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A Multi-attribute Decision-making Method for Interval Rough Number Considering Distribution Types

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Abstract

This paper proposes a novel multi-attribute decision-making (MADM) method for interval rough numbers (IRNs) considering different distribution types, namely uniform, exponential, and normal distributions. Upper and lower approximate interval dominance degrees are defined and aggregated using dynamic weights to obtain pairwise comparisons of IRNs. The properties of dominance are verified, and an attribute weight determination method based on the dominance balance degree is introduced. The proposed MADM method is data-driven and does not rely on the subjective preferences of decision-makers. Case analysis demonstrates the effectiveness and rationality of the proposed method, revealing that the distribution type of IRNs significantly impacts decision results, potentially leading to reversed ranking outcomes. The proposed method offers a comprehensive framework for handling MADM problems with IRNs under different distributions.

Keywords: interval rough numbers; dominance degree; uniform distribution; exponential distribution; normal distribution; dynamic weights.

1 Introduction

Uncertain decision theory is widely used in the fields of attribute reduction, artificial intelligence, data mining, and other fields [1, 2, 3, 4, 5]. Compared with the other theories mentioned above, fuzzy set theory and rough set theory [6, 7, 8, 9, 10] have the most interest. The interval rough number (IRN) is derived from the concept of random variable proposed by Professor B. Liu [11] combined with rough set theory and uncertainty variable in 2002. The IRN is a form of uncertainty data, which consists of a lower approximation interval and an upper approximation interval. Compared with uncertain data forms such as intervals, set values, and fuzzy numbers, interval rough numbers (IRNs) can reflect certain certainty in uncertainty when characterizing the uncertainty of data. It is more appropriate when dealing with certain real-world decision-making issues.

At present, the related research on IRN mainly focuses on two aspects: one is the comparison and ranking methods, and the other is the types of distribution. A series of studies have been carried out under the framework of multi-attribute decision-making theory based on these.

The research on the comparison and ranking methods of IRN has achieved rich results. For example, The literature [12] proposes a rough set model based on the λ-similarity relationship. The literature [13] proposes a possibility calculation based on the area and a ranking method of expectation and variance. It tries to provide a rough set model based on dominance relationships according to dominance thresholds [14]. The literature [15] uses the weighted average operator of Bonferroni and establishes a DEMATEL-VIKOR multi-attribute optimization decision model of the IRN. The literature [16] gives a parametric distancing degree in the form of an integral by defining the expectation and variance of the IRN with parameters. The literature [17] proposed the lower and upper approximation of the set dispersion and the minimum set dispersion of ξ the threshold and established a rough set model covering the minimum set based on the minimum set dispersion. The literature [18] proposes an IRN and establishes an IRN rough set model based on the equivalent class. The literature [19] proposes the concept of covering redundancy and β-equivalence classes, and on this basis, they proposed a property reduction method to keep β-equivalence classes unchanged. The literature [20] constructs a distance measurement index and compares the interval rough fuzzy numbers by using the ideal point method, and applies it to multi-attribute decision-making. The above literature is based on the comparison of IRN, on this basis, the multi-attribute decision problem is discussed. The above results discussed the comparison and sorting of rough numbers from expectation-square difference, equivalent relationship, and dominant relationship. None of the above discussions quantify comparison of IRNs.

In the study of the data distribution type of IRN, although the distribution type of classical IRN is not clearly defined by the distribution function, it can be defaulted to uniform distribution according to its calculation formula. Based on this, the problem of rough number ranking in the two stages of evenly distributed in two stages is discussed [21]. While the IRN obeys the normal distribution, a multi-attribute decision-making method based on the connection number is proposed [22]. The probability density function of IRNS obeying normal distribution is proposed to calculate the expectation, and a decision model of IRN based on the k-dominance relationship is established [23]. The ranking method of IRN under the two-stage uniform distribution, normal distribution, exponential distribution, and binomial distribution is discussed separately through MATLAB data simulation, but the conclusions are only drawn through experimental simulation without rigorous mathematical reasoning [24]. In terms of distribution type, the attribute data in the model obey the single distribution type and does not fully discuss the discussion according to the characteristics of the data.

To solve the above problems, This paper mainly discusses the MADM method for IRNs under multiple distribution types. The main contributions are : (1) extending the IRN theory by considering multiple distribution types and defining new dominance degrees; (2) developing a comprehensive MADM method for IRNs that integrates dynamic weights and dominance balance degree; (3) investigating the impact of distribution types on decision results and highlighting the importance of considering this factor in IRN-based decision-making.

The rest of this paper is organized as follows. In Section 2, the basic concepts of the IRN are briefly reviewed. In Section 3, dominance degrees are defined under the uniform distribution, exponential distribution, and normal distribution separately. In Section 4, steps of a multi-attribute decision-making method of IRN. In Section 5, an example is used to compare the method in this paper with other methods. Section 6 is a discussion, corresponding instructions to this method. Finally, the whole paper is concluded by the outlook for further research in Section 7.

2 Basic concept of IRN

Some basic concepts and properties of IRN are briefly introduced in this section.

Definition 1. *[25] Where U is a non-empty set called the universe, the lower approximation and the upper approximation are defined separately:*

$$\underline{X} = \left\{ x \in U | R^{-1}(x) \subseteq X \right\}, \bar{X} = \underset{x \in X}{\cup} R(x),$$

Where $R(x) = \{y \in U | y \cong x\}, R^{-1}(x) = \{y \in U | x \cong y\}$, $\cong$ express a reflexivity.

Definition 2. *[26] The whole of all sets with the same lower approximation and upper approximations is called a rough set, recorded as $(\underline{X}, \bar{X})$.*

Definition 3. *[26] An IRN is a rough set in which both the lower approximation and upper approximations are intervals, and are recorded as $([a, b], [c, d])$, where $c < a < b < d$.*

For example, the amount of investment in a project can be represented by the IRN as ([4,6],[3,7]). From the perspective of the investor, it can be understood that if the investment amount of the project is between 40,000 ~ 60,000 dollars, it is acceptable, and if the investment amount is between 30,000 ~ 70,000 dollars, it is possible to accept. That is, it is expressed in the semantics of "certain" and "probable", which is consistent with the lower and upper approximations of the rough set.

Definition 4. *Let two IRNs $\xi_1 = ([a_1, b_1], [c_1, d_1])$ and $\xi_2 = ([a_2, b_2], [c_2, d_2])$, there are:*
(1) $\xi_1 + \xi_2 = ([a_1 + a_2, b_1 + b_2], [c_1 + c_2, d_1 + d_2])$ [26];
(2) $\xi_1 - \xi_2 = ([a_1 - b_2, b_1 - a_2], [c_1 - d_2, d_1 - c_2])$;
(3) $\beta\xi = \begin{cases} ([\beta a, \beta b], [\beta c, \beta d]) & \beta \geq 0 \\ ([\beta b, \beta a], [\beta d, \beta c]) & \beta < 0 \end{cases}$ [26];
(4) $\xi^\beta = ([a^\beta, b^\beta], [c^\beta, d^\beta])$ $(\beta > 0)$ [26].

3 Different distribution types of IRN

At present, the distribution types of the two intervals in the IRN are considered to be relatively simple, and most of the cases are assumed to obey the uniform distribution. However, different IRNs may correspond to different distribution types in practical problems. For example, in the queuing system, the customer arrival rate generally obeys the exponential distribution or Poisson distribution, the passenger waiting time obeys the uniform distribution, and the test score generally obeys the normal distribution. Thus the calculation and comparison methods of the IRNs in different distribution should be different When comparing with different IRNs.

In this section, we first give the probability density function of the IRN under uniform distribution, exponential distribution, and normal distribution, and then introduce the concept of dominance degree. Based on them, we further present the detailed procedure of comparison of IRNs.

3.1 Uniform distribution

If the random variable X follows a uniform distribution over the interval $[a, b]$, then its probability density function is

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & else \end{cases}$$

Moreover, let $\xi = ([a, b], [c, d])$ follows a uniform distribution, then the probability density function of the upper approximate interval on it is:

$$f(x) = \begin{cases} \frac{1}{d-c}, & c < x < d \\ 0, & else \end{cases}$$

From the definition of the IRN, the probability density of the lower approximation interval should be greater than that of the upper approximation interval. It is clear that the proportion of the lower approximate interval to the upper approximate interval is $(b-a)/(d-c)$, so the probability density function of the upper approximation interval is:

$$f(x)=\begin{cases}\frac{1}{d-c}/\frac{b-a}{d-c}, & a<x<b\\ 0, & else\end{cases}=\begin{cases}\frac{1}{b-a}, & a<x<b\\ 0, & else\end{cases}$$

Definition 5. *Let $\xi_1=([a_1,b_1],[c_1,d_1])$ and $\xi_2=([a_2,b_2],[c_2,d_2])$ obey the uniform distribution. The dominance degree of ξ_1 over ξ_2 in the lower approximate interval can be denoted as:*

$$\underline{p}\{\xi_1\geq\xi_2\}=\iint_{\underline{D}} f(\underline{\xi_1})f(\underline{\xi_2})dxdy=\iint_{\underline{D}}\frac{1}{b_1-a_1}\frac{1}{b_2-a_2}dxdy \tag{1}$$

Similarly, the dominance degree of ξ_1 over ξ_2 in the upper approximate interval can be defined as:

$$\overline{p}\{\xi_1\geq\xi_2\}=\iint_{\overline{D}} f(\overline{\xi_1})f(\overline{\xi_2})dxdy=\iint_{\overline{D}}\frac{1}{d_1-c_1}\frac{1}{d_2-c_2}dxdy \tag{2}$$

Where $f(\overline{\xi})=\frac{1}{d-c}$, $f(\underline{\xi})=\frac{1}{b-a}$ represent the probability density of lower and upper approximately interval. $\underline{D},\overline{D}$ indicate that the area ξ_1 is greater than ξ_2 on the lower and upper approximately interval.

