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A Multi-Attribute Decision-Making Method Based on Entanglement Dominance Relation

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Abstract

To address the limitations of traditional dominance relation-based rough sets in handling information systems with partial order relations, this paper proposes a multi-attribute decision-making method based on entanglement dominance relation. First, the definition of the existing entanglement domain is revised, and the concept of entanglement dominance relation is introduced. The properties of the entanglement domain and its computation method-the improved Bron-Kerbosch algorithm are discussed. On this basis, a secondary ranking method is proposed, in which the subsets of the entanglement domain are first externally ordered, and then the objects within each subset are internally ordered according to specified rules. Finally, an empirical analysis of logistics supply chain performance evaluation is conducted to verify the effectiveness and dominance of the proposed method. The results demonstrate that this method can more accurately reflect the superiority and inferiority relation among objects, providing a new perspective and solution for multi-attribute decision-making problems.

Keywords: entanglement domain, entanglement dominance relation, maximal clique, uncertainty domain, multi-attribute decision-making.

1 Introduction

Rough set theory [1] is a mathematical tool for handling uncertain, imprecise, and inconsistent data. Since it was proposed by Professor Pawlak in the 1980s, it has been continuously developed in theoretical research. To date, rough set theory has been widely applied in fields such as machine learning, data mining, and knowledge discovery [2, 3, 4, 5, 6, 7]. Classical rough sets partition the objects in a universe into equivalence classes and approximate target decisions through lower and upper approximation sets. However, for information systems with partial order relations, the classical rough set cannot handle such problems effectively. Greco et al. [8] proposed a rough set model based on dominance relation. The dominance relation is an asymmetric relation used to describe the preference or ordering among objects. In the rough set model based on dominance relation, the dominance relation among objects replaces the equivalence relation in classical rough sets. This relation allows for the ranking or comparison of objects within a set, making it more suitable for handling ordered data. According to the attribute values of objects, this method partitions the objects into the superiority set and inferiority set and establishes upward unions (downward unions) through the superiority set (inferiority set) to achieve approximate descriptions.

In traditional dominance relation-based rough set theory, when comparing objects and alternatives, it is required that "object x is superior to object y if and only if x is superior to y in every attribute" for a dominance relation to be established. This condition is overly stringent, as it neglects situations where an object may perform better on some attributes while performing worse on others, leading to incomparability between objects and further affecting the extraction of decision rules. Therefore, it is necessary to improve the traditional dominance relation.

In recent years, scholars at home and abroad have conducted extensive research and extensions on multi-attribute decision-making based on dominance relation. Wang [9] utilized the ideal-degree to obtain the state set and the conditional probability, which has the same drawbacks as the distance based method. Weng [10] introduced a dominance degree threshold β and defined a dominance relation-based rough set model based on the dominance degree, proposing a ranking method. Lin [11] determined the weights of each evaluation index using the similarity of dominance granular structures and obtained the comprehensive ability rankings of the evaluated objects through aggregated group decision-making results. Mondal [12] proposed a novel three-way multi-attribute decision-making (3WMADM) model by combining the 3WDM theory and regret theory under an incomplete information system to realize the function of classifying and sorting alternatives. Wang [13] proposed a three-way decision-based multi-attribute decision-making (3WD-MADM) method, in which two state sets are developed through probabilistic dominance classes. Yu [14] addressed group-oriented multi-attribute decision-making problems by introducing advantage and disadvantage neighborhoods, computing local dominance degrees, and investigating their properties. Based on this, they designed a novel multi-attribute decision-making method. Qin [15] proposed a three-valued multi-attribute decision-making method based on an interval-valued intuitionistic fuzzy set model. This method calculates the conditional probability of each alternative using probabilistic dominance relations and constructs a loss function from the perspective of an ideal solution, thereby enabling ranking and classification of alternatives. Keshavarz [16] extended the EDAS (Evaluation based on Distance from Average Solution) method to handle multi-attribute decision-making problems under fuzzy environments and demonstrated the applicability of the proposed method through a supplier selection case study. Gao [17] proposed a rescheduling decision selection method, the G1-improved entropy method and the improved TOPSIS method are combined to evaluate rescheduling schemes. Wu [18] proposed a new σ superiority and inferiority relation entropy based on fuzzy classes of σ superiority and inferiority relation, and finally presented two multi-attribute decision-making methods based on σ superiority and inferiority relation entropy. Peng [19] presented a novel fuzzy three-way multi-attribute decision-making approach with the probabilistic dominance relation, the method was able to rank and classify the alternatives into the positive region, the boundary region. Huang [20] proposed an intuitionistic fuzzy probabilistic dominance relation model and construct the MADM method under the probabilistic dominance relation. Xue [21] introduced set-pair dominance relation into multi-granulation decision rough sets, improved the dominance degree, and established a multi-granulation decision rough set model based on set-pair dominance relation. Weng [22] addressed the multi-attribute decision-making problem under interval rough fuzzy numbers by defining concepts such as the optimal solution, worst solution, positive distance, negative distance, and dominance degree of the information system, determined attribute weights using the entropy weight method, and provided the weighted comprehensive dominance degree and corresponding ranking results. Weng [23] proposed a rough set model based on probabilistic dominance relation and further provided a ranking method. Liu [24] addressed the problem of multi-attribute decision-making under interval rough numbers with different distributions by defining upper and lower approximate interval dominance degrees, proposing an attribute weight determination method based on dominance balance degrees, and, on this basis, proposed a multi-attribute decision-making method based on interval rough numbers. Liu [25] proposed a rough set model based on entanglement domains, in which objects with similar attributes and functional approximations are integrated through entanglement domains, fully considering the influence of elements that cannot be included in the superiority set and inferiority set on the current object.

In summary, current research on multi-attribute decision-making based on dominance relation has yielded fruitful results, and researchers have proposed various derivative relations, such as entanglement dominance relation, neighborhood relation, and three-way decision-making. Traditional dominance relation requires that an object must be superior to or inferior to another object across all attributes, a condition that is overly stringent. This leads to the exclusion of objects that are superior in some attributes but inferior in others, thereby reducing the universality of the model. In view of the existing shortcomings in the current studies on entanglement dominance relation, such as the insufficient integration of objects with indistinguishable relations and the distinguishability of some objects within the entanglement domain - this paper extends and supplements the dominance relation, revises the definition of the entanglement domain, and expands the concept of entanglement dominance relation. Entanglement dominance relation integrates objects with similar attributes and functional approximations through entanglement domains, fully considering the influence of objects that cannot be included in the superiority or inferiority sets on the current sample, thus avoiding decision-making errors caused by neglecting these objects. Finally, a multi-attribute decision-making method based on entanglement dominance relation is proposed, providing a new perspective and solution for multi-attribute decision-making problems.

