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A new ranking method for trapezoidal intuitionistic fuzzy numbers and its application to multi-criteria decision making

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Abstract

The ranking of intuitionistic fuzzy numbers is paramount in the decision making process in a fuzzy and uncertain environment. In this paper, a new ranking function is defined, which is based on Robust's ranking index of the membership function and the non-membership function of trapezoidal intuitionistic fuzzy numbers. The mentioned function also incorporates a parameter for the attitude of the decision factors. The given method is illustrated through several numerical examples and is studied in comparison to other already-existent methods. Starting from the new classification method, an algorithm for solving fuzzy multi-criteria decision-making (MCDM) problems is proposed. The application of said algorithm implies accepting the subjectivity of the deciding factors, and offers a clear perspective on the way in which the subjective attitude influences the decision-making process. Finally, a MCDM problem is solved to outline the advantages of the algorithm proposed in this paper.

Keywords: ranking method; trapezoidal intuitionistic fuzzy number; multi-criteria decision making.

1 Introduction

The intuitionistic fuzzy sets were introduced by K.T. Atanassov [2] as a means of generalization of the fuzzy sets defined by L.A. Zadeh in 1965 [28]. Unlike the fuzzy sets, which are characterized solely by the membership function, the intuitionistic fuzzy sets are also characterized by the non-membership function. For this reason, the intuitionistic fuzzy sets are more useful in expressing uncertainty and vagueness. Thus, defining the ranking of intuitionistic fuzzy numbers has represented a significant scientific pursuit for researchers because they play an essential role, especially in the problems of Multi-Criteria Decision Making from various domains, e.g., Economics, Social Sciences, and Engineering.

Since the inception of the intuitionistic fuzzy numbers, these have been approached in various ways, and many classification methods have been proposed. In 2003, P. Grzegrorzewski [7] proposed a ranking method for intuitionistic fuzzy numbers by using the expected interval of an intuitionistic fuzzy number. In 2004, H.B. Mitchell [12] proposed a ranking of intuitionistic fuzzy numbers by regarding them as an ensemble of fuzzy numbers. Two years later, in 2006, V.L.G. Nayagam, G. Venkateshwari and G. Sivaraman [15] generalized Chen and Hwang's method, expending the scope of the method from fuzzy to intuitionistic fuzzy numbers. In 2010, D.F. Li [10], defined a new ranking function for intuitionistic fuzzy numbers as the ratio between the value index and the ambiguity index. The function mentioned above was also based on the value and the ambiguity of triangular intuitionistic fuzzy numbers. In 2012, P.K. De and D. Das [4] proposed a ranking method for trapezoidal intuitionistic fuzzy numbers, also making use of the value and ambiguity indices but instead considering their sum as the ranking function. In 2013, S. Rezvani [22] defined the values and ambiguities of the membership degree and the non-membership degree for the trapezoidal intuitionistic fuzzy numbers and developed a ranking method based on the value-index and ambiguity-index. In the same year, S.S. Roseline and E.C.H. Amirtharaj [23] defined the magnitude of membership and non-membership functions for intuitionistic fuzzy numbers and ordered them based on the intuitionistic fuzzy numbers. Moreover, in 2013, E. Jafarian and M.A. Rezvani [9] proposed a ranking method according to a crisp value associated with an intuitionistic fuzzy number related to the spread value concept defined. In the following period, many papers addressing this research topic were published ([19], [16], [3], [24], [26], [1], [13]).

The purpose of classifying the intuitionistic fuzzy numbers is to facilitate their utilization in real-life problems, especially in decision-making problems in which information is uncertain. Thus, the ranking methods proposed by J. Whang and Z. Zhang [25] in 2009, D.F. Li, J.X. Nan and M.J. Zhang [11] in 2010 and J. Ye [8] in 2011 have been applied in solving Multi-Criteria Decision-Making (MCDM) problems in an intuitionistic fuzzy environment. In the subsequent years, the known methods of solving MCDM problems were approached in an intuitionistic fuzzy context [17] and were applied in various fields. [21], [27], [29], [6], [5] there are only a few recent examples.

There are cases in which applying the different classification methods for IFN leads to different ranking relationships. A possible explanation for this difference in outcome is the human factor. Thus, the question of how much the human factor influences the value of the ranking function. In other words, how much do the decision factors matter in the decision-making processes with IFN? To address the aforementioned question, in this paper, the proposed classifying function includes a parameter that represents the subjectivity of the decision maker. Moreover, the examples provided illustrate that the attitude of the decision-making factors can sometimes change the hierarchy.

The remainder of this paper is organized as follows. The core concepts of intuitionistic fuzzy sets are provided in Section 2. In Section 3, a new ranking function for trapezoidal intuitionistic fuzzy numbers is defined, starting from Robust's ranking index used in ordering fuzzy numbers, which it is applied for the membership function, as well as for the non-membership function of trapezoidal intuitionistic fuzzy numbers. Section 4 provides examples that illustrate the given method and a study of the proposed ranking method in comparison with other methods. In section 5, given the new classification method of TrIFN, an algorithm for solving Multi-Criteria Decision Making problems in an intuitionistic fuzzy context is proposed. The efficiency of the algoritm is emphasized in section 6 by solving a selection problemm which allows intuitionistic fuzzy modelling. Section 7 provides relevant conclusions.

2 Preliminaries

The core concepts presented in this section are as defined in the current literature, i.e., [2], [7], [8], [18], [17], [25]. Said notions are fundamental for the research that follows.

Definition 1. *Let X denote a universal set. An intuitionistic fuzzy set (IFS) in X is a subset*

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \,; x \in X\},$$

where $\mu_{\tilde{A}} : X \to [0,1]$ and $\nu_{\tilde{A}} : X \to [0,1]$ represent the membership function and the non-membership function of the elements of X to $\tilde{A}$, respectively. $\mu_{\tilde{A}}$ and $\nu_{\tilde{A}}$ satisfy the condition:

$$\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leqslant 1, \ (\forall)x \in X.$$

$\pi_{\tilde{A}} : X \to [0,1],\ \pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$ *is called hesitation margin of* x *in* $\tilde{A}$.