There are 6 cases of the positional relationship between the lower approximate interval of the two IRNs that obey uniform distribution, as illustrated in Fig. 1:

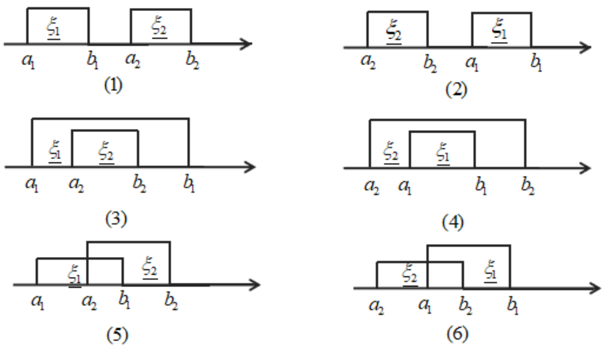

Figure 1: Lower approximate interval position graph

Similarly, the position relationship between the upper approximate interval of the two IRNs is also divided into 6 cases.

Then the specific calculation formula and simplified form of the lower approximate dominance degree $\underline{p}\{\xi_1\geq\xi_2\}$ is defined as

$$\underline{p}\{\xi_1\geq\xi_2\}=\begin{cases}0 & b_1<a_2\\ 1 & b_2<a_1\\ \int_{a_2}^{b_2} f(\xi_1)\,\mathrm{d}y\int_{y}^{b_1} f(\xi_2)\,\mathrm{d}x & a_1<a_2<b_2<b_1\\ \int_{a_1}^{b_1} f(\xi_1)\,\mathrm{d}x\int_{a_2}^{x} f(\xi_2)\,\mathrm{d}y & a_2<a_1<b_1<b_2\\ \int_{a_2}^{b_1} f(\xi_1)\,\mathrm{d}x\int_{a_2}^{x} f(\xi_2)\,\mathrm{d}y & a_1<a_2<b_1<b_2\\ 1-\int_{a_1}^{b_2} f(\xi_1)\,\mathrm{d}x\int_{x}^{b_2} f(\xi_2)\,\mathrm{d}y & a_2<a_1<b_2<b_1\end{cases}=\begin{cases}0 & b_1<a_2\\ 1 & b_2<a_1\\ \frac{(2b_1-a_2-b_2)}{2(b_1-a_1)} & a_1<a_2<b_2<b_1\\ \frac{(a_1+b_1-2a_2)}{2(b_2-a_2)} & a_2<a_1<b_1<b_2\\ \frac{(b_1-a_2)^2}{2(b_1-a_1)(b_2-a_2)} & a_1<a_2<b_1<b_2\\ 1-\frac{(b_2-a_1)^2}{2(b_1-a_1)(b_2-a_2)} & a_2<a_1<b_2<b_1\end{cases} \tag{3}$$

In the same way, the specific calculation formula and simplified form of the upper approximate dominance degree $\overline{p}\{\xi_1\geq\xi_2\}$ can be obtained as (4):

$$\overline{p}\{\xi_1\geq\xi_2\}=\begin{cases}0 & d_1<c_2\\ 1 & d_2<c_1\\ \int_{c_2}^{d_2} f(\xi_1)\,\mathrm{d}y\int_{y}^{d_1} f(\xi_2)\,\mathrm{d}x & c_1<c_2<d_2<d_1\\ \int_{c_1}^{d_1} f(\xi_1)\,\mathrm{d}x\int_{c_2}^{x} f(\xi_2)\,\mathrm{d}y & c_2<c_1<d_1<d_2\\ \int_{c_2}^{d_1} f(\xi_1)\,\mathrm{d}x\int_{c_2}^{x} f(\xi_2)\,\mathrm{d}y & c_1<c_2<d_1<d_2\\ 1-\int_{c_1}^{d_2} f(\xi_1)\,\mathrm{d}x\int_{x}^{d_2} f(\xi_2)\,\mathrm{d}y & c_2<c_1<d_2<d_1\end{cases}=\begin{cases}0 & d_1<c_2\\ 1 & d_2<c_1\\ \frac{(2d_1-c_2-d_2)}{2(d_1-c_1)} & c_1<c_2<d_2<d_1\\ \frac{(c_1+d_1-2c_2)}{2(d_2-c_2)} & c_2<c_1<d_1<d_2\\ \frac{(d_1-c_2)^2}{2(d_1-c_1)(d_2-c_2)} & c_1<c_2<d_1<d_2\\ 1-\frac{(d_2-c_1)^2}{2(d_1-c_1)(d_2-c_2)} & c_2<c_1<d_2<d_1\end{cases} \tag{4}$$

To further understand the dominance degree calculation formula, we take the case where the IRNs obey the uniform distribution in the case of $a_1 < a_2 < b_2 < b_1$ about the lower approximate interval as an example. According to the dominance degree formula: $\underline{p}\{\xi_1 \geq \xi_2\} = \int_{a_2}^{b_1} \frac{1}{b_1-a_1} \mathrm{d}x \int_{a_2}^{x} \frac{1}{b_2-a_2} \mathrm{d}y = \frac{\sigma}{(b_1-a_1)(b_2-a_2)}$ (σrepresents area). From the knowledge of geometry, it can be seen that the area below $y = x$ represents $x > y$, so σ is the area of the area under $y = x$, as shown in Fig. 2: then there is

$$\underline{p}\{\xi_1 \geq \xi_2\} = \int_{a_2}^{b_1} \frac{1}{b_1-a_1} \mathrm{d}x \int_{a_2}^{x} \frac{1}{b_2-a_2} \mathrm{d}y = \frac{\triangle\, EFB}{\square ABCD}$$

.

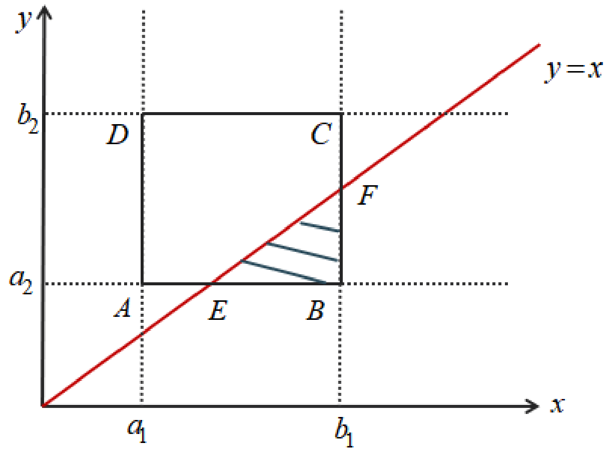

Figure 2: Area notation of upper approximate dominance degree

Therefore, $\underline{p}\{\xi_1 \geq \xi_2\}$ of uniform distribution can be intuitively understood as the proportion of the area that ξ_1 is better than ξ_2 in the total area.

The Calculations of dominance degree for other distributions can be understood similarly.

Definition 6. *Let $\xi_1 = ([a_1, b_1], [c_1, d_1])$ and $\xi_2 = ([a_2, b_2], [c_2, d_2])$ obey the uniform distribution, the dominance degree of ξ_1 over ξ_2 can be recorded as $p\{\xi_1 \geq \xi_2\}$:*

$$p\{\xi_1 \geq \xi_2\} = \alpha \underline{p}\{\xi_1 \geq \xi_2\} + (1-\alpha)\, \overline{p}\{\xi_1 \geq \xi_2\} \tag{5}$$

Where, $\alpha \in (0, 1)$ is a dynamic weight which means that the proportion of dominance degree in the lower approximate interval, and depends on the proportion of the sum of the lower approximate interval lengths to the sum of the upper approximate interval lengths between the two IRNs, then:

$$\alpha = \frac{(b_1 - a_1 + b_2 - a_2)/(d_1 - c_1 + d_2 - c_2)}{1 + (b_1 - a_1 + b_2 - a_2)/(d_1 - c_1 + d_2 - c_2)} \tag{6}$$

The dynamic weights will be different if the two IRNs are different. Compared with the subjective determination of the parameters in Ref. [7, 9, 13], the advantage of the dynamic weight in this paper is that it does not depend on the subjective preference of the decision-maker, and is objectively calculated from the upper and down approximate interval widths of the IRNs.

If $p\{\xi_1 \geq \xi_2\} > 0.5$, there is $\xi_1 \succ \xi_2$; if $p\{\xi_1 \geq \xi_2\} < 0.5$, then $\xi_2 \succ \xi_1$; if $p\{\xi_1 \geq \xi_2\} = 0.5$, there is $\xi_2 \sim \xi_1$ (where "$\succ$" means "superior", "$\sim$" means "no difference").

3.2 Exponential distribution

If a random variable obeys the exponential distribution, the probability density function is

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & else \end{cases}$$

.

The probability density function of $\xi = ([a, b], [c, d])$ regarded as a standard exponential distributed in the horizontal direction to the right level c-unit,

$$f_\xi(x) = \begin{cases} \lambda e^{-\lambda(x-c)}, & c < x < d \\ 0, & else \end{cases}$$

.

It can be presented in the form of Fig. 3:

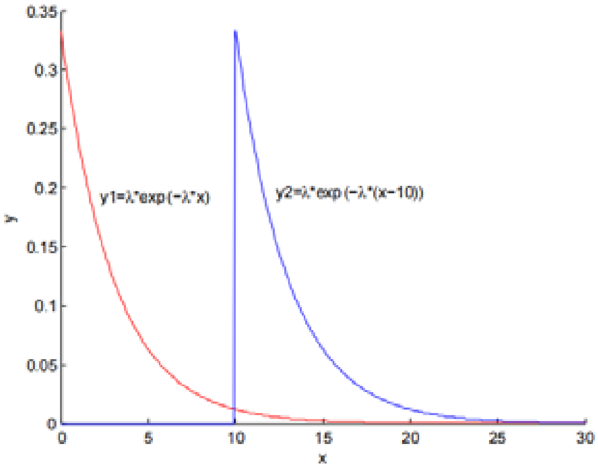

Figure 3: Standard exponential distribution and interval rough digital obedience to index distribution(c=10)

The IRN will only be taken on the interval $[c, +\infty]$, and only in a neighborhood to the right of the c value, which meets the basic requirement that the IRN value is greater than c .