2 Basic Concepts

2.1 Dominance Relation

To facilitate the exposition, some basic theories of dominance relation-based rough set models are introduced.

Definition 1 ([1]). *An information system $S = (U, A, V, f)$ is defined as an information system, where $U = \{x_1, x_{2,\dots}, x_m\}$ is a finite set of objects, $A = \{a_1, a_{2,\dots}, a_n\}$ is a finite set of conditional attributes, V is the set of attribute values, and f is a mapping: $U \times A \to V$, representing the relationship between objects and attribute values. If the values under all attributes possess an order relation, meaning that they can be compared in size or priority, then the information system $S = (U, A, V, f)$ is called an ordered information system.*

Definition 2 ([5]). *In the ordered information system $S = (U, A, V, f)$, for any $\forall B \subset A$, the dominance relation under B is defined as $R_B^{\gtrsim} = \{(x, y) \in U \times U \mid f(x, a) \geq f(y, a), \forall a \in B\}$. The dominance relation satisfies reflexivity, transitivity, and antisymmetry.*

Definition 3 ([26]). *Let $S = (U, A, V, f)$, where $B \subset A$, and $R_B^{\gtrsim}$ is the dominance relation of S under B. For $\forall x, y \in U$, the dominance sets of x is defined as: $R_B^{\gtrsim}(x) = \left\{y \in U \mid (y, x) \in R_B^{\gtrsim}\right\}$, where $R_B^{\gtrsim}(x)$ denotes the set of objects that are superior to x under attribute set B ; and $R_B^{\lesssim}(x) = \left\{y \in U \mid (x, y) \in R_B^{\gtrsim}\right\}$, where $R_B^{\lesssim}(x)$ denotes the set of objects that are inferior to x under attribute set B.*

2.2 Entanglement Domain

To fully consider the objects that cannot be included in the superiority set or the inferiority set, some scholars have proposed the use of entanglement domains to represent the set of objects that do not have a strictly unidirectional dominance relation with a given object.

Definition 4 ([25]). *For $\forall x \in U$, the uncertainty domain of the object x denoted as S_x: $S_x = U - R_B^{\gtrapprox}(x) - R_B^{\lessapprox}(x)$, where $R_B^{\gtrapprox}(x)$ and $R_B^{\lessapprox}(x)$ represent the superiority set and inferiority set of object x, respectively.*

It can be inferred that the uncertainty domain of the object x is the set of objects that cannot be included in either the superiority set or the inferiority set x, that is, the set of objects with which a definite dominance relation x cannot be established. In terms of attribute values, it is not difficult to observe that elements in S_x may be superior to x in some attributes while inferior to x in others.

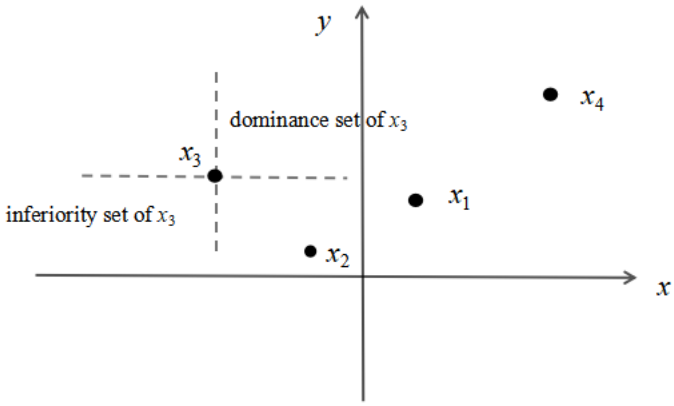

Figure 1: The uncertainty domain of x_3

As shown in Figure 1, suppose there are four two-dimensional data objects x_1, x_2, x_3, x_4, where larger coordinate values represent better performance of the corresponding object in the respective attributes. It can be observed that x_3 and x_4 have definite dominance relation (x_4 is superior to x_3), while no definite dominance relation can be established between x_3 and x_1, nor between x_3 and x_2. Therefore, for x_3, both x_1, x_2 are included in its uncertainty domain, that is $S_{x_3} = \{x_1, x_2\}$.

Definition 5 ([25]). *For $\forall x \in U$, the entanglement domain of the object x, denoted as Q_x, is defined as $Q_x = \{y \in U : S_x \cap S_y \neq \varnothing\} \cup \{x\}$. (The original definition is $Q_x = \{y \in U : S_x \cap S_y \neq \varnothing\} \cup x$).*

For data samples x and y, if there exists an observed sample q such that both x and y fall into the uncertainty domain of q, then x and y are said to be externally indistinguishable, and thus x and y are defined to have an entanglement dominance relation. The set of elements in the domain U that have a data entanglement dominance relation with x is defined as the entanglement domain of x. In the entanglement domain Q_x of object x, the relations between x and all elements are not definite.

It can be inferred from Definition 5 that the relations between the object x and all elements in its entanglement domain Q_x are not definite. However, there are two deficiencies in this definition. On the one hand, the uncertainty domain S_x of the object x should be included in Q_x, but according to the computational formula of Q_x in Definition 5, there may be cases where Q_x does not completely include S_x. On the other hand, according to the computational formula in Definition 6, the elements contained in Q_x are not necessarily externally indistinguishable from x, which is inconsistent with the original intention and interpretation of defining the entanglement domain.

For example, $S_{x_1} = \{x_3, x_6\}, S_{x_2} = \{x_4, x_5\}, S_{x_3} = \{x_1, x_4, x_6\}, S_{x_4} = \{x_2, x_3, x_6\}$, $S_{x_5} = \{x_2\}, S_{x_6} = \{x_1, x_3, x_4\}$. According to the uncertainty domain definition, $S_{x_2} = \{x_4, x_5\}$, indicating that object x_2 cannot be distinguished from x_4, x_5 based on dominance relation. Thus, it should be that $x_4, x_5 \in Q_{x_2}$. However, according to the computation formula given in Definition 6, the entanglement domain $Q_{x_2} = \{x_2, x_3, x_6\}$, and objects $x_4, x_5 \notin Q_{x_2}$, which is inconsistent with the expected interpretation of the entanglement domain. In addition, $Q_{x_3} = \{x_1, x_4, x_6\}, [x_1]^{\leq}_{A} = \{x_1, x_2, x_4, x_5\}$, where it is clear that $x_4 \in [x_1]^{\leq}_{A}$, meaning that x_4 can be distinguished x_1 in terms of dominance relation. Thus, some objects within the entanglement domain are distinguishable from x_1 based on dominance relation.