Definition 2. *An intuitionistic fuzzy set of the real line,* $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle ; x \in \mathbb{R}\}$ *is called intuitionistic fuzzy number (IFN) if the following conditions hold true:*

(i) $\tilde{A}$ *is IF - normal, i.e., there exists two elements* $x_0, x_1 \in \mathbb{R}$ *such that* $\mu_{\tilde{A}}(x_0) = 1$ *and* $\nu_{\tilde{A}}(x_1) = 1$;

(ii) $\tilde{A}$ *is IF - convex, i.e.,* $\mu_{\tilde{A}}(x)$ *is fuzzy convex and* $\nu_{\tilde{A}}(x)$ *is concave;*

(iii) $\mu_{\tilde{A}}(x)$ *is upper semicontinuous and* $\nu_{\tilde{A}}(x)$ *is lower semicontinuous;*

(iv) The support of $\tilde{A}$ *is bounded.*

The definition (2) implies that for any $\tilde{A} \in IFN$, there exist $a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4 \in \mathbb{R}$ with $a'_1 \leqslant a_1 \leqslant a'_2 \leqslant a_2 \leqslant a_3 \leqslant a'_3 \leqslant a_4 \leqslant a'_4$ and the functions $f_{\tilde{A}}, g_{\tilde{A}}, h_{\tilde{A}}, k_{\tilde{A}} : \mathbb{R} \to [0,1]$, where $f_{\tilde{A}}, k_{\tilde{A}}$ are nondecreasing functions and $g_{\tilde{A}}, h_{\tilde{A}}$ are nonincreasing functions, such that the membership and non-membership function of $\tilde{A}$ can be written as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & if \quad x < a_1 \\ f_{\tilde{A}} & if \quad a_1 \leqslant x < a_2 \\ 1 & if \quad a_2 \leqslant x < a_3; \\ g_{\tilde{A}} & if \quad a_3 \leqslant x < a_4 \\ 0 & if \quad a_4 \leqslant x \end{cases} \quad \nu_{\tilde{A}}(x) = \begin{cases} 0 & if \quad x < a'_1 \\ h_{\tilde{A}} & if \quad a'_1 \leqslant x < a'_2 \\ 1 & if \quad a'_2 \leqslant x < a'_3 \\ k_{\tilde{A}} & if \quad a'_3 \leqslant x < a'_4 \\ 0 & if \quad a'_4 \leqslant x. \end{cases}$$

Definition 3. *An intuitionistic fuzzy number having membership and non-membership functions of form:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & if \quad a_1 \leqslant x < a_2 \\ 1 & if \quad a_2 \leqslant x < a_3; \\ \frac{a_4-x}{a_4-a_3} & if \quad a_3 \leqslant x < a_4 \\ 0 & oterwise \end{cases} \quad \nu_{\tilde{A}}(x) = \begin{cases} \frac{a'_2-x}{a'_2-a'_1} & if \quad a'_1 \leqslant x < a'_2 \\ 0 & if \quad a'_2 \leqslant x < a'_3; \\ \frac{x-a'_3}{a'_4-a'_3} & if \quad a'_3 \leqslant x < a'_4 \\ 1 & oterwise, \end{cases}$$

where $a'_1 \leqslant a_1 \leqslant a'_2 \leqslant a_2 \leqslant a_3 \leqslant a'_3 \leqslant a_4 \leqslant a'_4$, *denoted by* $\tilde{A} = \{(a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)\}$ *is called trapezoidal intuitionistic fuzzy number (TrIFN).*

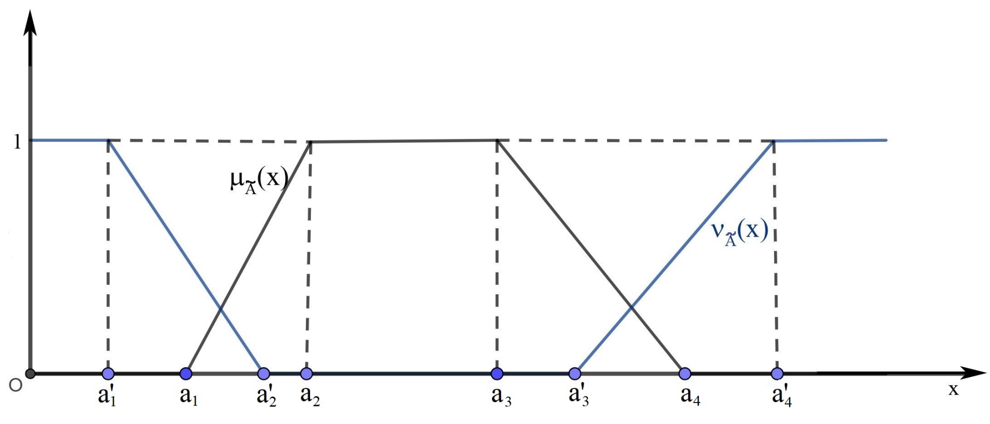

Figure 1: TrIFN

Remark 4. *A special case of the (3) is* $\tilde{A} = \{(a_1, a_2, a_4); (a'_1, a'_2, a'_4)\}$ *which is called triangular intuitionistic fuzzy number (TIFN) and is obtained when* $a_2 = a_3$ *and* $a'_2 = a'_3$.

Definition 5. *Let* $\tilde{A} = \{(a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)\}$ *and* $\tilde{B} = \{(b_1, b_2, b_3, b_4);\ (b'_1, b'_2, b'_3, b'_4)\}$ *be two trapezoidal intuitionistic fuzzy numbers, then the arithmetic operations of addition of* $\tilde{A}$ *and* $\tilde{B}$ *and scalar multiplication* $k\tilde{A}$, $k \in \mathbb{R}$ *are defined as follows:*

(i) $\tilde{A} + \tilde{B} = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); (a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3, a'_4 + b'_4)\}$

(ii) $k\tilde{A} = \begin{cases} \{(ka_1, ka_2, ka_3, ka_4); (ka'_1, ka'_2, ka'_3, ka'_4)\}, \; if \; k > 0 \\ \{(ka_4, ka_3, ka_2, ka_1); (ka'_4, ka'_3, ka'_2, ka'_1)\}, \; if \; k < 0. \end{cases}$

Definition 6. *Let $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle ; x \in \mathbb{R}\}$ an IFN. Then α-cut of $\tilde{A}$, $\alpha \in (0, 1]$ is a set defined as:*

$$\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geqslant \alpha, \nu_{\tilde{A}}(x) \leqslant 1 - \alpha\}.$$

Thus, α-cut of $\tilde{A}$ ultimately consists of two classic α-cuts:

$$\begin{aligned} \left(\tilde{A}^+\right)_\alpha &= \{x \in \mathbb{R}/\mu_{\tilde{A}}(x) \geqslant \alpha\} \\ \left(\tilde{A}^-\right)_\alpha &= \{x \in \mathbb{R}/\nu_{\tilde{A}}(x) \leqslant 1 - \alpha\}. \end{aligned}$$