To further determine the values of the parameter λ according to the interval width $[c, d]$, the confidence level $1 - \alpha$ is used to ensure that the IRN $\xi = ([a, b], [c, d])$ has a high probability on $[c, d]$, that is:

$$P\{c < X < d\} = \int_c^d \lambda e^{-\lambda(x-c)} \mathrm{d}x = e^{\lambda c}\left(-e^{-\lambda x}\right)|_c^d = 1 - \alpha$$

Then $\lambda = \ln(1/\alpha)/(d - c)$. To simplify the calculation, the general statistical law is followed, take $\alpha = 0.05$,

$$\lambda = \frac{\ln 20}{d - c} \tag{7}$$

Definition 7. *Let $\xi_1 = ([a_1, b_1], [c_1, d_1])$ and $\xi_2 = ([a_2, b_2], [c_2, d_2])$ obey the exponential distribution. The dominance degree of ξ_1 over ξ_2 in the lower approximate interval can be denoted as:*

$$\underline{p}\{\xi_1 \geq \xi_2\} = \frac{\iint\limits_{\underline{D}} f_{\xi_1}(x) f_{\xi_2}(y)\, dx\, dy}{\int_{a_1}^{b_1} f_{\xi_1}(x)\, dx \int_{a_2}^{b_2} f_{\xi_2}(y)\, dy} \tag{8}$$

The denominator term in the above equation is meant to normalize the dominance degree over the entire interval to 1. The principle of similar treatment is the same, and the explanation will not be repeated.

Similarly, the dominance degree of ξ_1 over ξ_2 in the upper approximate interval can be denoted as:

$$\overline{p}\{\xi_1 \geq \xi_2\} = \frac{\iint\limits_{\overline{D}} f_{\xi_1}(x) f_{\xi_2}(y) \mathrm{d}x \mathrm{d}y}{\int_{c_1}^{d_1} f_{\xi_1}(x) \mathrm{d}x \int_{c_2}^{d_2} f_{\xi_2}(y) \mathrm{d}y} \tag{9}$$

Where, the probability density function of ξ_1,ξ_2 are $f_{\xi_1}(x) = \begin{cases} \lambda_1 e^{-\lambda_1(x-c_1)}, & c_1 < x < d_1 \\ 0, & else \end{cases}$, . $f_{\xi_2}(y) = \begin{cases} \lambda_2 e^{-\lambda_2(y-c_2)}, & c_2 < y < d_2 \\ 0, & else \end{cases}$ respectively, $\underline{D}$, $\overline{D}$ denotes the planar region where ξ_1 is greater than ξ_2 on the upper and lower approximate intervals. Then the specific calculation formula of lower approximate dominance degree $\underline{p}\{\xi_1 \geq \xi_2\}$ and upper approximate dominance degree $\overline{p}\{\xi_1 \geq \xi_2\}$ are defined, as follows:

$$\underline{p}\{\xi_1 \geq \xi_2\} = \begin{cases} 0, & b_1 < a_2 \\ 1, & b_2 < a_1 \\ (\int_{a_2}^{b_2} f_{\xi_2}(y)\mathrm{d}y \int_{y}^{b_1} f_{\xi_1}(x)\mathrm{d}x)/(\int_{a_1}^{b_1} f_{\xi_1}(x)\mathrm{d}x \int_{a_2}^{b_2} f_{\xi_2}(y)\mathrm{d}y), & a_1 < a_2 < b_2 < b_1 \\ (\int_{a_1}^{b_1} f_{\xi_1}(x)\mathrm{d}x \int_{a_2}^{x} f_{\xi_2}(y)\mathrm{d}y)/(\int_{a_1}^{b_1} f_{\xi_1}(x)\mathrm{d}x \int_{a_2}^{b_2} f_{\xi_2}(y)\mathrm{d}y), & a_2 < a_1 < b_1 < b_2 \\ (\int_{a_2}^{b_1} f_{\xi_1}(x)\mathrm{d}x \int_{a_2}^{x} f_{\xi_2}(y)\mathrm{d}y)/(\int_{a_1}^{b_1} f_{\xi_1}(x)\mathrm{d}x \int_{a_2}^{b_2} f_{\xi_2}(y)\mathrm{d}y), & a_1 < a_2 < b_1 < b_2 \\ 1 - (\int_{a_1}^{b_2} f_{\xi_1}(x)\mathrm{d}x \int_{x}^{b_2} f_{\xi_2}(y)\mathrm{d}y)/(\int_{a_1}^{b_1} f_{\xi_1}(x)\mathrm{d}x \int_{a_2}^{b_2} f_{\xi_2}(y)\mathrm{d}y), & a_2 < a_1 < b_2 < b_1 \end{cases} \tag{10}$$

$$\overline{p}\{\xi_1 \geq \xi_2\} = \begin{cases} 0, & d_1 < c_2 \\ 1, & d_2 < c_1 \\ (\int_{c_2}^{d_2} f_{\xi_2}(y)\mathrm{d}y \int_{y}^{d_1} f_{\xi_1}(x)\mathrm{d}x)/(\int_{c_1}^{d_1} f_{\xi_1}(x)\mathrm{d}x \int_{c_2}^{d_2} f_{\xi_2}(y)\mathrm{d}y), & c_1 < c_2 < d_2 < d_1 \\ (\int_{c_1}^{d_1} f_{\xi_1}(x)\mathrm{d}x \int_{c_2}^{x} f_{\xi_2}(y)\mathrm{d}y)/(\int_{c_1}^{d_1} f_{\xi_1}(x)\mathrm{d}x \int_{c_2}^{d_2} f_{\xi_2}(y)\mathrm{d}y), & c_2 < c_1 < d_1 < d_2 \\ (\int_{c_2}^{d_1} f_{\xi_1}(x)\mathrm{d}x \int_{c_2}^{x} f_{\xi_2}(y)\mathrm{d}y)/(\int_{c_1}^{d_1} f_{\xi_1}(x)\mathrm{d}x \int_{c_2}^{d_2} f_{\xi_2}(y)\mathrm{d}y), & c_1 < c_2 < d_1 < d_2 \\ 1 - (\int_{c_1}^{d_2} f_{\xi_1}(x)\mathrm{d}x \int_{x}^{d_2} f_{\xi_2}(y)\mathrm{d}y)/(\int_{c_1}^{d_1} f_{\xi_1}(x)\mathrm{d}x \int_{c_2}^{d_2} f_{\xi_2}(y)\mathrm{d}y), & c_2 < c_1 < d_2 < d_1 \end{cases} \tag{11}$$

The dominance degree $p\{\xi_1 \geq \xi_2\}$ of the IRNs ξ_1 relative to ξ_2 is the same as that of equation (5).

3.3 Normal distribution

If the value of the IRN $\xi = ([a, b], [c, d])$ shows the characteristics of "high in the middle and low at both ends", it can be assumed that approximately obeys the normal distribution $N(\mu, \sigma^2)$. In the case of normal distribution, compared with the literature [23], the method of dominance degree used in this section is easier to calculate and avoids the complex operation of the probability density function of the two-dimensional normal distribution.

In the literature [22], adhering to criterion 3a of normal distribution, the expectations and variance of the IRN are proposed as:

$$\begin{cases} \mu = E(\xi) = \dfrac{a+b}{2} \\ \sigma = \sqrt{D(\xi)} = \dfrac{d-c}{6} \end{cases} \tag{12}$$

When comparing the IRNs ξ_1 and ξ_2 , if $a_1 + b_1 = a_2 + b_2$,$d_1 - c_1 = d_2 - c_2$, there is no way to distinguish between ξ_1 and ξ_2 using formula (14).

To avoid the above situation, and consider the size of the upper and lower approximate interval, the 3σ criterion is also observed, and the improved expectations and variance of IRN are as follows:

$$\begin{cases} \mu = E(\xi) = \dfrac{a+b+c+d}{4} \\ \sigma = \sqrt{D(\xi)} = \dfrac{d-c}{6} \end{cases} \tag{13}$$

Let the IRNs ξ_1 and ξ_2 obey the normal distribution, there is $Z = \xi_1 - \xi_1 \sim N(\mu_1 - \mu_2, {\sigma_1}^2 + {\sigma_2}^2)$. Then according to the definition of the subtraction of the IRN, $\xi_1 - \xi_2 = ([a_1 - b_2, b_1 - a_2], [c_1 - d_2, d_1 - c_2])$, that is, $[a_1 - b_2, b_1 - a_2]$,$[c_1 - d_2, d_1 - c_2]$ are the difference interval between ξ_1 and ξ_2 in the lower approximate interval and upper approximate interval.

Definition 8. *Let $\xi_1 = ([a_1, b_1], [c_1, d_1])$ and $\xi_2 = ([a_2, b_2], [c_2, d_2])$ obey the normal distribution. The dominance degree of ξ_1 over ξ_2 in the lower approximate interval can be denoted as:*

$$\underline{p}\{\xi_1 \geq \xi_2\} = \begin{cases} 1, & a_1 - b_2 \geq 0 \\ \frac{p(0 < Z < b_1 - a_2)}{p(a_1 - b_2 < Z < b_1 - a_2)}, & a_1 - b_2 < 0 \\ 0, & b_1 - a_2 \leq 0 \end{cases} \tag{14}$$

Where the distribution of $Z = \xi_1 - \xi_2$,$p\left(0 < Z < b_1 - a_2\right)$ denotes the probability of occurrence of $Z > 0$ distributed over the difference interval of lower approximate; then $p\left(a_1 - b_2 < Z < b_1 - a_2\right)$ denotes the probability of occurrence in the whole difference interval of lower approximate.