Therefore, the entanglement domain obtained according to Definition 5 neither sufficiently integrates objects with indistinguishable relations nor ensures that all objects within the entanglement domain are pairwise indistinguishable. Furthermore, it does not fully consider the influence of objects that cannot be included in the superiority set or inferiority set on the current sample. To address these issues, this paper proposes to revise the definition of the entanglement domain and apply it to multi-attribute decision-making.

3 Decision-Making Method Based on Entanglement Dominance Relation

This section mainly aims to improve the expression and computation of the entanglement domain in Definition 5 and to represent the set of objects that do not have a strictly unidirectional dominance relation with a given object based on the revised entanglement domain.

Definition 6. *The entanglement domain Q_U over the domain U is defined as $Q_U = \{Q_1, Q_2 \ldots Q_i \ldots Q_k\}$, where $Q_i \subset U$, and the following three conditions must be satisfied:*

(1) For $\forall x_m, x_n \in Q_i$, it holds that $x_m \in S_{x_n}, x_n \in S_{x_m}$;

(2) For $\forall x_m \in Q_i, x_n \in Q_k, x_m \notin Q_k, x_n \notin Q_i$, it holds that $x_m \notin S_{x_n}, x_n \notin S_{x_m}$;

(3) If we define $Q_k = Q_k \cup \{x_m\}, x_m \notin Q_k$, then there must exist $\exists x_n \in Q_k$ such that $x_m \in [x_n]^{\leq}_{A}$ or $x_m \in [x_n]^{\geq}_{A}$, which results in Q_k no longer satisfying conditions (1) and (2).

According to the definition, the entanglement domain is a collection of subsets of the domain U, and the objects within each subset are pairwise indistinguishable in terms of dominance relation. If two objects do not belong to the same subset within the entanglement domain, then these two objects are distinguishable.

Definition 7. *Given the entanglement domain $Q_U = \{Q_1, Q_2 \ldots Q_i \ldots Q_k\}$ over the domain U, where $Q_i = \{x_{i1}, x_{i2} \ldots \ldots\}$, the entanglement dominance relation system is constructed as $R^{Q}_{B} = \{(x_{i1}, x_{i2}) \mid x_{i1}, x_{i2} \in Q_i, i = 1, 2 \ldots k\}$.*

The computation of entanglement domains exhibits a fundamental connection with the maximal clique enumeration problem in graph theory. For the convenience of readers, the definitions of cliques and maximal cliques are briefly introduced below. Clique: A clique is a subset of vertices in a graph such that every two distinct vertices are adjacent, that is, the subset forms a complete subgraph. Maximal Clique: A maximal clique is a clique that cannot be extended by including any additional adjacent vertex from the graph without losing its completeness.

Based on the definitions of the entanglement domain, clique, and maximal clique, the theoretical relationship between maximal cliques and entanglement domains is established.

Construct a graph $G = (V, E)$, where $V = U$ (the set of objects in the domain), and the edge set E is defined such that for any $x, y \in U, (x, y) \in E$ if and only if x, y are incomparable, i.e., $x \in S_y, y \in S_x$. Since the incomparability relation is symmetric, the graph $G = (V, E)$ is undirected.

According to Condition (1) in Definition 7 of the entanglement domain, for any $\forall x_m, x_n \in Q_i$, it holds that $x_m \in S_{x_n}, x_n \in S_{x_m}$, which implies that x_m, x_n are adjacent (i.e., connected by an edge) in the graph. Therefore, any two distinct vertices in Q_i are adjacent, satisfying the definition of a clique. Thus, each Q_i is a clique in the graph $G = (V, E)$.

According to Condition (3) of the entanglement domain definition, suppose $Q_k = Q_k \cup \{x_m\}, x_m \notin Q_k$, where $\exists x_n \in Q_k$. Then there must exist $x_m \in [x_n]^{\leq}_{A}$ or $x_m \in [x_n]^{\leq}_{A}$, causing Q_k to no longer satisfy Conditions (1) and (2). In other words, after adding x_m, the set $Q_k \cup \{x_m\}$ is no longer a clique, as it contains at least one pair of non-adjacent vertices. This satisfies the definition of a maximal clique (i.e., cannot be further extended). Therefore, each Q_i is a maximal clique in the graph $G = (V, E)$.

Let $Q_U = \{Q_1, Q_2 \ldots Q_i \ldots Q_k\}$, where each Q_i is a maximal clique. According to Condition (2) of the entanglement domain definition, for $\forall x_m \in Q_i, x_n \in Q_k$, with $x_m \notin Q_k, x_n \notin Q_i$, it holds that $x_m \notin S_{x_n}, x_n \notin S_{x_m}$; in graph G, this implies that vertices from different maximal cliques are not adjacent (i.e., no edge exists between them). This conforms to a basic property of maximal cliques: if two vertices belong to different maximal cliques, they must not be adjacent (otherwise, they should belong to the same maximal clique). Condition (3) ensures that the maximality of each clique guarantees Q_U includes all possible maximal cliques (i.e., no omissions). Therefore, Q_U is the complete set of all maximal cliques in graph G.

Therefore, the Bron-Kerbosch algorithm from graph theory can be employed to find entanglement domains. However, the classical Bron-Kerbosch algorithm may involve redundant computations; thus, the optimized version with pivot selection is adopted to reduce the number of recursive calls. Pivot selection significantly decreases the number of recursive branches and greatly shortens the computation time, making it more suitable for solving small- to medium-sized graphs.

Algorithm Description for Constructing the Entanglement Domain Q_U over Domain U:

Input: The uncertainty domain S_{x_i} for each object x_i in U.

Output: The entanglement domain Q_U.

The steps for constructing the entanglement domain Q_U over domain U are as follows:

(1) According to Definition 5, for each object $x_i\,(i = 1, 2 \ldots |U|)$, obtain its uncertainty domain $S_{x_i} = \{x_{i1} \ldots x_{ij} \ldots\}$ for each x_i, construct pairwise indistinguishable subsets based on the uncertainty domains. Remove duplicate subsets, and for symmetric subsets (e.g., $\{x_i, x_{ij}\}, \{x_{ij}, x_i\}$), retain only the subset where the object index appearing first is smaller than the latter. Thus, the resulting sets $\{x_i, x_{i1}\} \ldots \{x_i, x_{ij}\} \ldots$ form the entanglement dominance relation system $R_B^Q = \{\{x_i, x_{i1}\} \ldots \{x_i, x_{ij}\} \ldots\}$.