$\left(\tilde{A}^+\right)_\alpha$ and $\left(\tilde{A}^-\right)_\alpha$ are closed intervals, as given below:

$$\begin{aligned} \left(\tilde{A}^+\right)_\alpha &= \left[\tilde{A}^+_L(\alpha), \tilde{A}^+_U(\alpha)\right] \\ \left(\tilde{A}^-\right)_\alpha &= \left[\tilde{A}^-_L(\alpha), \tilde{A}^-_U(\alpha)\right] \end{aligned}$$

where,

$$\begin{aligned} \tilde{A}^+_L(\alpha) &= \inf\{x \in \mathbb{R}/\mu_{\tilde{A}}(x) \geqslant \alpha\} \\ \tilde{A}^+_U(\alpha) &= \sup\{x \in \mathbb{R}/\mu_{\tilde{A}}(x) \geqslant \alpha\} \\ \tilde{A}^-_L(\alpha) &= \inf\{x \in \mathbb{R}/\nu_{\tilde{A}}(x) \leqslant 1 - \alpha\} \\ \tilde{A}^-_U(\alpha) &= \sup\{x \in \mathbb{R}/\nu_{\tilde{A}}(x) \leqslant 1 - \alpha\}. \end{aligned}$$

Generally, for an IFN $\tilde{A}$, for which $f_{\tilde{A}}, g_{\tilde{A}}, h_{\tilde{A}}, k_{\tilde{A}}$ from (2) are strictly monotone functions, $\tilde{A}^+_L(\alpha)$, $\tilde{A}^+_U(\alpha)$, $\tilde{A}^-_L(\alpha)$, $\tilde{A}^-_L(\alpha)$ can be computed as follows:

$$\tilde{A}^+_L(\alpha) = f^{-1}_{\tilde{A}}(\alpha); \; \tilde{A}^+_U(\alpha) = g^{-1}_{\tilde{A}}(\alpha); \; \tilde{A}^-_L(\alpha) = h^{-1}_{\tilde{A}}(\alpha); \tilde{A}^-_U(\alpha) = k^{-1}_{\tilde{A}}(\alpha).$$

If $\tilde{A}$ is a TrIFN, $\tilde{A} = \{(a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)\}$ with $a'_1 \leqslant a_1 \leqslant a'_2 \leqslant a_2 \leqslant a_3 \leqslant a'_3 \leqslant a_4 \leqslant a'_4$, then

$$\begin{aligned} \tilde{A}^+_L(\alpha) &= a_1 + (a_2 - a_1)\alpha \\ \tilde{A}^+_U(\alpha) &= a_4 - (a_4 - a_3)\alpha \\ \tilde{A}^-_L(\alpha) &= a'_1 + (a'_2 - a'_1)\alpha \\ \tilde{A}^-_U(\alpha) &= a'_4 - (a'_4 - a'_3)\alpha. \end{aligned}$$

It is often the case that for comparing fuzzy numbers, ranking functions are required. Robust's ranking index has proven to be, in the case of fuzzy numbers, a method of defuzzification which leads to results which are in accordance with human intuition [14], [20].

Definition 7. *Let $\tilde{a}$ be a fuzzy number. The Robust's Ranking index is defined as follows:*

$$\mathrm{R}(\tilde{a}) = \int_0^1 0.5(a^l_\alpha, a^u_\alpha)d\alpha,$$

where (a^l_α, a^u_α) is α-level cut of the fuzzy number $\tilde{a}$.

Remark 8. $\mathrm{R}(\tilde{a})$ *is a function which satisfies compensation, linearity, and additivity properties.*

For some fuzzy multi-criteria decision making (MCDM) problem, assume that there are m alternatives $A = \{A_1, A_2, ..., A_m\}$, n decision criteria $C = \{C_1, C_2, \ldots, C_n\}$ and the corresponding weight coefficients are $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, $\sum_{j=1}^{n} \omega_j = 1$ is the weight vector.

If the value of alternative $A_i, i = \overline{1,m}$ on the criteria $C_j, j = \overline{1,n}$ is intuitionistic trapezoidal fuzzy number $\widetilde{A}_{ij}$, then the MCDM problem can be expressed through a decision matrix:

$$D_{ij} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} & \ldots & \widetilde{A}_{1n} \\ \widetilde{A}_{21} & \widetilde{A}_{22} & \ldots & \widetilde{A}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{A}_{m1} & \widetilde{A}_{m2} & \ldots & \widetilde{A}_{mn} \end{bmatrix}.$$

Definition 9. *Let $\tilde{A}_j$, $j = \overline{1,n}$ be a set of intuitionistic trapezoidal fuzzy numbers, and $f : \Omega^n \to \Omega$:*

$$f_\omega(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \sum_{j=1}^{n} \omega_j \tilde{A}_j, \tag{1}$$

where Ω is the set of all intuitionistic trapezoidal fuzzy numbers, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weight vector of $\tilde{A}j$, $j = \overline{1,n}$, $\sum_{j=1}^{n} \omega_j = 1$ then, f_ω is called the weighted arithmetic average operator on intuitionistic trapezoidal fuzzy numbers.

3 A new ranking method for TrIFN

Let $\tilde{A}$ be a TrIFN, $\tilde{A} = \{(a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)\}$, $a'_1 \leqslant a_1 \leqslant a'_2 \leqslant a_2 \leqslant a_3 \leqslant a'_3 \leqslant a_4 \leqslant a'_4$. According to the definition (7), in the case of IFNs, Robust's ranking index can be defined for the membership function, as well as for the non-membership function as given below:

$$\mathrm{R}(\tilde{A}^+)_\alpha = \int_0^1 0.5 \left(\tilde{A}_L^+, \tilde{A}_U^+\right) d\alpha \tag{2}$$

$$\mathrm{R}(\tilde{A}^-)_\alpha = \int_0^1 0.5 \left(\tilde{A}_L^-, \tilde{A}_U^-\right) d\alpha, \tag{3}$$

where $(\tilde{A}_L^+, \tilde{A}_U^+) = (\tilde{A}^+)_\alpha$ and $(\tilde{A}_L^-, \tilde{A}_U^-) = (\tilde{A}^-)_\alpha$ are α- cuts of $\tilde{A}$.