With the same method, the dominance degree of ξ_1 over ξ_2 in the upper approximate interval is obtained:

$$\overline{p}\left\{\xi_1 \geq \xi_2\right\} = \begin{cases} 1, & c_1 - d_2 \geq 0 \\ \frac{p(0<Z<d_1-c_2)}{p(c_1-d_2<Z<d_1-c_2)}, & c_1 - d_2 < 0 \\ 0, & d_1 - c_2 \leq 0 \end{cases} \tag{15}$$

Where the distribution of $Z = \xi_1 - \xi_2$, $p\left(0 < Z < b_1 - a_2\right)$ denotes the probability of occurrence of $Z > 0$distributed over the difference interval of upper approximate; then $p\left(a_1 - b_2 < Z < b_1 - a_2\right)$ denotes the probability of occurrence in the whole difference interval of upper approximate.

The specific calculation formulas of equations (14) and (15) are as follows:

$$\underline{p}\left\{\xi_1 \geq \xi_2\right\} = \begin{cases} 1, & a_1 - b_2 \geq 0 \\ \dfrac{\Phi\left(\frac{b_1-a_2-(\mu_1-\mu_2)}{\sqrt{\sigma_1{}^2+\sigma_2{}^2}}\right)-\Phi\left(\frac{0-(\mu_1-\mu_2)}{\sqrt{\sigma_1{}^2+\sigma_2{}^2}}\right)}{\Phi\left(\frac{b_1-a_2-(\mu_1-\mu_2)}{\sqrt{\sigma_1{}^2+\sigma_2{}^2}}\right)-\Phi\left(\frac{a_1-b_2-(\mu_1-\mu_2)}{\sqrt{\sigma_1{}^2+\sigma_2{}^2}}\right)}, & a_1 - b_2 < 0 \\ 0, & b_1 - a_2 \leq 0 \end{cases} \tag{16}$$

$$\overline{p}\left\{\xi_1 \geq \xi_2\right\} = \begin{cases} 1, & c_1 - d_2 \geq 0 \\ \dfrac{\Phi\left(\frac{d_1-c_2-(\mu_1-\mu_2)}{\sqrt{\sigma_1{}^2+\sigma_2{}^2}}\right)-\Phi\left(\frac{0-(\mu_1-\mu_2)}{\sqrt{\sigma_1{}^2+\sigma_2{}^2}}\right)}{\Phi\left(\frac{d_1-c_2-(\mu_1-\mu_2)}{\sqrt{\sigma_1{}^2+\sigma_2{}^2}}\right)-\Phi\left(\frac{c_1-d_2-(\mu_1-\mu_2)}{\sqrt{\sigma_1{}^2+\sigma_2{}^2}}\right)}, & c_1 - d_2 < 0 \\ 0, & d_1 - c_2 \leq 0 \end{cases} \tag{17}$$

Based on them, the table can be looked up to obtain the specific values of the dominance degree of ξ_1 over ξ_2 in the lower and upper approximate intervals. The dominance degree $p\left\{\xi_1 \geq \xi_2\right\}$ of the IRNs ξ_1 and ξ_2 can also be obtained by equation (5).

3.4 Properties of dominance degree of the IRNs

Property 1. *Let the IRNs* $\xi_1 = ([a_1, b_1], [c_1, d_1])$ *and* $\xi_2 = ([a_2, b_2], [c_2, d_2])$, *no matter whether they obey any one of the uniform distribution, exponential distribution, or normal distribution, their dominance degree satisfy the following properties:*

(1.1) $0 \leq p\left\{\xi_1 \geq \xi_2\right\} \leq 1$.

(1.2) $p\left\{\xi_1 \geq \xi_2\right\} = 1$, *if and only if* $d_2 \leq c_1$.

(1.3) $p\left\{\xi_1 \geq \xi_2\right\} = 0$, *if and only if* $d_1 \leq c_2$.

(1.4) $p\left\{\xi_1 \geq \xi_2\right\} + p\left\{\xi_2 \geq \xi_1\right\} = 1$.

(1.5) $p\left\{\xi_1 \geq \xi_1\right\} = 0.5$.

Proof. The following is an example of the IRN obeying a uniform distribution, when the IRN obeys the exponential distribution and the normal distribution, the proof method is similar, and it will not be repeated.

(1.1) For $0 \leq \underline{p}\left\{\xi_1 \geq \xi_2\right\} \leq 1$, $0 \leq \overline{p}\left\{\xi_1 \geq \xi_2\right\} \leq 1$, by equation (5), there is $0 \leq p\left\{\xi_1 \geq \xi_2\right\} \leq 1$.

(1.2) For $p\left\{\xi_1 \geq \xi_2\right\} = 1$, then there is $\underline{p}\left\{\xi_1 \geq \xi_2\right\} = \overline{p}\left\{\xi_1 \geq \xi_2\right\} = 1$, and $b_2 < a_1, d_2 < c_1$, vice versa.

(1.3) Can be proved as the same as (1.2).

(1.4) According to equation (5), that is, $p\left\{\xi_1 \geq \xi_2\right\}+p\left\{\xi_2 \geq \xi_1\right\} = \alpha\underline{p}\left\{\xi_1 \geq \xi_2\right\}+(1-\alpha)\overline{p}\left\{\xi_1 \geq \xi_2\right\}+ \alpha\underline{p}\left\{\xi_2 \geq \xi_1\right\} + (1-\alpha)\overline{p}\left\{\xi_2 \geq \xi_1\right\}$,

Where $\underline{p}\left\{\xi_1 \geq \xi_2\right\} + \underline{p}\left\{\xi_2 \geq \xi_1\right\} = \frac{\sigma_{\underline{D}}}{(b_1-a_1)(b_2-a_2)} + \frac{(b_1-a_1)(b_2-a_2)-\sigma_{\underline{D}}}{(b_1-a_1)(b_2-a_2)} = 1$,

by the same reason, $\overline{p}\left\{\xi_1 \geq \xi_2\right\} + \overline{p}\left\{\xi_2 \geq \xi_1\right\} = 1$ can be obtained, then there is $p\left\{\xi_1 \geq \xi_2\right\} + p\left\{\xi_2 \geq \xi_1\right\} = 1$.

(1.5) For $\underline{p}\left\{\xi_1 \geq \xi_1\right\} = \frac{(b_1-a_1)^2}{2(b_1-a_1)^2} = \frac{1}{2}$, $\overline{p}\left\{\xi_1 \geq \xi_1\right\} = \frac{(d_1-c_1)^2}{2(d_1-c_1)^2} = \frac{1}{2}$, it is easy to get:

$p\left\{\xi_1 \geq \xi_1\right\} = \alpha\underline{p}\left\{\xi_1 \geq \xi_1\right\} + (1-\alpha)\overline{p}\left\{\xi_1 \geq \xi_1\right\} = 0.5$, proven. □

Property 2. *Let the IRNs $\xi_1 = ([a_1, b_1], [c_1, d_1])$ and $\xi_2 = ([a_2, b_2], [c_2, d_2])$, no matter whether they obey any one of the uniform distribution, exponential distribution, or normal distribution, their dominance degree satisfy the following properties:*

(2.1) If $p\{\xi_1 \geq \xi_2\} = 1$, $p\{\xi_2 \geq \xi_3\} = 1$, then $p\{\xi_1 \geq \xi_3\} = 1$.

(2.2) If $p\{\xi_1 \geq \xi_2\} = 1$, then $p\{\xi_1 \geq \xi_3\} \geq p\{\xi_2 \geq \xi_3\}$.

(2.3) If $p\{\xi_1 \geq \xi_2\} \geq 0.5$, $p\{\xi_2 \geq \xi_3\} \geq 0.5$, then $p\{\xi_1 \geq \xi_3\} \geq 0.5$.

Proof. The IRN obeys a uniform distribution as an example, and the proof method is similar when the interval roughness number obeys the exponential distribution and the normal distribution.

(2.1) If $p\{\xi_1 \geq \xi_2\} = 1, p\{\xi_2 \geq \xi_3\} = 1$, by the property (1.2), $c_2 \leq d_2 \leq c_1$, $d_3 \leq c_2$, it is obvious that $d_3 \leq c_1$, That is $p\{\xi_1 \geq \xi_3\} = 1$.

(2.2) Because of $p\{\xi_1 \geq \xi_2\} = 1$, it is clear that $\underline{p}\{\xi_1 \geq \xi_2\} = 1$, $\overline{p}\{\xi_1 \geq \xi_2\} = 1$, and $d_2 \leq c_1$. The following is a discussion of the comparison of the dominance degree betweenξ_3,ξ_2, and$\xi_1$under the six positional relationships of the upper and lower approximate intervals.

a) For $d_1 \leq c_3$, there is $d_2 \leq c_3$, it is evident that$p\{\xi_1 \geq \xi_3\} = p\{\xi_2 \geq \xi_3\} = 0$.

b) For $d_3 \leq c_2$, so $d_3 \leq c_1$, therefore $p\{\xi_1 \geq \xi_3\} = p\{\xi_2 \geq \xi_3\} = 1$.

c) $d_2 < c_3, d_3 < c_1$, there is $p\{\xi_1 \geq \xi_3\} = 1$, $p\{\xi_2 \geq \xi_3\} = 0$, $p\{\xi_1 \geq \xi_3\} > p\{\xi_2 \geq \xi_3\}$ holds. □

Next, let's discuss the case of lower approximate interval crossing underξ_1, ξ_2, ξ_3. The dominance degree of the upper approximate intervals can be obtained for the same reason, known $d_2 \leq c_1$.

d) If $a_2 < b_3 < a_1$, then $\underline{p}\{\xi_1 \geq \xi_3\} = 1 >\underline{p}\{\xi_2 \geq \xi_3\}$.