(2) Based on the entanglement dominance relation system R_B^Q, construct an undirected graph $G = (V, E)$, where $V = U$ is the vertex set and E is the set of edges. That is, $E = \{\{x_i, x_{i1}\} \ldots \{x_i, x_{ij}\} \ldots\}$. The adjacency list of the undirected graph $G = (V, E)$ is thus obtained from the vertex set and edge set.

(3) Use the classical recursive Bron-Kerbosch algorithm with Pivot selection optimization to find the maximal cliques of the graph $G = (V, E)$. The core idea of the optimization is to select the vertex with the maximum degree as the pivot to reduce the number of recursive calls.

(4) Output the sets of all maximal cliques as the entanglement domain.

The pseudocode for finding the entanglement domain of universe U is as follows:

Input: Uncertainty domains S_{x_i} of each object x_i in U

Output: Entanglement domain Q_U

$R_B^Q = \varnothing$

for $x_i \in U$

for $x_j \in S(x_i)$

if $i<j$

$R_B^Q = R_B^Q \cup \{(x_i, x_j)\}$

end if

end for

end for

$V = U$

$E = R_B^Q$

$Q_U = \varnothing$

BronKerbosch1 (R, P, X) # R: The regiment that is currently being constructed. P: Candidate vertex set. X: Excluded vertex set.

if P and X are both empty:

report R as a maximal clique

max_degree_vertex = find_vertex_with_max_degree(P) # Find the vertex with the highest degree in P

sorted_P = sort_P_vertices_by_degree_descending(P) # Sort the vertices from the degrees in P from largest to smallest *for each vertex v in sorted_P:*

BronKerbosch 1 $(R \cup \{v\}, P \cap N(v), X \cap N(v))$ #$N(v)$ *is the set of adjacency points with v*

$P = P \mid \{v\}$

$X = X \cup \{v\}$

else

$R_U = R_U U R$

Output R_U

Example 1. *Suppose the uncertainty domains of five objects are given as follows:* $S_{x_1} = \{x_2, x_5\}$, $S_{x_2} = \{x_1, x_3, x_4, x_5\}$, $S_{x_3} = \{x_2, x_4\}$, $S_{x_4} = \{x_2, x_3, x_5\}$, $S_{x_5} = \{x_1, x_2, x_4\}$.

According to Step 1, the entanglement dominance relation system is obtained:

$$R_B^Q = \{\{x_1, x_2\}, \{x_1, x_5\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_4, x_5\}\}$$

According to Step 2, the undirected graph $G = (V, E)$ is constructed, where the vertex set is $V = \{x_1, x_2, x_3, x_4, x_5\}$, and the edge set is:

$$E = \{\{x_1, x_2\}, \{x_1, x_5\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_4, x_5\}\}.$$

The adjacency list of the undirected graph is then obtained based on the vertex and edge sets.

Table 1: Adjacency tables for undirected graphs

Vertex	Adjacent Vertices	Description
x_1	$[x_2, x_5]$	Vertex x_1 is adjacent to vertices x_2 and x_5
x_2	$[x_1, x_3, x_4, x_5]$	Vertext x_2 is adjacent to vertices x_1, x_3, x_4 , and x_5
x_3	$[x_2, x_4]$	Vertex x_3 is adjacent to vertices x_2 and x_4
x_4	$[x_2, x_3, x_5]$	Vertext x_4 is adjacent to vertices x_2, x_3 , and x_5
x_5	$[x_1, x_2, x_4]$	Vertex x_5 is adjacent to vertices x_1, x_2 , and x_4

The visualization of the undirected graph is shown below:

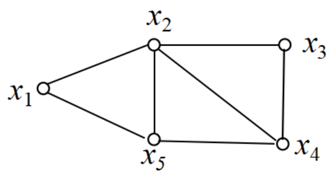

Figure 2: Undirected Graph 1

According to Step 3, since the vertex x_2 has the maximum degree, we start from x_2. The initial clique is $\{x_2\}$, and the neighboring vertices are x_4, x_5, x_1, x_3. Among them, x_4 and x_5 have the highest degrees, so we first expand to $\{x_2, x_4\}$. The neighbors of vertex x_4 are x_3, x_5, leading to two possible expansions: $\{x_2, x_4, x_3\}$ and $\{x_2, x_4, x_5\}$. For $\{x_2, x_4, x_3\}$, since $x_2 - x_3$ is adjacent, the set $\{x_2, x_4, x_3\}$ forms a clique. The neighbor set of x_3 is x_2, x_4 and no further expansion is possible, indicating that $\{x_2, x_4, x_3\}$ is a maximal clique. For $\{x_2, x_4, x_5\}$, since $x_2 - x_5$ are adjacent, the set $\{x_2, x_4, x_5\}$ forms a clique. Attempting to add x_1, a neighbor of x_5, is unsuccessful because $x_1 - x_4$ are not adjacent, thus no further expansion is possible. Therefore, $\{x_2, x_4, x_5\}$ is also a maximal clique. In conclusion, two maximal cliques are obtained: $\{x_2, x_4, x_3\}$ and $\{x_2, x_4, x_5\}$.

Starting from the initial clique $\{x_2\}$, we expand to $\{x_2, x_5\}$. The neighbors of vertex x_5 are x_1, x_4, leading to expansions $\{x_2, x_5, x_1\}$ and $\{x_2, x_5, x_4\}$. Considering $\{x_2, x_5, x_1\}$, since $x_2 - x_1$ are adjacent, $\{x_2, x_5, x_1\}$ forms a clique. The neighbors set of x_1 are x_2, x_5, and no further expansion is possible, indicating that $\{x_2, x_5, x_1\}$ is a maximal clique. Among them, $\{x_2, x_4, x_5\}$ has already been identified as a maximal clique and will not be listed again.

Starting again from the initial clique $\{x_2\}$, we expand to $\{x_2, x_1\}$. The neighbors of the vertex x_1 is x_5, and $\{x_2, x_1, x_5\}$ has already been discussed as a maximal clique. Starting from the initial clique

$\{x_2\}$, we expand to $\{x_2, x_3\}$. The neighbor of the vertex x_3 is x_4, leading to the clique $\{x_2, x_3, x_4\}$, which has already been discussed. By arranging the elements of the obtained maximal cliques in ascending order according to their indices, the final maximal cliques are: $\{x_1, x_2, x_5\}, \{x_2, x_3, x_4\}$, $\{x_2, x_4, x_5\}$.