Making use of Robust's ranking index, $\mathrm{R}(\tilde{A}^+)_\alpha$ and $\mathrm{R}(\tilde{A}^-)_\alpha$ can be defined as a new ranking function.

Definition 10. *Let $\tilde{A}$ be a TrIFN, then define a ranking function as follows:*

(i) $\mathrm{R}(\tilde{A}, \lambda) = \lambda \cdot \mathrm{R}(\tilde{A}^+)_\alpha + (1-\lambda) \cdot \mathrm{R}(\tilde{A}^-)_\alpha$ *if* $a_1 + a_2 + a_3 + a_4 \geqslant a'_1 + a'_2 + a'_3 + a'_4$;

(ii) $\mathrm{R}(\tilde{A}, \lambda) = (1-\lambda) \cdot \mathrm{R}(\tilde{A}^+)_\alpha + \lambda \cdot \mathrm{R}(\tilde{A}^-)_\alpha$ *if* $a_1 + a_2 + a_3 + a_4 < a'_1 + a'_2 + a'_3 + a'_4$,

where $\lambda \in [0, 1]$ is a parameter with a value that represents the subjective attitude of the decision marker (DM) as given below:

$$\begin{cases} \lambda \in \left[0, \frac{1}{2}\right) & \textit{for a pessimistic attitude;} \\ \frac{1}{2} & \textit{for a neutral attitude;} \\ \lambda \in \left(\frac{1}{2}, 1\right] & \textit{for an optimistic attitude.} \end{cases}$$

Remark 11. *In the case of an indifferent attitude of the DM, when $\lambda = \frac{1}{2}$, the ranking function becomes:*

$$\mathrm{R}(\tilde{A}) = \frac{\mathrm{R}(\tilde{A}^+)_\alpha + \mathrm{R}(\tilde{A}^-)_\alpha}{2}. \tag{4}$$

As $\tilde{A}$ is a TrIFN, $\tilde{A} = \{(a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)\}$ with $a'_1 \leqslant a_1 \leqslant a'_2 \leqslant a_2 \leqslant a_3 \leqslant a'_3 \leqslant a_4 \leqslant a'_4$, as defined in (2) and (3), the following is obtained:

$$\mathrm{R}(\tilde{A}^+)_\alpha = \int_0^1 0.5 \cdot [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha] d\alpha =$$
$$= \int_0^1 \frac{a_1 + (a_2 - a_1)\alpha + a_4 - (a_4 - a_3)\alpha}{2} d\alpha = \frac{a_1 + a_2 + a_3 + a_4}{4}$$

and similarly,

$$\mathrm{R}(\tilde{A}^-)_\alpha = \frac{a'_1 + a'_2 + a'_3 + a'_4}{4}.$$

In the special case when the subjectivity parameter is $\lambda = \frac{1}{2}$, then

$$\mathrm{R}(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4 + a'_1 + a'_2 + a'_3 + a'_4}{8}, \tag{5}$$

which is the same as the *expected value* utilised by J. Ye in [8] for classifying TrINFs.

Remark 12. *If $\tilde{A} = \{(a_1, a_2, a_3); (a'_1, a'_2, a'_3)\}$ is a TIFN, then*

$$\mathrm{R}(\tilde{A}^+)_\alpha = \frac{a_1 + 2a_2 + a_3}{4} \quad \textit{and} \quad \mathrm{R}(\tilde{A}^-)_\alpha = \frac{a'_1 + 2a'_2 + a'_3}{4}$$

Moreover, if $\lambda = \frac{1}{2}$, then

$$\mathrm{R}(\tilde{A}) = \frac{a_1 + 2a_2 + a_3 + a'_1 + 2a'_2 + a'_3}{8}. \tag{6}$$

Remark 13. *In the special case when A is a trapezoidal fuzzy number, it can be written as an intuitionistic fuzzy trapezoidal number $\tilde{A} = \{(a_1, a_2, a_3, a_4); (a_1, a_2, a_3, a_4)\}$ and*

$$\mathrm{R}(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4}{4}.$$

Definition 14. *Let $\tilde{A} = \{(a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)\}$ and $\tilde{B} = \{(b_1, b_2, b_3, b_4); (b'_1, b'_2, b'_3, b'_4)\}$ be two TrIFN. For $\lambda \in [0, 1]$, λ-fixed, a new ranking method can be defined as follows:*

$$\begin{aligned}
\tilde{A} >_\lambda \tilde{B} &\quad \textit{if and only if} \quad \mathrm{R}(\tilde{A}, \lambda) > \mathrm{R}(\tilde{B}, \lambda);\\
\tilde{A} <_\lambda \tilde{B} &\quad \textit{if and only if} \quad \mathrm{R}(\tilde{A}, \lambda) < \mathrm{R}(\tilde{B}, \lambda);\\
\tilde{A} \approx_\lambda \tilde{B} &\quad \textit{if and only if} \quad \mathrm{R}(\tilde{A}, \lambda) = \mathrm{R}(\tilde{B}, \lambda).
\end{aligned}$$

4 Examples and comparative study

The examples provided below illustrate the variation in the ranking function in terms of the parameter λ, where $\lambda \in [0, 1]$ is a parameter with a value that represents the subjective attitude of the decision marker. Moreover, the ranking is determined for some given TrINFs.

Example 15. *Let $\tilde{A} = \{(4, 5, 5.2, 6); (3, 4.8, 5.4, 6.5)\}$ and $\tilde{B} = \{(4, 5, 5.5, 6);\ (4, 5, 5.5, 6.1)\}$.*

Example 16. *Let $\tilde{A} = \{(0.5, 1, 1.5, 2); (0, 0.6, 1.5, 2.5)\}$ and $\tilde{B} = \{(0.5, 1, 1.3, 1.8); (0.2, 0.8, 1.5, 2.5)\}$.*

Example 17. *Let $\tilde{A} = \{(0.2, 0.25, 0.35, 0.4); (0.1, 0.25, 0.35, 0.75)\}$ and $\tilde{B} = \{(0.2, 0.3, 0.4, 0.5); (0.1, 0.3, 0.4, 0.7)\}$.*