e) If $b_2 < a_3 < b_1$, then $\underline{p}\{\xi_1 \geq \xi_3\} > \underline{p}\{\xi_2 \geq \xi_3\} = 0$.

f) If $a_2 < a_3 < b_2 < a_1 < b_3 < b_1$, by the equation (3), $\underline{p}\{\xi_1 \geq \xi_3\} = 1 - \frac{(b_3-a_1)^2}{2(b_1-a_1)(b_3-a_3)}$, apparently $b_3 - a_1 < b_1 - a_1$, $b_3 - a_1 < b_3 - a_3$, then there is $\underline{p}\{\xi_1 \geq \xi_3\} > \frac{1}{2}$, for $\underline{p}\{\xi_2 \geq \xi_3\} = \frac{(b_2-a_3)^2}{2(b_2-a_2)(b_3-a_3)}$ obvious $b_2 - a_3 < b_2 - a_2$,$b_2 - a_3 < b_3 - a_3$, therefore $\underline{p}\{\xi_2 \geq \xi_3\} < \frac{1}{2}$, proven.

g) If $a_3 < a_2 < b_2 < a_1 < b_1 < b_3$, then $\underline{p}\{\xi_1 \geq \xi_3\} = \frac{(a_1-a_3+b_1-a_3)}{2(b_3-a_3)}$, $\underline{p}\{\xi_2 \geq \xi_3\} = \frac{(a_2-a_3+b_2-a_3)}{2(b_3-a_3)}$, obviously, $a_2 - a_3 < a_1 - a_3$, $b_2 - a_3 < b_1 - a_3$, then $\underline{p}\{\xi_1 \geq \xi_3\} > \underline{p}\{\xi_2 \geq \xi_3\}$.

h) If $a_2 < a_3 < b_2 < a_1 < b_1 < b_3$, then$\underline{p}(\xi_1 \geq \xi_3) = \frac{(a_1-a_3+b_1-a_3)}{2(b_3-a_3)}$, $\underline{p}\{\xi_2 \geq \xi_3\} = \frac{(b_2-a_3)^2}{2(b_3-a_3)(b_2-a_2)}$, for $\frac{\underline{p}\{\xi_1 \geq \xi_3\}}{\underline{p}\{\xi_2 \geq \xi_3\}} = \frac{(a_1-a_3+b_1-a_3)(b_2-a_2)}{(b_2-a_3)^2}$, it is clear that $b_2 - a_3 < a_1 - a_3$ and $b_2 - a_3 < b_2 - a_2$, so $\frac{\underline{p}\{\xi_1 \geq \xi_3\}}{\underline{p}\{\xi_2 \geq \xi_3\}} > 1$, that is $\underline{p}\{\xi_1 \geq \xi_3\} > \underline{p}\{\xi_2 \geq \xi_3\}$.

i) If $a_3 < a_2 < b_2 < a_1 < b_3 < b_1$, then $\underline{p}\{\xi_1 \geq \xi_3\} = 1 - \frac{(b_3-a_1)^2}{2(b_1-a_1)(b_3-a_3)}$, $\underline{p}\{\xi_2 \geq \xi_3\} = \frac{a_2-a_3+b_2-a_3}{2(b_3-a_3)}$, for$\underline{p}\{\xi_1 \geq \xi_3\} - \underline{p}\{\xi_2 \geq \xi_3\} = 1 - \frac{(b_3-a_1)^2}{2(b_1-a_1)(b_3-a_3)} - \frac{a_2-a_3+b_2-a_3}{2(b_3-a_3)} = 1 - \frac{(b_3-a_1)^2+(a_2-a_3+b_2-a_3)(b_1-a_1)}{2(b_1-a_1)(b_3-a_3)} = 1 - \left(\frac{(b_3-a_1)^2}{2(b_1-a_1)(b_3-a_3)} + \frac{(a_2-a_3+b_2-a_3)(b_1-a_1)}{2(b_1-a_1)(b_3-a_3)}\right)$, it is obvious that $b_3 - a_1 < b_1 - a_1, b_3 - a_1 < b_3 - a_3$, then $\frac{(b_3-a_1)^2}{2(b_1-a_1)(b_3-a_3)} < \frac{1}{2}$. It is evident that $\frac{(a_2-a_3+b_2-a_3)(b_1-a_1)}{2(b_1-a_1)(b_3-a_3)} = \frac{a_2-a_3+b_2-a_3}{2(b_3-a_3)} = \frac{a_2-a_3+b_2-a_2+a_2-a_3}{2(a_2-a_3+b_2-a_2+b_3-a_2)}$,

For $p\{\xi_1 \geq \xi_2\} = 1$, because of b_3 much greater than a_2, then $a_2 - a_3 < b_3 - a_2$, that is $\frac{(a_2-a_3+b_2-a_3)(b_1-a_1)}{2(b_1-a_1)(b_3-a_3)} < \frac{1}{2}$. In summary $\underline{p}\{\xi_1 \geq \xi_3\} - \underline{p}\{\xi_2 \geq \xi_3\} > 0$, proven.

The above (d)-(i) are all cases of the lower approximate interval crossing, and it is verified that $\underline{p}\{\xi_1 \geq \xi_3\} > \underline{p}\{\xi_2 \geq \xi_3\}$is satisfied in all 6 cases. Similarly, the property of the upper approximate $\overline{p}\{\xi_1 \geq \xi_3\} > \overline{p}\{\xi_2 \geq \xi_3\}$ interval can be obtained for the same reason. According to equation (5), the dominance degrees of IRNs satisfy $p\{\xi_1 \geq \xi_3\} > p\{\xi_2 \geq \xi_3\}$. Proven.

(2.3) According to Fig. 1, To dominance to be greater than 0.5, only (3), (4), and (6) are met, among them, where case (6) is greater than 0.5 constantly. Therefore, to discuss the comparison of the dominance degree of the three IRNs, it is necessary to divide the discussion into $A_3^3 = 27$ cases, there are 9 cases of constant existence including (6), and the remaining 18 cases are discussed.

a) The position between ξ_1 and ξ_2 satisfies the third case, the ξ_2 and ξ_3 satisfies the fourth case, and the ξ_1 and ξ_3 position satisfies the third case.

For $\underline{p}\{\xi_1 \geq \xi_3\} = \frac{0.5(2b_1-b_2-a_2)(b_2-a_2)}{(b_1-a_1)(b_2-a_2)} \geq 0.5$, then $2b_1 - b_2 - a_2 \geq b_1 - a_1$,

For $\underline{p}\{\xi_2 \geq \xi_3\} = \frac{0.5(b_2+a_2-2a_3)(b_2-a_2)}{(b_2-a_2)(b_3-a_3)} \geq 0.5$, then $a_2 + b_2 - 2a_3 \geq b_3 - a_3$.

To prove $\underline{p}\{\xi_1 \geq \xi_3\} = \frac{0.5(2b_1-b_3-a_3)(b_3-a_3)}{(b_1-a_1)(b_3-a_3)} \geq 0.5$, that is, to prove $2b_1 - b_3 - a_3 \geq b_1 - a_1$, since $2b_1 - b_3 - a_3 \geq 2b_1 + 2a_3 - a_2 - b_3 \geq 2b_1 + 2a_3 + b_1 - a_1 - 2b_1 \geq b_1 - a_1$, proven.

b) The position between ξ_1 and ξ_2 satisfies the fourth case, the $\xi_2$and ξ_3 satisfies the third case, and the ξ_1 and ξ_3 position relationship satisfies the fourth case.

For $\underline{p}\{\xi_1 \geq \xi_2\} = \frac{0.5(b_1+a_1-2a_2)(b_1-a_1)}{(b_1-a_1)(b_2-a_2)} \geq 0.5$, then $b_1 + a_1 - 2a_2 \geq b_2 - a_2$, $b_1 + a_1 \geq b_2 + a_2$,

For $\underline{p}\{\xi_2 \geq \xi_2\} = \frac{0.5(2b_2-b_3-a_3)(b_3-a_3)}{(b_2-a_2)(b_3-a_3)} \geq 0.5$, then $2b_2 - b_3 - a_3 \geq b_2 - a_2$, $b_2 + a_2 \geq b_3 + a_3$,

To prove $\underline{p}\{\xi_1 \geq \xi_3\} = \frac{0.5(b_1+a_1-2a_3)(b_1-a_1)}{(b_1-a_1)(b_3-a_3)} \geq 0.5$, that is to prove $b_1 + a_1 - 2a_3 \geq b_3 - a_3$, $b_1 + a_1 \geq b_3 + a_3$, that is true, proven.

In other cases, the position between ξ_1 and ξ_2 satisfies any combination of (3) and (4), and the proof method is similar. In the same way, the dominance degree of ξ_1 and ξ_2 in the upper approximate interval can also be obtained by considering 18 cases. According to equation (5), the dominance degrees of IRNs satisfy $p\{\xi_1 \geq \xi_3\} > p\{\xi_2 \geq \xi_3\}$. Proven.

In summary, property 2 is verified.

Therefore, when comparing and ranking multiple TRNs in MADM, the TRNs satisfying properties 1 and 2 can be compared and ranked according to the conclusion without specific calculations, which reduces the amount of computation.

3.5 Example verification

In this section, to discuss the advantages and disadvantages of IRNs in different distribution types, now use the dominance degree to compare and analyze IRNs. And the two IRNs ξ_1=([8, 9], [7, 10]), ξ_2=([7, 10], [6, 14]) are assumed.

(1) ξ_1, ξ_2 obey uniform distribution

$\underline{p}\{\xi_1 \geq \xi_2\} = \int_8^9 \mathrm{d}x \int_7^x \frac{\mathrm{d}y}{3} = \frac{1}{2}$, $\overline{p}\{\xi_1 \geq \xi_2\} = \int_7^{10} \frac{\mathrm{d}x}{3} \int_6^x \mathrm{d}y = \frac{5}{16}$, $\alpha = \frac{4/11}{1+4/11} = \frac{4}{15}$.

