (student 4) | L | VL | SL | L | L | AL |
| Test 6 (student 5) | SL | L | VL | L | VL | L |
| Test 7 (student 6) | M | AL | SL | VL | AL | AL |
| Test 8 (student 7) | M | SL | AL | AL | L | AL |
| Test 9 (client 1) | SH | VL | L | AL | AL | VL |
| Test 10 (client 2) | L | SH | L | AL | AL | VL |
| Test 11 (client 3) | M | L | VL | VL | L | AL |
| Test 12 (client 4) | L | L | AL | M | VL | AL |
| Test 13 (client 5) | L | VL | L | VL | L | L |
| Test 14 (client 6) | L | L | AL | AL | L | L |

Alternatives A1, A5 and A6 maintain the same position throughout all tests, so it can be said that their results are stable and not very sensitive to changes in weight. Meanwhile, alternatives A3

and A4 are the most sensitive to changes in ratings, while A2 and A7 exhibit slightly smaller but still noticeable fluctuations.

5 Conclusions

This study presents a new framework of SWARA DS and CoCoSo DS methods as a new decision analysis method that incorporates modelling ambiguous and imprecise initial information. Despite uncertainty and incomplete information, the evaluation of solar panel installation for renovated single-family houses can be effectively conducted applying the proposed approach. To work on this challenge, a PFS can be applied to model the vague information. Application of the Dempster-Shafer theory (DST) in a picture fuzzy set environment appraises incomplete information in decision making. This framework also allows us to overcome the limitations of the traditional algebraic laws incorporating probability measures in criteria values in the decision-making process.

The characteristics of the novel proposed SWARA DS and CoCoSo DS methods are studied using a real-life numerical example devoted to selecting the best roof design for installing solar panels. Moreover, this study offers the single-family home construction industry a unique opportunity to prioritize sustainability, competitiveness, and adaptability to changes in energy resources, thereby contributing to increased awareness and responsibility.

The criteria weights were determined by the SWARA DS method. The subjective opinions of experts in various fields were used to determine the weightings of the criteria. The estimations of experts from different fields for each parameter were combined using the PF-weighted geometric operator, and the resulting aggregated values were then defuzzified to convert PF data into standard crisp data.

Applying the DST framework in PFNs enables the quantification of incomplete data in our analyses while overcoming the limitations of algebraic operational laws. This is achieved by incorporating probability measures into argument values and utilizing probability theory to determine the values associated with different alternatives. The proposed approach offers several advantages:

- The impact of biased criteria values is reduced by utilizing power weights in the PFS.
- The constraints of incomplete data and the limitations of algebraic t-norm and t-conorm based operational laws through DST are addressed.

The study showed that gable roofs are the most rational solution for a single-family house being renovated, as they combine technical, economic, energy and aesthetic aspects, and the final choice between the two best alternatives should be determined by the customer's priorities - whether they are aiming for maximum energy efficiency (45°) or cost-effectiveness and more convenient maintenance (30°). The scope of the analysis could have been expanded further by considering a wider range of alternatives, but the number chosen was sufficient for methodological demonstration and practical design situations.

The robustness of the model is confirmed by sensitivity analysis. This indicates the stability and consistency of the proposed operators. The results also confirm that the developed operators are more universal, consistent, and stable compared to the existing aggregation operators analyzed in the CoCoSo DS environment. Due to these properties, they are more effective in solving various MCDM problems in real conditions. Although a clearly defined and consistent conversion operator was used in this study, the application of other transformation schemes could be investigated in the future to assess the robustness of the method under different conversion procedures.

Author contributions

The authors contributed equally to this work.

Conflict of interest

The authors declare no conflict of interest.

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