Table 1: Ranking of INF's for values of λ

λ	$R(\tilde{A})$	$R(\tilde{B})$	
$\lambda = 0$	$R(\tilde{A}) = 4.925$	$R(\tilde{B}) = 5.125$	
$\lambda = 0.1$	$R(\tilde{A}) = 4.9395$	$R(\tilde{B}) = 5.1275$	
$\lambda = 0.2$	$R(\tilde{A}) = 4.95$	$R(\tilde{B}) = 5.13$	
$\lambda = 0.3$	$R(\tilde{A}) = 4.9625$	$R(\tilde{B}) = 5.1325$	
$\lambda = 0.4$	$R(\tilde{A}) = 4.975$	$R(\tilde{B}) = 5.135$	
$\lambda = 0.5$	$R(\tilde{A}) = 4.9875$	$R(\tilde{B}) = 5.1375$	$\tilde{A} < \tilde{B}$
$\lambda = 0.6$	$R(\tilde{A}) = 5$	$R(\tilde{B}) = 5.14$	
$\lambda = 0.7$	$R(\tilde{A}) = 5.0125$	$R(\tilde{B}) = 5.1425$	
$\lambda = 0.8$	$R(\tilde{A}) = 5.025$	$R(\tilde{B}) = 5.145$	
$\lambda = 0.9$	$R(\tilde{A}) = 5.0375$	$R(\tilde{B}) = 5.1475$	
$\lambda = 1$	$R(\tilde{A}) = 5.05$	$R(\tilde{B}) = 5.15$	

Table 2: Ranking of INF's for values of λ

λ	$R(\tilde{A})$	$R(\tilde{B})$	
$\lambda = 0$	$R(\tilde{A}) = 1.15$	$R(\tilde{B}) = 1.15$	
$\lambda = 0.5$	$R(\tilde{A}) = 1.2$	$R(\tilde{B}) = 1.2$	$\tilde{A} \approx \tilde{B}$
$\lambda = 1$	$R(\tilde{A}) = 1.25$	$R(\tilde{B}) = 1.25$	

Table 3: Ranking of INF's for values of λ

λ	$R(\tilde{A})$	$R(\tilde{B})$	
$\lambda = 0.2$	$R(\tilde{A}) = 0.3125$	$R(\tilde{B}) = 0.355$	
$\lambda = 0.5$	$R(\tilde{A}) = 0.3313$	$R(\tilde{B}) = 0.3625$	$\tilde{A} < \tilde{B}$
$\lambda = 0.8$	$R(\tilde{A}) = 0.35$	$R(\tilde{B}) = 0.37$	

Table 4: Ranking of INF's for values of λ

λ	$R(\tilde{A})$	$R(\tilde{B})$	
$\lambda = 0.2$	$R(\tilde{A}) = 0.35$	$R(\tilde{B}) = 0.25$	
$\lambda = 0.5$	$R(\tilde{A}) = 0.35$	$R(\tilde{B}) = 0.25$	$\tilde{A} > \tilde{B}$
$\lambda = 0.9$	$R(\tilde{A}) = 0.35$	$R(\tilde{B}) = 0.25$	

Table 5: Ranking of INF's for values of λ

λ	$R(\tilde{A})$	$R(\tilde{B})$	
$\lambda = 0.1$	$R(\tilde{A}) = 4.5743$	$R(\tilde{B}) = 4.5754$	
$\lambda = 0.5$	$R(\tilde{A}) = 4.5813$	$R(\tilde{B}) = 4.5834$	$\tilde{A} < \tilde{B}$
$\lambda = 1$	$R(\tilde{A}) = 4.59$	$R(\tilde{B}) = 4.5933$	

Example 18. *Let* $\tilde{A} = \{(0.2, 0.3, 0.4, 0.5); (0.1, 0.3, 0.4, 0.6)\}$ *and* $\tilde{B} = \{(0.1, 0.2, 0.3, 0.4); (0, 0.2, 0.3, 0.5)\}$ *(the numerical example is taken from [13]).*

Example 19. *Let* $\tilde{A} = \{(4.02, 4.72, 4.83); (4, 4.72, 4.92)\}$ *and* $\tilde{B} = \{(4.021, 4.721, 4.831); (4.01, 4.721, 4.921)\}$ *(the numerical example is taken from [1]).*

Table 6: Ranking of IFN's for values of λ

λ	$R(\tilde{A})$	$R(\tilde{B})$	
$\lambda = 0$	$R(\tilde{A}) = 6.8625$	$R(\tilde{B}) = 7.0625$	
$\lambda = 0.1$	$R(\tilde{A}) = 6.9075$	$R(\tilde{B}) = 7.0713$	
$\lambda = 0.2$	$R(\tilde{A}) = 6.9525$	$R(\tilde{B}) = 7.08$	$\tilde{A} < \tilde{B}$
$\lambda = 0.3$	$R(\tilde{A}) = 6.9975$	$R(\tilde{B}) = 7.0888$	
$\lambda = 0.4$	$R(\tilde{A}) = 7.0425$	$R(\tilde{B}) = 7.0975$	
$\lambda = 0.5$	$R(\tilde{A}) = 7.0875$	$R(\tilde{B}) = 7.1063$	
$\lambda = 0.6$	$R(\tilde{A}) = 7.1325$	$R(\tilde{B}) = 7.115$	
$\lambda = 0.7$	$R(\tilde{A}) = 7.1775$	$R(\tilde{B}) = 7.1238$	$\tilde{A} > \tilde{B}$
$\lambda = 0.8$	$R(\tilde{A}) = 7.2225$	$R(\tilde{B}) = 7.1325$	
$\lambda = 0.9$	$R(\tilde{A}) = 7.2675$	$R(\tilde{B}) = 7.1413$	
$\lambda = 1$	$R(\tilde{A}) = 7.3125$	$R(\tilde{B}) = 7.15$	

Example 20. *Let* $\tilde{A} = \{(5.25, 7, 7.05, 8.15); (4.85, 6.95, 7.5, 9.95)\}$ *and* $\tilde{B} = \{(6.35,\ 7.15, 7.2, 7.9); (5, 7, 7.25, 9)\}$.

This example proves that the ranking relationship of two intuitionistic fuzzy numbers can, in some cases, change according to the attitude of the decision-making factors.

Example 21. *Let* $\tilde{A}_1 = \{(2.95, 5.1, 6.65); (2.25, 5.1, 9.65)\}$, $\tilde{A}_2 = \{(3.95, 5.15, 6.2);\ (2.325, 5.15, 8.75)\}$ *and* $\tilde{A}_3 = \{(3.85, 5.15, 6.4); (2.35, 5.15, 9.05)\}$.