According to equation (5), get:$p\{\xi_1 \geq \xi_2\} = 0.363$, then $\xi_2 \succ \xi_1$.

Similarly: $\underline{p}\{\xi_2 \geq \xi_1\} = \int_8^9 \mathrm{d}y \int_y^{10} \frac{\mathrm{d}x}{3} = \frac{1}{2}$, $\overline{p}\{\xi_2 \geq \xi_1\} = \int_7^{10} \frac{\mathrm{d}y}{3} \int_y^{14} \frac{\mathrm{d}x}{8} = \frac{11}{16}$.

Then $p\{\xi_2 \geq \xi_1\} = 0.637$, $\xi_2 \succ \xi_1$, and satisfies $p\{\xi_1 \geq \xi_2\} + p\{\xi_2 \geq \xi_1\} = 1$.

(2) ξ_1, ξ_2 obey exponential distribution

According to Equation (7), $\lambda_1 = \frac{\ln 20}{3}$, $\lambda_2 = \frac{\ln 20}{8}$, the probability density functions of ξ_1 and ξ_2 are $f_{\xi_1}(x) = \begin{cases} \lambda_1 e^{-\lambda_1(x-7)}, & 7 < x < 10 \\ 0, & else \end{cases}$ and $f_{\xi_2}(y) = \begin{cases} \lambda_2 e^{-\lambda_2(y-6)}, & 6 < x < 14 \\ 0, & else \end{cases}$ respectively.

Then there are $\underline{p}\{\xi_1 \geq \xi_2\} = \frac{\int_8^9 \frac{\ln 20}{3} e^{-\frac{\ln 20}{3}(x-7)} \mathrm{d}x \int_7^x \frac{\ln 20}{8} e^{-\frac{\ln 20}{8}(y-6)} \mathrm{d}y}{\int_8^9 \frac{\ln 20}{3} e^{-\frac{\ln 20}{3}(x-7)} \mathrm{d}x \int_7^{10} \frac{\ln 20}{8} e^{-\frac{\ln 20}{8}(y-6)} \mathrm{d}y} = 0.606$,

$\overline{p}\{\xi_1 \geq \xi_2\} = \frac{\int_7^{10} \frac{\ln 20}{3} e^{-\frac{\ln 20}{3}(x-7)} \mathrm{d}x \int_6^x \frac{\ln 20}{8} e^{-\frac{\ln 20}{8}(y-6)} \mathrm{d}y}{\int_7^{10} \frac{\ln 20}{3} e^{-\frac{\ln 20}{3}(x-7)} \mathrm{d}x \int_6^{14} \frac{\ln 20}{8} e^{-\frac{\ln 20}{8}(y-6)} \mathrm{d}y} = 0.507$,

and $\alpha = \frac{4}{15}$, then $p\{\xi_1 \geq \xi_2\} = 0.533$, that is $\xi_1 \succ \xi_2$. Similarly there are:

$\underline{p}\{\xi_2 \geq \xi_1\} = \frac{\int_8^9 \frac{\ln 20}{3} e^{-\frac{\ln 20}{3}(y-7)} \mathrm{d}y \int_y^{11} \frac{\ln 20}{8} e^{-\frac{\ln 20}{8}(x-6)} \mathrm{d}x}{\int_7^{10} \frac{\ln 20}{8} e^{-\frac{\ln 20}{8}(x-6)} \mathrm{d}x \int_8^9 \frac{\ln 20}{3} e^{-\frac{\ln 20}{3}(y-7)} \mathrm{d}y} = 0.394$,

$\overline{p}\{\xi_2 \geq \xi_1\} = \frac{\int_7^{10} \frac{\ln 20}{3} e^{--\frac{\ln 20}{3}(y-7)} \mathrm{d}y \int_y^{10} \frac{\ln 20}{8} e^{-\frac{\ln 20}{8}(x-6)} \mathrm{d}x}{\int_6^{14} \frac{\ln 20}{8} e^{-\frac{\ln 20}{8}(x-6)} \mathrm{d}x \int_7^{10} \frac{\ln 20}{3} e^{-\frac{\ln 20}{3}(y-7)} \mathrm{d}y} = 0.493$,

get $p\{\xi_2 \geq \xi_1\} = 0.467$, then $\xi_1 \succ \xi_2$, and meet $p\{\xi_1 \geq \xi_2\} + p\{\xi_2 \geq \xi_1\} = 1$.

(3) ξ_1, ξ_2 obey normal distribution

According to equation (13), $\mu_1 = \frac{34}{4}$, $\sigma_1 = \frac{1}{2}$, $\mu_2 = \frac{37}{4}$, $\sigma_2 = \frac{4}{3}$, get $\mu_1 - \mu_2 = -\frac{3}{4}$, $\sqrt{\sigma_1{}^2 + \sigma_2{}^2} = \frac{\sqrt{73}}{6}$, and $\xi_1 - \xi_2$= ([−2, 2], [−7, 4]). Then:

$\underline{p}\{\xi_1 \geq \xi_2\} = \frac{\Phi\left(\frac{(2+3/4)}{\sqrt{73}/6}\right) - \Phi\left(\frac{(0+3/4)}{\sqrt{73}/6}\right)}{\Phi\left(\frac{(2+3/4)}{\sqrt{73}/6}\right) - \Phi\left(\frac{(-2+3/4)}{\sqrt{73}/6}\right)} = 0.346$, $\overline{p}\{\xi_1 \geq \xi_2\} = \frac{\Phi\left(\frac{(4+3/4)}{\sqrt{73}/6}\right) - \Phi\left(\frac{(0+3/4)}{\sqrt{73}/6}\right)}{\Phi\left(\frac{(4+3/4)}{\sqrt{73}/6}\right) - \Phi\left(\frac{(-7+3/4)}{\sqrt{73}/6}\right)} = 0.28$,

and $\alpha = \frac{4}{15}$, get $p\{\xi_1 \geq \xi_2\} = 0.298$, then there is $\xi_2 \succ \xi_1$.

Similarly: $\xi_2 - \xi_1$=([−2, 2], [−4, 7]), $\mu_2 - \mu_1 = \frac{3}{4}$, then

$\underline{p}\{\xi_2 \geq \xi_1\} = \frac{\Phi\left(\frac{(2-3/4)}{\sqrt{73}/6}\right) - \Phi\left(\frac{(0-3/4)}{\sqrt{89}/6}\right)}{\Phi\left(\frac{(2-3/4)}{\sqrt{73}/6}\right) - \Phi\left(\frac{(-2-3/4)}{\sqrt{73}/6}\right)} = 0.654$, $\overline{p}\{\xi_2 \geq \xi_1\} = \frac{\Phi\left(\frac{(7-3/4)}{\sqrt{73}/6}\right) - \Phi\left(\frac{(0-3/4)}{\sqrt{73}/6}\right)}{\Phi\left(\frac{(7-3/4)}{\sqrt{73}/6}\right) - \Phi\left(\frac{(-4-3/4)}{\sqrt{73}/6}\right)} = 0.72$,

get $p\{\xi_2 \geq \xi_1\} = 0.702$, then $\xi_2 \succ \xi_1$, and meet $p\{\xi_1 \geq \xi_2\} + p\{\xi_2 \geq \xi_1\} = 1$.

Sorting out the results of the above examples, it can be obtained that when the value IRNs is the same but the distribution type is different, the comparison results between the two are shown in Table 1:

Table 1: Comparison of sort results under different distribution

Distribution	$\underline{p}(\xi_1 \geq \xi_2)$	$\overline{p}(\xi_1 \geq \xi_2)$	$p(\xi_1 \geq \xi_2)$	Ranking Result
uniform distribution	0.5	0.313	0.363	$\xi_2 \succ \xi_1$
exponential distribution	0.606	0.507	0.533	$\xi_1 \succ \xi_2$
normal distribution	0.346	0.280	0.298	$\xi_2 \succ \xi_1$

The examples discussed the comparative results of the dominance degree of uniform distribution, exponential distribution, and normal distribution, respectively, and it can be seen that the properties $0 \leq p(\xi_1 \geq \xi_2) \leq 1$, $p\{\xi_1 \geq \xi_2\} + p\{\xi_2 \geq \xi_1\} = 1$ are satisfied under the three distributions. In addition, from the comparison results, the ranking results are $\xi_2 \succ \xi_1$ when the IRNs obeyed uniform distribution and normal distribution, and the ranking result when obeying the exponential distribution is $\xi_1 \succ \xi_2$, which has an interesting reverse order phenomenon. This shows that when the IRNs obey different distributions, the dominance degree of the same IRNs will also be different, and there will even be completely different ranking results. Therefore, it is necessary to discuss the type of distribution of IRNs.