The simplified visualization of the branching recursion tree is as follows:

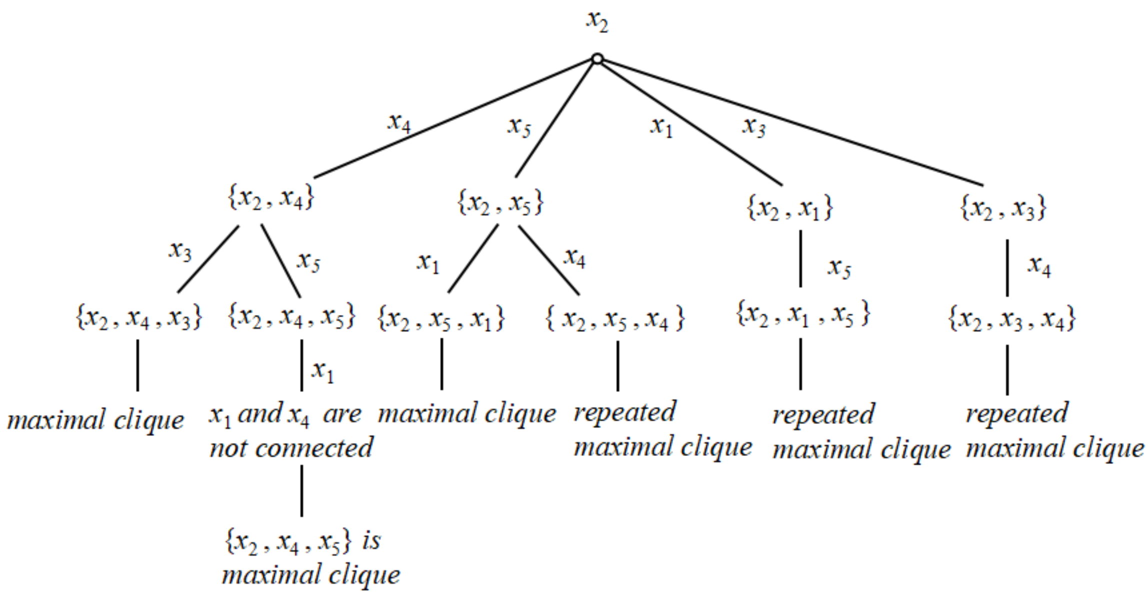

Figure 3: Simplified Branch Recursion Tree

The entanglement domain Q_U is obtained as: $Q_U = \{\{x_1, x_2, x_5\}, \{x_2, x_3, x_4\}, \{x_2, x_4, x_5\}\}$.

After obtaining the entanglement domain Q_U of the domain U, the dominance relation between entanglement domain subsets can be further determined by comparison.

Definition 8. *Given the entanglement domain* $Q_U = \{Q_1, Q_2 \ldots Q_i \ldots Q_k\}$, *let* $Q_i = \{x_{i1}, x_{i2} \ldots \ldots\}$, $Q_k = \{x_{k1}, x_{k2}, \ldots \ldots\}$, $i \neq k$, *define* $DU(Q_i) = [x_{i1}]^{\leq}_{A} \cup [x_{i2}]^{\leq}_{A} \ldots \ldots, DU(Q_k) = [x_{k1}]^{\leq}_{A} \cup [x_{k2}]^{\leq}_{A} \ldots \ldots$ *if* $DU(Q_i) \supset DU(Q_k)$, *then* Q_i *is said to be superior to* Q_k, *denoted as* $Q_i \succ Q_k$.

By applying Definition 9, the dominance comparison between entanglement domain subsets Q_i, Q_k can be determined, thereby establishing the external dominance ranking among the entanglement domain subsets.

Property 1. *For different subsets in the entanglement domain, the union of the superiority set (inferiority set) of the objects exhibits a necessary inclusion relationship.*

Proof. Let $Q_i = \{x_{i1}, x_{i2} \ldots \ldots\}, Q_k = \{x_{k1}, x_{k2} \ldots \ldots\}, i \neq k$, where the objects within each of Q_i and Q_k are pairwise distinguishable. Suppose that the union of the inferiority sets of the objects in different subsets of the entanglement domain does not exhibit a necessary inclusion relationship. Take $x_{i1} \in [x_{k1}]^{\leq}_{A}$ and $x_{k2} \in [x_{i2}]^{\leq}_{A}$ as an example. Since Q_i and Q_k are distinguishable, that is, x_{i1} is distinguishable from x_{k1}, x_{k2}, and x_{i2} is distinguishable from x_{k1}, x_{k2}, two situations arise: ① If $x_{i1} \in [x_{k2}]^{\leq}_{A}$ and $x_{k2} \in [x_{i2}]^{\leq}_{A}$, then $x_{i1} \in [x_{i2}]^{\leq}_{A}$, which contradicts the assumption. ① If $x_{k2} \in [x_{i1}]^{\leq}_{A}$ and $x_{i1} \in [x_{k1}]^{\leq}_{A}$, then $x_{k2} \in [x_{k1}]^{\leq}_{A}$, which also contradicts the assumption. Thus, if the assumption that $DU(Q_i), DU(Q_k)$ do not have a necessary inclusion relationship is invalid, it follows that either $DU(Q_i) \supset DU(Q_k)$ or $DU(Q_i) \subset DU(Q_k)$. This property clarifies that external ordering among the subsets of the entanglement domain. The next section will discuss the internal ordering objects within the subsets. □

Definition 9. *Given the entanglement domain* $Q_U = \{Q_1, Q_2 \ldots Q_i \ldots Q_k\}$ *over domain* U, *let* $Q_i = \{x_{i1}, x_{i2} \ldots \ldots\}$. *If there exists* $x_k \in U - Q_i$ *such that either* $x_{i1} \in [x_k]^{\leq}_{A}, x_{i2} \notin [x_k]^{\leq}_{A}$ *or* $x_{i1} \notin [x_k]^{\geq}_{A}, x_{i2} \in [x_k]^{\geq}_{A}$, *then* $x_{i2} \succ x_{i1}$.

Based on the above content, the steps for the multi-attribute decision-making method based on the entanglement dominance relation are summarized as follows:

(1) For $\forall x \in U$, calculate its superiority set $R_B^{\geqslant}(x)$ and inferiority set $R_B^{\leqslant}(x)$;

(2) According to Definition 4, solve for the uncertainty domain S_{x_i} of each object;

(3) According to Definition 6 and by using the improved Bron-Kerbosch algorithm, compute the entanglement domain Q_U ;

(4) Based on Step (1), apply Definition 8 to perform external ranking among the subsets of the entanglement domain;

(5) According to Definition 9, perform internal ranking within each subset of the entanglement domain;

(6) Integrate the results from Steps (4) and (5) to obtain the overall ranking of all objects in the domain.