Table 7: Ranking of IFN's for values of λ

λ	$R(\tilde{A}_1)$	$R(\tilde{A}_2)$	$R(\tilde{A}_3)$	
$\lambda = 0$	$R(\tilde{A}_1) = 4.9525$	$R(\tilde{A}_2) = 5.1125$	$R(\tilde{A}_3) = 5.1375$	
$\lambda = 0.1$	$R(\tilde{A}_1) = 5.0096$	$R(\tilde{A}_2) = 5.1356$	$R(\tilde{A}_3) = 5.1663$	
$\lambda = 0.2$	$R(\tilde{A}_1) = 5.067$	$R(\tilde{A}_2) = 5.1588$	$R(\tilde{A}_3) = 5.195$	$\tilde{A}_3 > \tilde{A}_2 > \tilde{A}_1$
$\lambda = 0.3$	$R(\tilde{A}_1) = 5.1243$	$R(\tilde{A}_2) = 5.1819$	$R(\tilde{A}_3) = 5.2238$	
$\lambda = 0.4$	$R(\tilde{A}_1) = 5.1815$	$R(\tilde{A}_2) = 5.205$	$R(\tilde{A}_3) = 5.2525$	
$\lambda = 0.5$	$R(\tilde{A}_1) = 5.2388$	$R(\tilde{A}_2) = 5.2281$	$R(\tilde{A}_3) = 5.2813$	$\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2$
$\lambda = 0.6$	$R(\tilde{A}_1) = 5.296$	$R(\tilde{A}_2) = 5.2513$	$R(\tilde{A}_3) = 5.31$	
$\lambda = 0.7$	$R(\tilde{A}_1) = 5.3533$	$R(\tilde{A}_2) = 5.2744$	$R(\tilde{A}_3) = 5.3388$	
$\lambda = 0.8$	$R(\tilde{A}_1) = 5.4105$	$R(\tilde{A}_2) = 5.2975$	$R(\tilde{A}_3) = 5.3675$	$\tilde{A}_1 > \tilde{A}_3 > \tilde{A}_2$
$\lambda = 0.9$	$R(\tilde{A}_1) = 5.4678$	$R(\tilde{A}_2) = 5.3206$	$R(\tilde{A}_3) = 5.3963$	
$\lambda = 1$	$R(\tilde{A}_1) = 5.525$	$R(\tilde{A}_2) = 5.3438$	$R(\tilde{A}_3) = 5.425$	

As it can be seen in this example, in the case of several TrIFNs, the subjective attitude of the decision-makers plays an essential role in determining the ranking.

The aforementioned examples are used in a comparative study of the given research method and several other IFN ranking methods already existent in the current literature. The study is summarized below:

Table 8 shows that the proposed method provides similar results to the other methods when the result is the same for all methods (the examples 15 - 19). But when the other methods give contradictory results, the proposed method includes and explains these results, through the influence of the subjective attitude of the decision-maker on the ranking function (the examples 20 and 21).

Table 8: Comparison of proposed method with other ranking methods for IFNs

$\tilde{A}, \tilde{B}$ from example	Ye's method [8]	Rezvani's method [22]	Roseline and Amirtharaj's method [23]	Bharati's method [3]	Mohan's method [13]	Proposed method
15	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$
16	$\tilde{A} \approx \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} \approx \tilde{B}$	$\tilde{A} \approx \tilde{B}$
17	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$
18	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$
19	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$
20	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$ $\tilde{A} > \tilde{B}$
21	$\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2$	$\tilde{A}_3 > \tilde{A}_2 > \tilde{A}_1$	$\tilde{A}_3 > \tilde{A}_2 > \tilde{A}_1$	$\tilde{A}_1 > \tilde{A}_3 > \tilde{A}_2$	$\tilde{A}_3 > \tilde{A}_2 > \tilde{A}_1$	$\tilde{A}_3 > \tilde{A}_2 > \tilde{A}_1$ $\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2$ $\tilde{A}_1 > \tilde{A}_3 > \tilde{A}_2$

5 Algorithm for solving MCDM based on proposed ranking method

In this section, the aforementioned TrIFN classification method will be applied to solve MCDM problems in which evaluating the alternatives and the weights of the criteria are expressed with the help of TrIFN.

We consider an MCDM problem in which decision makers are asked to make judgements and assess through linguistic terms a series of alternatives in relation to a plethora of pre-determined criteria. The importance of the criteria can differ, but can also be imprecise, i.e., their weights are also expressed with the help of linguistic terms. It is assumed that the scale of TrIFN values corresponding to linguistic terms is known.

The notations used to describe these problems are: $E = \{E_1, E_2, \ldots, E_p\}$ - set of experts taking part in the decision making process; $A = \{A_1, A_2, \ldots, A_m\}$ - set of possible alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ - set of criteria used in evaluating the alternatives.

Each expert E_k, $k = \overline{1,p}$ evaluates all alternatives A_i, $i = \overline{1,m}$ in relation to each criterion C_j, $j = \overline{1,n}$ using the linguistic terms which are then converted in TrIFN $\tilde{A}_{ij} = \{(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}); (a'_{ij1}, a'_{ij2}, a'_{ij3}, a'_{ij4})\}$. The weights of the criteria C_j, $j = \overline{1,n}$ is expressed through the TrIFN $\tilde{C}_j = \{(c_{j1}, c_{j2}, c_{j3}, c_{j4}); (c'_{j1}, c'_{j2}, c'_{j3}, c'_{j4})\}$, and will consider the subjective assessments of the decision making factors, with respect the condition of normalization.

In this context, the proposed algorithm for solving MCDM problems with TrIFN it is summarized as follows:

Step 1 Identify the atitude of the experts regarding the evaluation criteria, and for each expert E_k, $k = \overline{1,p}$ the normalized weigth for each critria is computed C_j, $j = \overline{1,n}$ using the formula:

$$\omega_{kj} = \frac{R_k(C_j)}{\sum\limits_{j=1}^{n} R_k(C_j)}, \tag{7}$$

where $R_k(C_j)$ is computed according to the definition 10.

Step 2 For each expert E_k, $k = \overline{1,p}$ the decision matrix $(D_{ij})_{m\times n}$ is filled in which leads to TrIFN $\widetilde{W}_{ki}$ which expresses the opinion of the expert k in relation to the alternative i:

$$\widetilde{W}_{ki} = \left\{ \left(\sum_{j=1}^{n} \omega_{kj} a_{ij1}, \sum_{j=1}^{n} \omega_{kj} a_{ij2}, \sum_{j=1}^{n} \omega_{kj} a_{ij3}, \sum_{j=1}^{n} \omega_{kj} a_{ij4} \right); \left(\sum_{j=1}^{n} \omega_{kj} a'_{ij1}, \sum_{j=1}^{n} \omega_{kj} a'_{ij2}, \sum_{j=1}^{n} \omega_{kj} a'_{ij3}, \sum_{j=1}^{n} \omega_{kj} a'_{ij4} \right) \right\}. \tag{8}$$

Denote $\widetilde{W}_{ki} = \{(w_{ki1}, w_{ki2}, w_{ki3}, w_{ki4}); (w'_{ki1}, w'_{ki2}, w'_{ki3}, w'_{ki4})\}$.