Different distributions result in different sorting results. To further verify the rationality of the assumptions of different distribution types, the values of ξ_2=([7, 10], [6, 14]) obeys the exponential distribution and normal distribution, respectively, as shown in Fig. 4 and Fig. 5:

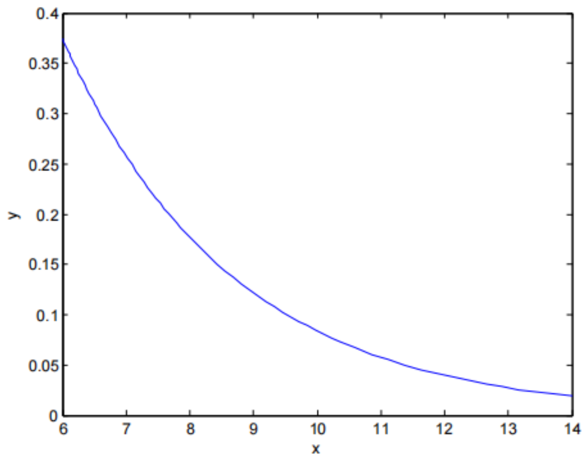

Figure 4: The value of ξ_2 in the exponential distribution

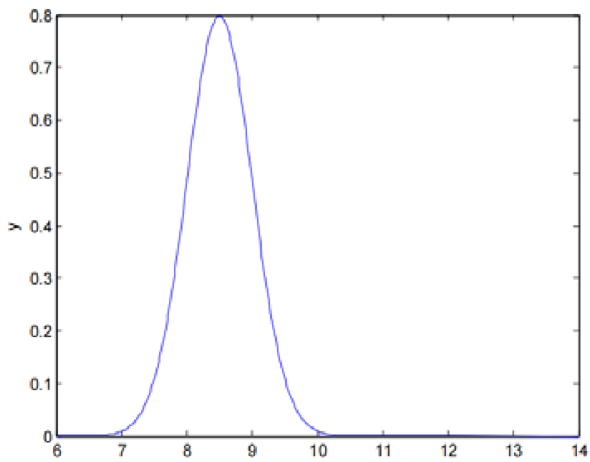

Figure 5: The value of ξ_2 in the exponential distribution

Obviously, although the values of ξ_2 are all taken between [6,14], the concentration interval of the values is different due to different distribution types. In the exponential distribution, the values of ξ_2 are mainly concentrated in [6,8]; while in the normal distribution, the values of ξ_2 are mainly concentrated in [8,9], which is symmetrical. Therefore, in the study of IRN, it is very necessary to make reasonable assumptions about the distribution type obeyed by the IRN according to the actual situation.

4 Multi-attribute decision-making method for IRNs

This section proposes a new multi-attribute decision-making (MADM) method based on dominance degree.

In MADM, there are multiple schemes, each scheme has multiple evaluation indicators, and the values of each scheme under different indicators are in the form of IRNs. According to the description of the rough set theory, MADM problems can be converted into a solution problem in a sequence information system. Let $S = (U, A, V, f)$ be the information system, where $U = \{x_1, x_{2,\cdots,}x_m\}$ is the set of objects (schemes), $A = \{a_1, a_{2,\dots,} a_n\}$is the set of attributes (indicators), V is the set of values of objects under attributes, and each attribute value is the IRN, and f is the mapping of $U \times A \to V$.

Two problems need to be solved in MADM: one is the dominance degree of different objects under the same attribute, and the other is the data aggregation problem of a single object under multiple attributes. Due to the uncertainty of the IRN and the complexity of the objective world, the attribute weight information is often unknown, so it is important to determine the attribute weight. This paper determines the key point to determine the weight of an attribute, the more distinguishable the attribute is, the more important the attribute is, and the greater the weight should be. According to the previous article, whether the IRN can be distinguished or the degree of distinction depends on the dominance degree. The greater the difference between the IRN dominance degree, the more important the attribute is. We define the difference in the dominance degree of IRN with the dominance balance degree. Compared with other methods, the attribute weight of the literature [13] depends on the subjective preferences of decision-makers; the literature [21] uses the entropy right method to obtain the attribute weight. The advantages of the dominance balance degree proposed in this article to calculate the attribute weight method are easier and more objective.

Therefore, in the MADM of IRNs, based on the dominance degree of IRNs, the dominance balance degree ξ_i relative to ξ_j is defined as:

$$Dp_{ij} = |p_{ij} - p_{ji}| \tag{18}$$

Where $p_{ij} = p\{\xi_i \geq \xi_j\}, p_{ji} = p\{\xi_j \geq \xi_i\} \, (i, j = 1, 2, ..., m)$, then the sum of the dominance balance degree of IRNs under a single attribute is:

$$Dp_{a_k} = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} Dp_{ij} \tag{19}$$

So the sum of the dominance balance degree under all attributes is:

$$Dp = \sum_{k=1}^{n} Dp_{a_k} \tag{20}$$

Normalize the DP to get the weight of each attribute as:

$$\omega_{a_k} = \frac{Dp_{a_k}}{Dp} \, (k = 1, 2, ...n) \tag{21}$$

So the greater the dominance balance degree of the IRNs under a certain attribute, the more the attribute can distinguish between them, and the greater the weight of the attribute. There are two extremes of the value of ω_{a_k}:

(1) If the dominance matrix elements of pairwise comparison of the attribute a_k are both 0.5, then $\omega_{a_k} = 0$;

(2) If the dominance matrix elements of pairwise comparison under a certain attribute a_k are both 0 and 1 (excluding diagonal elements), then ω_{a_k} reach the maximum value.

Based on the previous discussion on the distribution types of IRNs and the calculation of attribute weights by using dominance balance degree, a new MADM method based on dominance degree is obtained.

Step 1: According to the type of distribution of the IRN, the corresponding probability density function is obtained, and the corresponding dynamic weight α_{ij} is calculated.

Step 2: Through the relevant formulas, calculate the upper and lower approximate dominance degrees in each attribute: that is, $\underline{p}_{a_k}\{\xi_i \geq \xi_j\}$,$\overline{p}_{a_k}\{\xi_i \geq \xi_j\}$,$(i,j=1,2,...,m,k=1,2,...,n)$, and then obtain the corresponding dominance degrees table $p_{a_k}\{\xi_i \geq \xi_j\}(p_{a_k}\{\xi_j \geq \xi_i\})$.

Step 3: Depending on equation (18)-(21), the weight of each attribute $\omega_{a_k}(k=1,2,......n)$ is calculated.

Step 4: Calculate the average dominance degree of the IRNs for all attributes:
$P_{ij}=\sum\limits_{j=1}^{m}\omega_{a_k}p_{a_k}\{\xi_i \geq \xi_j\}\,(i=1,2,...,m,j=1,2,...,m,k=1,2,...,n)$.

Step 5: Compute the composite average dominance degree of the IRN for each object: $z_i=\frac{1}{m}\sum\limits_{j=1}^{m}P_{ij},(i=1,2,...,m)$, to obtain the ranking of each scheme (object) for practical decision-making problems.

The time complexity of the dynamic weight is $O(|U|^2|A|)$, the time complexity of the attribute weight is $O(|U|^2|A|)$. To sum up, the time complexity of the algorithm is $O(|U|^2|A|)$.

5 Case Applications

In this section, we empirically evaluate the effectiveness and accuracy of the MADM method of the IRN based on the dominance degree of different distributions.

In terms of logistics and distribution, enterprises should choose the location of alternative transfer stations. There are 5 alternative addresses to choose from, and four indicators are mainly considered when choosing alternative addresses: cost savings, expected benefits, management benefits, and risk avoidance. Table 2 shows the corresponding survey data table in this example (for the convenience of data processing, the data in the table have been converted into benefit-based indicators). The data under each indicator is an estimate and therefore is represented as an IRN.

Table 2: Logistics Center site selection original data

U\A	a_1	a_2	a_3	a_4
x_1	([11,12],[9,13])	([4,5],[2,7])	([12,13],[11,14])	([1.3,1.4],[1.2,1.6])
x_2	([7,9],[6,11])	([2,3],[1,5])	([9,11],[8,13])	([0.3,0.4],[0.2,0.6])
x_3	([18,20],[15,22])	([7,8],[5,9])	([11,12],[10,13])	([0.7,0.8],[0.5,1.0])
x_4	([8,9],[7,11])	([3,4],[2,6])	([2,3],[1,4])	([0.4,0.5],[0.3,0.7])
x_5	([11,12],[10,13])	([6,7],[5,8])	([16,18],[14,20])	([1.6,1.7],[1.5,1.8])

According to the data characteristics of the indicator itself, and to further verify the rationality of the calculation of the dominance degree of different distributions proposed in this paper. It is assumed that cost savings (a_1) and expected benefits (a_2) obey uniform distribution, management benefits(a_3)obey normal distribution, and risk avoidance(a_4) obeys exponential distribution. According to the MADM method proposed in this paper, the calculation is as follows:

(1) Calculate the dynamic weight α_{ij} of the IRNs under each attribute.

Attribute of a_1: $\alpha_{12}=\frac{1}{4},\alpha_{13}=\frac{1}{4},\alpha_{14}=\frac{1}{5},\alpha_{15}=\frac{2}{9}$, $\alpha_{23}=\frac{1}{5},\alpha_{24}=\frac{2}{11},\alpha_{25}=\frac{3}{11},\alpha_{34}=\frac{3}{14},\alpha_{35}=\frac{3}{13},\alpha_{45}=\frac{2}{9}$.

Attribute of a_2: $\alpha_{12}=\frac{2}{11},\alpha_{13}=\frac{2}{11},\alpha_{14}=\frac{2}{11},\alpha_{15}=\frac{1}{5}$, $\alpha_{23}=\frac{1}{5},\alpha_{24}=\frac{1}{5},\alpha_{25}=\frac{2}{9},\alpha_{34}=\frac{1}{5},\alpha_{35}=\frac{2}{9},\alpha_{45}=\frac{2}{9}$.

Attribute of a_3: $\alpha_{12}=\frac{3}{11},\alpha_{13}=\frac{1}{4},\alpha_{14}=\frac{1}{4},\alpha_{15}=\frac{1}{4}$, $\alpha_{23}=\frac{1}{5},\alpha_{24}=\frac{3}{11},\alpha_{25}=\frac{4}{13},\alpha_{34}=\frac{1}{4},\alpha_{35}=\frac{1}{4},\alpha_{45}=\frac{1}{4}$.

Attribute of a_4: $\alpha_{12}=\frac{1}{5},\alpha_{13}=\frac{2}{11},\alpha_{14}=\frac{1}{5},\alpha_{15}=\frac{2}{9}$, $\alpha_{23}=\frac{2}{11},\alpha_{24}=\frac{1}{5},\alpha_{25}=\frac{2}{9},\alpha_{34}=\frac{2}{11},\alpha_{35}=\frac{1}{5},\alpha_{45}=\frac{2}{9}$.

(2) The lower approximate dominance degree and upper approximate dominance degree of the IRN under each indicator are calculated, and then the dominance degree is obtained in Table 3.