4 Case Study

4.1 Problem Description

With the rapid development of supply chains, the issue of logistics supply chain performance evaluation has increasingly attracted the attention of managers, becoming a research hotspot in logistics management. Reference [22] argues that collaborative development capability is the most critical aspect of logistics services based on supply chains. Therefore, this study focuses on evaluating various refined indicators under the overarching metric of collaborative development capability.

A logistics center needs to select the optimal integrated service provider from six candidate logistics companies, denoted as $x_1, x_2 \ldots\ldots, x_6$. The evaluation is based on eight secondary indicators under the overarching metric of collaborative development capability, represented as $a_1, a_2 \ldots\ldots, a_8$, where: a_1: In-process product ratio (%); a_2: Average inspection-free rate of goods (%) ; a_3: Average safe inventory reduction rate of goods (%) ; a_4: Cooperation trust level; a_5: Information transmission accuracy rate (%); a_6: Staff quality (scored out of 10); a_7: Average service market share (%); a_8: Service innovation capability (%). The original evaluation data of the integrated logistics service providers under these detailed indicators are shown in the following table.

Table 2: Raw data of the coordinate development capability index

UA	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
x_1	25	87	11	1.62	1.2	9	4.3	7
x_2	15	78	6.5	1.1	1.4	5	3	4
x_3	19	92	9.6	1.7	1	7	3.5	5
x_4	20	80	10	1.6	1.2	8	3.6	3
x_5	12	75	7	1.5	1.3	6	2.3	1
x_6	23	85	10.3	1.54	1.16	8	4.1	9

Among the indicators, the information transmission error rate is a cost-type attribute. It is normalized according to the following formula, yielding a new set of standardized data as shown in Table 3. (Note that benefit-type indicators do not require normalization, and the evaluation results are not affected. The normalization here is performed solely for consistency with the cost-type attributes.) Specifically, $\max(c_{ij}), \min(c_{ij})$ denotes the maximum and minimum values under the j -th attribute, respectively.

$$\begin{cases} c'_{ij} = \frac{c_{ij} - \min(c_{ij})}{\max(c_{ij}) - \min(c_{ij})}, & \text{benefit} - \text{type} \\ c'_{ij} = \frac{\min(c_{ij}) - c_{ij}}{\max(c_{ij}) - \min(c_{ij})}, & \text{cost} - \text{type} \end{cases}$$

Table 3: Standardized data of the coordinate development capability index

UA	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
x_1	1.00	0.71	1.00	0.87	1.00	1.00	0.75	1.00
x_2	0.23	0.18	0.00	0.00	0.00	0.35	0.38	0.23
x_3	0.54	1.00	0.69	1.00	0.50	0.60	0.50	0.54
x_4	0.62	0.29	0.78	0.83	0.75	0.65	0.25	0.62
x_5	0.00	0.00	0.11	0.67	0.25	0.00	0.00	0.00
x_6	0.85	0.59	0.84	0.73	0.75	0.90	1.00	0.85

4.2 Case Decision Analysis

For each attribute, a pairwise dominance comparison between objects is conducted, resulting in the following table:

Table 4: Comparison of the advantages and disadvantages of the two samples

UA	x_1	x_2	x_3	x_4	x_5	x_6
x_1	====	>>>	><><	>>>	>>>	>>>
	====	>>>	<>>>	=> > >	>>>	<>><
x_2	<<<<	====	<<<<	<<<<	>>>	<<<<
	<<<<	====	<<<<	<<<>	<<>>>	<<<<
x_3	<><>	>>>	====	<><>	>>>	<><>
	><<<	>>>	====	><<>	>>>	><<<
χ_4	<<<<	>>>	><><	====	>>>	<<<>
	=< < <	>>>	<>><	====	>>>	< =< <
x_5	<<<<	<<>>	<<<<	<<<<	====	<<<<
	< < < <	<<>>	<<<<	<<<<	= = = =	< < < <
	< < < <	>><<	< < < <	< < < <	= = = =	< < < <
x_6	< < < <	>>>>	><><	>>><	>>>>	= = = =
	><<>	>>>>	<>>>	>=>>	>>>>	= = = =

Note: In Table 4, each cell contains eight comparison symbols, each representing the pairwise comparison results of two objects under the eight attributes. Specifically: " = " indicates $f(x_i, a) = f(x_j, a)$, "$>$" indicates $f(x_i, a) > f(x_j, a)$, "$<$" indicates $f(x_i, a) < f(x_j, a)$.

(1) Based on Table 4, the dominance and inferiority sets of each object can be calculated as follows:

$[x_1]_A^{\geq} = \{x_1\}$ $[x_1]_A^{\leq} = \{x_1, x_2, x_4, x_5\}$
$[x_2]_A^{\geq} = \{x_1, x_2, x_3, x_6\}$ $[x_2]_A^{\leq} = \{x_2\}$
$[x_3]_A^{2} = \{x_3\}$ $[x_3]_A^{4} = \{x_2, x_3, x_5\}$
$[x_4]_A^{\geq} = \{x_1, x_4\}$ $[x_4]_A^{\leq} = \{x_4, x_5\}$
$[x_6]_A^{\geq} = \{x_6\}$ $[x_6]_A^{\leq} = \{x_2, x_5, x_6\}$

(2) According to Definition 5, calculate the uncertainty domain of each object:

$S_{x_1} = \{x_3, x_6\}$ $S_{x_2} = \{x_4, x_5\}$
$S_{x_3} = \{x_1, x_4, x_6\}$ $S_{x_4} = \{x_2, x_3, x_6\}$
$S_{x_5} = \{x_2\}$ $S_{x_6} = \{x_1, x_3, x_4\}$

(3) According to Definition 7, the entangled domain of each object is further derived. Based on the uncertainty domains, the entanglement dominance relation is obtained as: $R_B^Q = \{\{x_1, x_3\}, \{x_1, x_6\}, \{x_3, x_6\}, \{x_3, x_4\}, \{x_2, x_4\}, \{x_2, x_5\}, \{x_4, x_6\}\}$, and the indistinguishability relations between objects can be represented by the undirected graph shown in Figure 4.

Based on the improved Bron-Kerbosch algorithm, the maximal cliques of the graph are computed, yielding the entangled domain: $Q_U = \{Q_1, Q_2, Q_3, Q_4\}$, where: $Q_1 = \{x_1, x_3, x_6\}$, $Q_2 = \{x_2, x_4\}$, $Q_3 = \{x_2, x_5\}$, $Q_4 = \{x_3, x_4, x_6\}$.