Step 3 The decision matrix $(D_{ki})_{p\times m}$ is filled in, which incorporates the opinions of p experts in relation to the m alternatives. Given that the evaluations given by the experts have the same weight in making the final decision, the TrIFN which represents the evaluation of the alternative A_i can be determined:

$$\tilde{A}_i = \left\{ \left(\frac{\sum_{k=1}^{p} w_{ki1}}{p}, \frac{\sum_{k=1}^{p} w_{ki2}}{p}, \frac{\sum_{k=1}^{p} w_{ki3}}{p}, \frac{\sum_{k=1}^{p} w_{ki4}}{p} \right); \left(\frac{\sum_{k=1}^{p} w'_{ki1}}{p}, \frac{\sum_{k=1}^{p} w'_{ki2}}{p}, \frac{\sum_{k=1}^{p} w'_{ki3}}{p}, \frac{\sum_{k=1}^{p} w'_{ki4}}{p} \right) \right\}. \tag{9}$$

Denote $\tilde{A}_i = \{(a_{i1}, a_{i2}, a_{i3}, a_{i4}); (a'_{i1}, a'_{i2}, a'_{i3}, a'_{i4})\}$.

Step 4 Applying the classification method proposed (10), and taking into consideration different levels of subjectivity in computing $R(\tilde{A}_i), i = \overline{1, m}$, the best alternative is determined.

6 An application to an selection problem

To exemlify the algorithm proposed in section 5, consider the following MCDM problem:

A company plans to hire a new department manager and are considering three candidates for the job in question (A_1, A_2, A_3), which will be evaluated independently by the three experts. As part of the application process, four indicators are assessed, i.e., professional know-how C_1, leadership ability C_2, moral quality C_3 and communication skills C_4 with the help of linguistic terms converted to TrINF, according to the scale:

Table 9: Lingvistic scale and its corresponding TrIFN

Linguistic term	Trapezoidal intuitionistic fuzzy number
Low (L)	$\{(0, 1, 1.5, 2.5); (0, 0.5, 2, 3)\}$
Fairly low (FL)	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$
Medium (M)	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$
Fairly high (FH)	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$
High (H)	$\{(8, 8.5, 9.5, 10); (7, 8.5, 9.5, 10)\}$

To these indicators are then assigned the following weights: M, FH, FL, L respectively. The purpose of the decision factor is to determine a hierarchy of the three candidates based on expert evaluations.

The evaluation made by the three experts of each candidate relative to the four indicators is provided in the tables below:

Table 10: Alternative evaluations in linguistic terms of experts

	Expertul E_1				Expertul E_2				Expertul E_3			
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
A_1	M	FH	FL	L	FL	H	M	L	M	FL	FL	FH
A_2	FH	FL	M	M	FH	M	FL	M	M	FL	FH	FH
A_3	FH	FL	FL	FH	FH	M	M	H	FH	FL	FL	M

Step 1 First, the experts express their subjective atitiude in relation to the weights M, FH, FL and L of the four indicators C_j, $j = \overline{1, 4}$ by fixing the subjective parameter. In the Table 11 are

summarized the values of these parameters λ_{kj}, $\lambda_{kj} \in [0,1]$, $k = \overline{1,3}$, $j = \overline{1,4}$, the weights of the indicators personalized for each expert are computed according to the new classification method of intuitionistic fuzzy numbers, as well as the normalized weights, ω_{kj}, $k = \overline{1,3}$, $j = \overline{1,4}$, computed according to the 7:

Table 11: The normalized weights of the criterias for each expert

	Parameters				Weights				Normalized weights			
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
E_1	0.2	0.8	0.7	0.5	4.9	6.975	3.5825	1.3125	0.2921	0.4158	0.2139	0.0782
E_2	0.5	0.5	0.5	0.5	4.9375	6.9375	3.5625	1.3125	0.2948	0.4142	0.2127	0.0783
E_3	0.6	0.9	0.3	0.7	4.95	6.9875	3.5375	1.3375	0.2944	0.4156	0.2104	0.0796

Step 2 In Tables 12, 13 and 14 the decision matrices corresponding to the three experts are presented. By applying the formula 8, the evaluations of the alternatives expressed through TrIFN are provided and can be found in Tables 15, 16 and 17.

Table 12: The decision making matrix for the E_1 expert

	C_1	C_2	C_3	C_4
A_1	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(0, 1, 1.5, 2.5); (0, 0.5, 2, 3)\}$
A_2	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$
A_3	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$

Table 13: The decision making matrix for the E_2 expert

	C_1	C_2	C_3	C_4
A_1	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(8, 8.5, 9.5, 10); (7, 8.5, 9.5, 10)\}$	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$	$\{(0, 1, 1.5, 2.5); (0, 0.5, 2, 3)\}$
A_2	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$
A_3	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$	$\{(8, 8.5, 9.5, 10); (7, 8.5, 9.5, 10)\}$

Table 14: The decision making matrix for the E_3 expert

	C_1	C_2	C_3	C_4
A_1	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$
A_2	$\{((3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$
A_3	$\{(5.5, 6.5, 7, 8.5); (5, 6, 8, 9)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(1.5, 3, 4.5, 5.5); (1, 2, 5, 6)\}$	$\{(3.5, 4.5, 5.5, 6.5); (2.5, 4, 6, 7)\}$

Table 15: Evaluations for alternative A_1

	A_{k1}
E_1	$\tilde{A}_{11} = \{(3.6301, 4.7371, 5.597, 6.8049); (3.0232, 4.1301, 6.3049, 7.3049)\}$
E_2	$\tilde{A}_{21} = \{(4.5003, 5.4406, 6.5488, 7.3417); (3.726, 5.0003, 6.8417, 7.6346)\}$
E_3	$\tilde{A}_{31} = \{(2.4072, 3.7202, 4.9934, 6.0332); (1.76, 2.9072, 5.5332, 6.5332)\}$