Table 3: Comparison dominance degree table

p_{ij}	x_1	x_2	x_3	x_4	x_5
x_1	0.5, 0.5, 0.5, 0.5	0.925, 0.816, 0.990, 1	0, 0.082, 0.941, 1	0.9, 0.673, 1, 1	0.402, 0.107, 0, 0.016
x_2	0.075, 0.184, 0.010, 0	0.5, 0.5, 0.5, 0.5	0, 0, 0.078, 0.01	0.373, 0.225, 1, 0.168	0.017, 0, 0, 0
x_3	1, 0.918, 0.059, 0	1, 1, 0.922, 0.99	0.5, 0.5, 0.5, 0.5	1, 0.975, 1, 0.948	1, 0.708, 0, 0
x_4	0.1, 0.327, 0, 0	0.627, 0.775, 0, 0.832	0, 0.025, 0, 0.052	0.5, 0.5, 0.5, 0.5	0.032, 0.032, 0, 0
x_5	0.592, 0.893, 1, 0.984	0.983, 1, 1, 1	0, 0.292, 1, 1	0.968, 0.968, 1, 1	0.5, 0.5, 0.5, 0.5

The data in the third column of the second row in the table (0.925, 0.816, 0.990, 1) represents the corresponding dominance degree of x_1 and x_2 under the four attribute indicators. Then $p_{a_1}\{x_1 \geq x_2\} = 0.925$, $p_{a_2}\{x_1 \geq x_2\} = 0.816$, $p_{a_3}\{x_1 \geq x_2\} = 0.990$, $p_{a_4}\{x_1 \geq x_2\} = 1$, other numerical representations are similar in the table. It is easy to get $p_{a_3}\{x_5 \geq x_3\} = 1$, $p_{a_3}\{x_3 \geq x_4\} = 1$, $p_{a_3}\{x_5 \geq x_4\} = 1$, that satisfies the property (2.1); from $p_{a_3}\{x_2 \geq x_4\} = 1$, $p_{a_3}\{x_2 \geq x_3\} = 0.078$, and $p_{a_3}\{x_4 \geq x_3\} = 0$, that meets property (2.2); Because of $p_{a_1}\{x_1 \geq x_2\} = 0.925 > 0.5$, and $p_{a_1}\{x_5 \geq x_2\} = 0.983 > 0.5$, satisfies the nature (2.3).

According to Table 3, the comparative dominance degree of pairs under a single attribute is shown in Table 4.

Table 4: Comparative dominance degree tables under a single attribute

			a_1						a_2		
p_{ij}	x_1	x_2	x_3	x_4	x_5	p_{ij}	x_1	x_2	x_3	x_4	x_5
x_1	0.5	0.925	0	0.9	0.402	x_1	0.5	0.816	0.082	0.673	0.107
x_2	0.075	0.5	0	0.373	0.017	x_2	0.184	0.5	0	0.225	0
x_3	1	1	0.5	1	1	x_3	0.918	1	0.5	0.975	0.708
x_4	0.1	0.625	0	0.5	0.032	x_4	0.327	0.775	0.025	0.5	0.032
x_5	0.592	0.983	0	0.968	0.5	x_5	0.893	1	0.292	0.968	0.5
			a_3						a_4		
p_{ij}	x_1	x_2	x_3	x_4	x_5	p_{ij}	x_1	x_2	x_3	x_4	x_5
x_1	0.5	0.990	0.941	1	0	x_1	0.5	1	1	1	0.016
x_2	0.010	0.5	0.078	1	0	x_2	0	0.5	0.01	0.168	0
x_3	0.059	0.922	0.5	1	0	x_3	0	0.99	0.5	0.948	0
x_4	0	0	0	0.5	0	x_4	0	0.832	0.052	0.5	0
x_5	1	1	1	1	0.5	x_5	0.984	1	1	1	0.5

(3) Calculate the attribute weight ω_{a_k}. Dominance balance degree under a single attribute is obtained firstly, then $Dp_{a_1} = 7.997$, $Dp_{a_2} = 6.452$, $Dp_{a_3} = 9.706$, $Dp_{a_4} = 9.508$, and then the attribute weights are obtained: $\omega_{a_1} = 0.238$, $\omega_{a_2} = 0.192$, $\omega_{a_3} = 0.288$, $\omega_{a_4} = 0.282$.

(4) Next, the attribute weights are used to calculate the average dominance degree of each object under the four attributes, and the composite average dominance degree table is obtained.

Table 5: The average dominance degree

p_{ij}	x_1	x_2	x_3	x_4	x_5
x_1	0.5	0.944	0.569	0.913	0.121
x_2	0.056	0.5	0.025	0.467	0.004
x_3	0.431	0.975	0.5	0.981	0.374
x_4	0.087	0.533	0.019	0.5	0.014
x_5	0.879	0.996	0.626	0.986	0.5

(5) The comprehensive dominance degree of each address is calculated $z_1 = 0.609$, $z_2 = 0.210$, $z_3 = 0.652$, $z_4 = 0.231$, $z_5 = 0.797$, and then the ranking result of the alternative addresses is $x_5 \succ x_3 \succ x_1 \succ x_4 \succ x_2$.

As shown in Table 6 (the metrics of all models are benefit-based), the method in this paper is compared with the other three models in terms of theory (attribute weight, distribution type) and empirical aspects (ranking accuracy). In literature [14] and [24], the comprehensive dominance degree of x_2 and x_4 is similar, and no more precise ranking results can be obtained. Referring to the method of this paper, there is a significant difference in the comprehensive dominance of x_2 and x_4, then the accurate ranking results can be obtained. In the process of comparing the INRs in pairs, the quantitative comparison of the dominance degree can not only intuitively see whether x_i is better than x_j, but also obtain the specific value that x_i is better than x_j. Compared with the literature [14, 24], the advantage of this method is that it can obtain an accurate ranking of the two schemes.

The data in Table 5 shows that the comprehensive dominance degree of x_2 is significantly lower than that of x_4. From the dominance degree of x_2 and x_4 compared with the other INRs, the dominance degree of x_2 is also lower than that of x_4.

The literature [14, 23] does not discuss the distribution of data in detail according to the characteristics of indicator data, which does not meet the actual needs. In addition, in the determination of attribute weights, [14, 23] relies on subjective will and lacks objectivity.

Table 6: Comparison of different model ranking results

Model	Ranking Results	Attribute Weight	Distribution	Sorting Accuracy
The model presented in this paper	$x_5 \succ x_3 \succ x_1 \succ x_4 \succ x_2$	objective	uniform, exponential, normal	accurate
IRN model based on dominance relation [14]	$x_5 \succ x_3 \succ x_1 \succ x_2 \sim x_4$	subjective	uniform	not accurate
IRN model of the normal distribution based on the dominant relation [23]	$x_5 \succ x_3 \succ x_1 \succ x_2 \succ x_4$	subjective	normal	accurate
IRN model based on the MATLAB software simulation [24]	$x_5 \succ x_3 \succ x_1 \succ x_2 \sim x_4$	—	uniform, exponential, normal	not accurate

In summary, the attribute weight proposed in this article does not depend on the subjective will of the decision maker, and refines the type of distribution according to the actual data characteristics, and gets more accurate sorting. This article proposes that the MADA method based on the IRNs in different distributions is effective and accurate.

6 Discussion

This paper presents a novel MADM method for IRNs considering different distribution types, namely uniform, exponential, and normal distributions. The proposed method defines upper and lower approximate interval dominance degrees and aggregates them using dynamic weights to obtain pairwise comparisons of IRNs. The properties of dominance are verified, and a data-driven attribute weight determination method based on dominance balance degree is introduced. The case study demonstrates the effectiveness and rationality of the proposed method and reveals that the distribution type of IRNs significantly impacts decision results, potentially leading to reversed ranking outcomes.

We discuss sensitivity about dynamic weights, attribute weights, distribution types, and the accuracy of index distribution. The result of dynamic weights and attribute weights is calculated by objective data, so there is no sensitivity problem. According to part 3.5, the sorting results are different when attribute data distribution types are different, so it is necessary to reasonably assume the distribution type according to the data characteristics. Then we discuss the accuracy $1-\alpha$ in the index distribution, and the two IRNs ξ_1=$([8,9],[7,10])$, ξ_2=$([7,10],[6,14])$ are assumed. Through calculation, we get that when the accuracy is $1-\alpha=90\%$, the sorting $\xi_2 \succ \xi_1$ is obtained, and then the accuracy of $1-\alpha$ are 91%, 95% , and 99%, the ranking sort is $\xi_1 \succ \xi_2$. Therefore, the index distribution requires greater than 90% accuracy selection.

However, the proposed method also has some limitations. First, the study only considers three specific distribution types (uniform, exponential, normal), and the applicability of the method to other distributions needs further investigation. Second, the method assumes that the distribution types of IRNs are known or can be accurately estimated, which may not always be the case in real-world decision-making problems. Third, the computational complexity and scalability of the method for large-scale problems with many attributes and alternatives require further analysis.

Future research directions include: (1) extending the proposed method to handle other distribution types and more complex decision-making scenarios [27, 28]; (2) developing methods for estimating or learning the distribution types of IRNs from data; (3) investigating the integration of the proposed method with other uncertainty theories (e.g., fuzzy sets, rough sets) to handle more diverse types of uncertain information; (4) applying the proposed method to real-world decision-making problems in various domains [29, 30, 31] and evaluating its performance in comparison with other state-of-the-art methods.

7 Conclusions

This paper provides a valuable contribution to the field of multi-attribute decision-making under uncertainty by proposing a novel method for handling interval rough numbers considering different distribution types. The proposed method offers a more flexible and realistic approach for modeling and analyzing decision problems with IRNs, and the results highlight the importance of considering distribution information in decision-making processes. With further extensions and applications, the proposed method has the potential to become a powerful tool for supporting complex decision-making under uncertainty.

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Conflict of interest

The authors declare no conflict of interest.

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