(4) The dominance relations among the entangled domain subsets are then compared to establish an external ranking:

$DU(Q_1) = [x_1]_A^{\leq} \cup [x_3]_A^{\leq} \cup [x_6]_A^{\leq} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$
$DU(Q_2) = [x_2]_A^{\leq} \cup [x_4]_A^{\leq} = \{x_2, x_4, x_5\}$
$DU(Q_3) = [x_2]_A^{\leq} \cup [x_5]_A^{\leq} = \{x_2, x_5\}$
$DU(Q_4) = [x_3]_A^{\leq} \cup [x_4]_A^{\leq} \cup [x_6]_A^{\leq} = \{x_2, x_3, x_4, x_5, x_6\}$

Clearly, the results above satisfy: $DU(Q_3) \subset DU(Q_2) \subset DU(Q_4) \subset DU(Q_1)$, which is consistent with Property 1. Therefore, the external ranking of the entangled domain subsets is: $Q_1 \succ Q_4 \succ Q_2 \succ Q_3$, i.e., $\{x_1, x_3, x_6\} \succ \{x_3, x_4, x_6\} \succ \{x_2, x_4\} \succ \{x_2, x_5\}$.

(5) Based on this, internal rankings within each subset are conducted as follows:

For $Q_3 = \{x_2, x_5\}$, using x_4 as a reference, since $x_5 \in [x_4]^{\leq}_{A}, x_2 \notin [x_4]^{\leq}_{A}$, it follows that $x_2 \succ x_5$.

For $Q_2 = \{x_2, x_4\}$, using x_3 as a reference, $x_2 \in [x_3]^{\leq}_{A}, x_4 \notin [x_3]^{\leq}_{A}$, hence $x_4 \succ x_2$.

For $Q_4 = \{x_3, x_4, x_6\}$, using x_1 as a reference, $x_4 \in [x_1]^{\leq}_{A}, x_6 \notin [x_1]^{\leq}_{A}$, thus $x_6 \succ x_4$.

For $Q_1 = \{x_1, x_3, x_6\}$, using x_4 as a reference, $x_1 \in [x_4]^{2}_{A}, x_3 \notin [x_4]^{2}_{A}$, so $x_1 \succ x_3$.

For the pair x_3, x_6, a direct comparison using this method is not feasible. However, referring to Table 3, it is observed that: x_3 outperforms x_6 in attributes a_2, a_4 and a_5; x_6 outperforms x_3 in the remaining five attributes. Additionally, the average value of x_3 across all attributes is 0.729, while that of x_6 is 0.801 . Therefore, we conclude that $x_6 \succ x_3$.Next, comparing x_3, x_4, x_3 outperforms x_4 in 26 attributes, while x_4 outperforms x_3 in 22 attributes. Therefore, $x_3 \succ x_4$. To sum up, the final ranking result is $x_1 \succ x_6 \succ x_3 \succ x_4 \succ x_2 \succ x_5$.

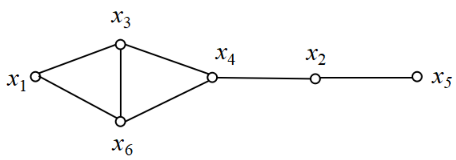

Figure 4: Undirected Graph 2

4.3 Comparison of Ranking Methods

To further evaluate the scientific validity and rationality of the proposed method, this study introduces the approaches from references [14, 23], and the classical TOPSIS method into the aforementioned case for comparison. All four models adopt equal weights for the attributes. According to the method in reference [14], the original case data is first converted into boolean values (in line with the threshold rule in the original paper: the median is selected, and values greater than or equal to the median are set to 1). Then, advantage and disadvantage neighborhoods are obtained, and local advantage-disadvantage degrees are computed for ranking the alternatives (the smaller the local disadvantage degree, the better the ranking). Reference [23] constructs an α -probabilistic dominance relation, where the parameter α is set to0.5,0.75, and 1 . The final comprehensive dominance degree is obtained by averaging the values corresponding to these parameters, which is then used to rank the alternatives. For the TOPSIS method, after normalizing the original data, the distance between each alternative and the ideal and negative-ideal solutions is calculated, and the alternatives are ranked based on their relative closeness. The evaluation results of the comparative methods are shown in Table 5.

Table 5: Comparison of Different Ranking Models

Model	Ranking Result	Ranking-Rationality	Method Used
Proposed Model	$x_1 \succ x_6 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	Reasonable	Entanglement Dominance Relation
Ranking Method Based on Probabilistic Dominance Relation [23]	$x_1 \succ x_6 \succ x_3 \succ x_4 \succ x_2 \succ x_5$	Reasonable	α-Probabilistic Dominance Relation
Ranking Method Based on Advantage (Disadvantage) Degree [14]	$x_1 \succ x_6 \succ x_4 \succ x_3 \succ x_2 \sim x_5$	Unreasonable	Boolean Data, Local Advantage- Disadvantage Degree
TOPSIS	$x_1 \succ x_6 \succ x_4 \succ x_3 \succ x_2 \succ x_5$	Unreasonable	Relative Closeness to Ideal Solution

As shown in Table 5, the ranking result of the proposed method is consistent with that of reference [23], but differs from those of references [14] and the TOPSIS method. The discrepancies are mainly concentrated in the comparison among x_2, x_5, x_3, x_4. The method in [14] relies on Boolean-transformed data, which imposes certain limitations: when converting other types of data into Boolean form, the precision is inevitably reduced.This paper argues that ranking $x_2 \succ x_5$ is more reasonable, for two main reasons. First, according to Table 3, in terms of attribute values, x_2, x_5 each dominate the other on half of the attributes, indicating they are approximately equivalent in performance - hence, they can be considered "indistinguishable." Second, by leveraging the disadvantage set of the reference object x_4, we find that $x_5 \in [x_4]_A^{\leq}, x_2 \notin [x_4]_A^{\leq}$, meaning x_5 belongs to the disadvantage set of x_4, while x_2 does not. Under the assumption of "no significant difference" between x_2, x_5, it is thus more reasonable to conclude $x_2 \succ x_5$. Regarding the comparison between x_3, x_4 dominates on 26 attributes while x_4 dominates on 22 attributes. Therefore, the conclusion $x_3 \succ x_4$ is considered more rational. Although the TOPSIS method yields slightly different results, this paper supports the judgment that $x_3 \succ x_4$ is more reasonable.