Table 16: Evaluations for alternative A_2

	A_{k2}
E_1	$\tilde{A}_{12} = \{(3.2526, 4.4605, 5.5236, 6.6684); (2.6066, 3.7526, 6.1684, 7.1684)\}$
E_2	$\tilde{A}_{22} = \{(3.6642, 4.7706, 5.7295, 6.8769); (2.918, 4.1642, 6.3769, 7.3769)\}$
E_3	$\tilde{A}_{32} = \{(3.2488, 4.4566, 5.5194, 6.6644); (2.6016, 3.7488, 6.1644, 7.1644)\}$

Step 3 Given the evaluations of the three experts, the TrIFN $\tilde{A}_i, i = \overline{1,m}$ are computed by applying

Table 17: Evaluations for alternative A_3

	A_{k3}
E_1	$\tilde{A}_{13} = \{(2.9812, 4.2961, 5.4258, 6.6109); (2.4812, 3.4812, 6.1109, 7.1109)\}$
E_2	$\tilde{A}_{23} = \{(4.442, 5.4028, 6.2554, 7.3637); (3.5894, 4.942, 6.8637, 7.8245)\}$
E_3	$\tilde{A}_{33} = \{(2.8368, 4.1498, 5.3156, 6.4628); (2.297, 3.3368, 5.9628, 6.9628)\}$

the formula 9, and express the performance of the alternative A_i:

$$\begin{aligned}
\tilde{A}_1 &= \{(3.5125, 4.6326, 5.7131, 6.7266); (2.8364, 4.025, 6.2266, 7.1576)\} \\
\tilde{A}_2 &= \{(3.3885, 4.5626, 5.5904, 6.7366); (2.7087, 3.8886, 6.2366, 7.2366)\} \\
\tilde{A}_3 &= \{(3.42, 4.6162, 5.6656, 6.8125); (2.7892, 3.92, 6.3125, 7.2994)\}.
\end{aligned}$$

Step 4 According to the new classification method of intuitionistic fuzzy numbers, and considering a neutral attitude in making the final decision ($\lambda = 0.5$), $\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2$ is obtained. Therefore, in the case of a neutral attitude of the decision factor, the best candidate is A_3.

When different values of the λ coefficient are considered, it can be seen that the ranking changes and the candidate A_1 can reach the first position in the hierarchy, as shown in Table 18.

Table 18: Ranking of candidates for different values of λ

$\lambda = 0$	$R(\tilde{A}_1) = 5.05828$	$R(\tilde{A}_2) = 5.0176$	$R(\tilde{A}_3) = 5.0803$	
$\lambda = 0.1$	$R(\tilde{A}_1) = 5.0671$	$R(\tilde{A}_2) = 5.02282$	$R(\tilde{A}_3) = 5.0851$	
$\lambda = 0.2$	$R(\tilde{A}_1) = 5.0759$	$R(\tilde{A}_2) = 5.028$	$R(\tilde{A}_3) = 5.0899$	$\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2$
$\lambda = 0.3$	$R(\tilde{A}_1) = 5.0847$	$R(\tilde{A}_2) = 5.0332$	$R(\tilde{A}_3) = 5.0948$	
$\lambda = 0.4$	$R(\tilde{A}_1) = 5.0934$	$R(\tilde{A}_2) = 5.0384$	$R(\tilde{A}_3) = 5.0996$	
$\lambda = 0.5$	$R(\tilde{A}_1) = 5.1022$	$R(\tilde{A}_2) = 5.0436$	$R(\tilde{A}_3) = 5.1044$	
$\lambda = 0.6$	$R(\tilde{A}_1) = 5.111$	$R(\tilde{A}_2) = 5.0488$	$R(\tilde{A}_3) = 5.1093$	
$\lambda = 0.7$	$R(\tilde{A}_1) = 5.1198$	$R(\tilde{A}_2) = 5.054$	$R(\tilde{A}_3) = 5.1141$	
$\lambda = 0.8$	$R(\tilde{A}_1) = 5.1286$	$R(\tilde{A}_2) = 5.0591$	$R(\tilde{A}_3) = 5.1189$	$\tilde{A}_1 > \tilde{A}_3 > \tilde{A}_2$
$\lambda = 0.9$	$R(\tilde{A}_1) = 5.1374$	$R(\tilde{A}_2) = 5.0643$	$R(\tilde{A}_3) = 5.1237$	
$\lambda = 1$	$R(\tilde{A}_1) = 5.1462$	$R(\tilde{A}_2) = 5.0695$	$R(\tilde{A}_3) = 5.1286$	

If in the first step of applying the algorithm, the subjective attitude of the decision makers in relation to the evaluating criteria is not taken into consideration, assuming they have a neutral attitude ($\lambda_{kj} = 0.5$), then in the table presented in the fourth step the following result is obtained $\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2$, except when $\lambda = 1$ and the ranking order changes, $\tilde{A}_1 > \tilde{A}_3 > \tilde{A}_2$. Therefore, in the given problem, the subjective attitude of the decision makers regarding the evaluated indicators can influence the final decision.

Thus, one of the benefits of applying the algorithm consists of the fact that the evaluation process undergone by an expert is truthful. It reflects the evaluation of a candidate's fulfillment of the criteria from his subjective perspective in relation to the importance attributed to the evaluation criteria.

Moreover, it is sufficient to analyse step 4 to observe that the subjective attitude of the decision factors can decisively influence the final ranking. The application of the proposed ranking method provides a clear image of the dimensions of said influence. Therefore, the final decision can be made fully taking into consideration the subjectivity of the decision maker.

7 Conclusion

In this paper, the Robust's ranking index is defined for the the membership function and the non-membership function of TrIFN, and subsequently a ranking function is defined for TrIFN. The method can also be applied in the special case of TIFN. For $\lambda = 0.5$, the same results are obtained as in [8]. It is worth emphasizing that the method uses a parameter which gauges the subjectivity of the decision maker. Thus, the variation of the ranking function value can be determined in relation to the decision maker's level of optimism, compared to the central value ($\lambda = 0.5$). The advantage of the method proposed in this paper is exactly this parameter which determines the degree to which the subjective attitude of the decision-making factors influences the ranking of TrIFN. The aforementioned examples illustrated that their order could be changed significantly.

The paper also proposes an algorithm for solving MCDM fuzzy problems which use the new ranking function of TrINF to stabilise the weight of the criteria separately for each expert, as well as to make the final decision by ranking the alternatives. By applying this algorithm in solving the decision making problems, the results can be easy to interpret and provide a clear perspective on the way the subjective attitude influences the decision.

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Not applicable.

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