5 Conclusion

This paper proposes a multi-attribute decision-making method based on entangled dominance relation within rough set theory, and validates its effectiveness through both theoretical analysis and empirical study. By defining the entangled domain and its computational procedure, and constructing the corresponding dominance relation, the proposed method addresses the limitations of traditional rough set-based dominance models in handling partially ordered data. In particular, it significantly alleviates the decision-making constraints caused by undersized dominance sets, offering a novel perspective and solution to multi-attribute decision problems. While this method is primarily designed for information systems within the framework of rough sets, its applicability to other data structures: such as fuzzy or interval-valued data will be explored in future research.

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References

[1] Pawlak, Z.: Rough sets. International Journal of Computer & Information Sciences, 1982,11(5): 341-356.

[2] Huang, Z.H., Li, J.J., Qian, Y H.: Noise-tolerant Fuzzy-β-covering-based Multi-granulation Rough Sets and Feature Subset Selection. IEEE Transactions on Fuzzy Systems, 2022, 30(7): 2721-2735.

[3] Ding, W.P., Lin, C.T., Pedrycz, W.: Multiple Relevant Feature Ensemble Selection Based on Multi-layer Co-evolutionary Consensus Map Reduce. IEEE Transactions on Cybernetics, 2020, 50(2): 425-439.

[4] Zhan, J.M., Zhang, X.H., Yao, Y.Y.: Covering-based Multi-granulation Fuzzy Rough Sets and Corresponding Applications. Artificial Intelligence Review, 2020, 53(2): 1093-1126.

[5] Almeida, V. N. d., Alegre, L. N., Bazzan, A. L. C.: Knowledge Transfer in Multi-Objective Multi-Agent Reinforcement Learning via Generalized Policy Improvement. Computer Science and Information Systems, 2024, 21(1), 335-362.

[6] Liu, J.Z., Qu, Q.L., Yang, H.Y., Zhang, J.M., et al.: Deep Learning-based Intelligent Fault Diagnosis for Power Distribution Networks. International Journal of Computers Communications & Control, 2024, 19(4), 6607.

[7] He, Y., Song, J., Ouyang, W., Li, Q.: Formation Mechanism and Implementation Path of a Digital Agriculture Innovation Ecosystem. Tehnicki vjesnik-Technical Gazette, 2024,31 (2), 402-411.

[8] Greco, S., Matarazzo, B., Slowinski, R.: Rough Approximation of a Preference Relation by Dominance Relations. European Journal of Operational Research,1999,117(1): 63-83.

[9] Wang, W., Zhan, J., Mi, J.: A Three-way Decision Approach with Probabilistic Dominance Relations under Intuitionistic Fuzzy Information. Information Sciences, 2022,582, 114-145.

[10] Weng, S.Z., Zhu, J., Wang, C., et al.: Dominance Relation Rough Set Model in Interval Rough Number Ordered Information System. Fuzzy Systems and Mathematics, 2021, 35 (03): 133-144.

[11] Lin, W.H, Li, J.J, Wu, Y.N.: Multi-attribute Group Decision-making Method Based on Dominance Relation. Computer Engineering and Applications, 2018, 54(09): 237-242.

[12] Mondal, A., Roy, S.K., Pamucar, D.: Regret-based Three-way Decision Making with Possibility Dominance and SPA Theory in Incomplete Information System. Expert Systems with Applications, 2023,211.

[13] Wang, W.J., Zhan, J.M. Zhang, C.: Three-way Decisions Based Multi-attribute Decision Making with Probabilistic Dominance Relations. Information Sciences. 2021(559):75-96.

[14] Yu, B., Xu, R.H., Fu, Y., Xu, Z.S.: A Novel Dominance-Based Rough Set Model with Advantage (Disadvantage) Neighborhoods and Its Applications to Assess Sales Group. International Journal of Fuzzy Systems. 2022.8(24):3501-3512.

[15] Qin, H.W., Peng, Q.W., Ma, X.Q.: A Novel Interval-valued Fermatean Fuzzy Three-way Decision Making Method with Probability Dominance Relations. Expect Systems With Applications.2023,242.

[16] Keshavarz, G.M., Zavadskas, E.K., Amiri M., et al.: Extended EDAS Method for Fuzzy Multicriteria Decision-making: An Application to Supplier Selection. International Journal of Computers Communications & Control, 2016,11(3), 358-371.

[17] Gao L. L., Zha J., Feng Z. Y., Liu S. F., et al.: Flexible Job Shop Rescheduling Scheme Selection Using Improved TOPSIS. Int. Journal of Simulation Modelling, 2024, 23(3), 507-518.

[18] Wu, J.M., Huang, Z.H., Li, J. J, et al.: σ dominance relation entropy and its application in multi-attribute decision-making . Control and Decision, 2024, 39(02): 613-624.

[19] Peng, Q.W., Ma, X., Q, H.W.: A novel Interval-valued Fermatean Fuzzy Three-way Decision Making Method with Probability Dominance Relations. Expert Systems with Applications. 2023,242.

[20] Huang, Z.W., Weng, S.Z., Lv, Y.J., Liu, H.Y.: Ranking Method of Intuitionistic Fuzzy Numbers and Multiple Attribute Decision Making Based on the Probabilistic Dominance Relationship. SYMMETRY-BASEL. 2023.15(5).

[21] Xue, Z.A., Zhang, M., Zhao, L.P, et al.: Variable three-way decision model of multi-granularity decision rough set under set pair dominance relation. Computer Science, 2021, 48(01): 157-166.

[22] Weng, S.Z., Lv, Y.J., Cao, Z.Q. Multi-attribute decision-making method based on distance measure of interval rough fuzzy numbers. Fuzzy Systems and Mathematics, 2022, 36(03): 131-144.

[23] Weng, S.Z., Lv, Y.J.: Ranking method based on probability dominance relation and its application. Journal of Shanxi University. 2015, 38(03): 439-446.

[24] Liu, H.M., Weng, S.Z.: A Multi-attribute Decision-making Method for Interval Rough Number Considering Distribution Types. International Journal of Computers Communications & Control, 2024,19(4), 6633.

[25] Liu, L.K., Li, J.L., Liu, J.L.: Variable Precision Dominance-based Rough Set Model Based on Entangled Dominance Relation. Computer Applications and Software, 2023, 40(02): 281-286+313.

[26] Greco, S., Matarazzo, B., Słowiński, R.: Dominance-based rough set approach as a proper way of handling graduality in rough set theory. Transactions on rough sets VII. Berlin: Springer, 2007: 36-52.

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