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A Modified Uncertainty Measure of Z-numbers

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Abstract

The Z-number is a more adequate construct for describing real-life information. While considering the uncertainty of the information, it also models the partial reliability of the information. It is a combination of probabilistic restriction and possibilistic restriction. In this paper, we modified the uncertainty measurement of the discrete Z-number and proposed the uncertainty measurement of the continuous Z-number. Some numerical examples are used to illustrate the calculation processes and advantages of the proposed method. An application of journey vehicle selection shows the effectiveness of the proposed uncertainty measurement in determining the weights of criteria.

Keywords: Uncertainty, discrete Z-numbers, continuous Z-numbers, fuzzy numbers, decision-making.

1 Instruction

Crisp numbers often cannot describe real-life information, it is insufficient. As the information is often uncertain, there are many mathematical models to handle such uncertain information, probability theory [18], fuzzy sets theory [29], Dempster-Shafer evidence theory [11, 22] and other soft constraints. However, the information also is partially reliable. In the above soft constraints, they default the handled information is completely reliable. Unfortunately, the information in practice is not so.

The Z-number, as a more sufficient formal construct for describing real-life information, models the uncertainty and partial reliability of the information together. A Z-number is an ordered pair of fuzzy numbers, $Z = (A, B)$, where A is a fuzzy constrict on the values of the variable, X, can take, B is also a fuzzy constrict on the reliability value of A. The reliability value is a probability of the event 'X is A'. Thus, a Z-number combines the possibilistic restriction and the probabilistic restriction.

Since the thought of Z-numbers was put forward in 2011 to now, the mathematical system and application of Z-numbers have become more and more complete. In applications of Z-number, The most widespread application areas are decision-making, such as selection problems [2, 10], fault diagnosis [13], pattern recognition [25], risk assessment [1]. The Z-number has been especially useful for describing calculations in natural language. For instance, Ref. [20] analyzed features and challenges of the Z-number approach to Computing With Words (CWW). In Ref. [9], Z-number is used as a tool for CWW and consequently subjective natural language understanding. In the mathematical system of Z-number, the basic computations were summarized in Ref. [8], including addition, subtraction, multiplication, division, square, square root, absolute value, ranking and distance of Z-numbers. On this basis, more advanced arithmetic is proposed, including Z-numbers based Linear Program (Z-LP) [4], Z-numbers function [6], Approximate Reasoning [7], the parametric form [21], the negation operator [17], total utility [14], Z-Differential equations [19], multidimensional Z-numbers [23], Z-number if-then rules [5], soft likelihood function [24], Z-mixture-numbers [27]. Ref. [3, 26] defined the concepts of specificity and horizontal membership functions of Z-numbers. Besides, Li et. al. proposed an uncertainty measure of discrete Z-number [16]. However, we find there is an unintuitive point in this uncertainty measurement. When the second fuzzy number of the Z-number becomes sharper, the reliability value of the first fuzzy number is more certain, the uncertainty of the Z-number should reduce. However, in the method of Li et. al. [16], the uncertainty increases slowly. In addition, there is no approach to measure the uncertainty of continuous Z-number.

In this paper, we modified the uncertainty measure of the discrete Z-number. The proposed measurement conforms to three intuitions about the uncertainty of Z-numbers:

(1) When the first fuzzy number becomes shaper, the value of the variable is more accurate, the uncertainty of Z-number should be smaller.

(2) When the second fuzzy number becomes sharper, the reliability value of the first fuzzy number is more certain, the uncertainty of Z-number should be smaller.

(3) When the first fuzzy number is more reliable, the uncertainty of Z-number should be smaller.

We also proposed an analytical solution of the uncertainty of continuous Z-number. If the analytical solution is difficult to obtain, we provided an appropriate method and set the precision to control the calculation consumption and accuracy of the result.

This paper is structured as follows. Section 2 lists the concepts of Z-numbers and other related backgrounds. In section 3, we amend the measuring the uncertainty of discrete Z-numbers. And the uncertainty measurement of continuous Z-numbers is proposed. Some numerical examples are used to demonstrate the advantages of the proposed methodology in Section 4. In Section 5, an application of journey vehicle selection shows the usefulness of the proposed uncertainty measurement in determining the weights of criteria. Finally, this paper is concluded in Section 5.

2 Preliminaries

In this section, some preliminaries about definitions of possibilistric restriction, probabilistric restriction, Z-numbers and Z-restriction are discussed.

Definition 1. *A restriction $R_{prob}(X)$ on real-valued uncertain variable X is a probabilistric restriction if*

$$R_{prob}(X): \ X \ is \ P, \tag{1}$$

if X is a continuous variable, P is called the probability density function of X,

$$Prob(u \leq X \leq u + du) = \int P(u)du. \tag{2}$$

If X is a discrete variable, P is called the probability distribution of X.

$$Prob(X = u) = P(X = u). \tag{3}$$

Definition 2. *A possibilistric restriction $R)poss(X)$ on variable X is denoted as*

$$R_{poss}(X): \ X \ is \ A, \tag{4}$$

where A is a fuzzy number in universe U with membership function $\mu_A : U \to [0, 1]$. The membership degree of a base value $u \in U$ can be seen as the possibility value of '$X = u$'. More explicitly,

$$R_{poss}(X) : Poss(X = u) = \mu_A(u) \tag{5}$$

Definition 3. *A Z-number is an ordered pair of fuzzy numbers, $Z = (A, B)$. Where A is a possibilistric restriction on values of variable X can take with membership function $\mu_A(u) : X \to [0, 1]$, B is another possibilistric restriction on reliability of A with membership function $\mu_B(v) : [0, 1] \to [0, 1]$ [28]. v is a base value of B and can be seen as the reliability value.*

Definition 4. *A Z-restriction combine a possibilistric restriction and a series of probabilistric restrictions [28],*

$$R_Z(X) : X \ is \ Z \to Prob(X \ is \ A) \ is \ B, \tag{6}$$

where $Prob(X \ is \ A)$ is the probability of the event 'X is A' occur and can be expressed as:

$$Prob(X \ is \ A) = \int_R \mu_A(u)p_X(u)du. \tag{7}$$

where p_X is the underlying probability distribution of X and is unknown. If A and p_X are matching, then the centroids of μ_A is same with the expected value of p_X,

$$\int up_X(u)du = \frac{\int u\mu_A(u)du}{\int \mu_A(u)du} \tag{8}$$

3 Uncertainty measure of Z-numbers

In this section, we propose a novel uncertainty measure of discrete and continuous Z-numbers. We try to find the underlying probability distribution for reliability value, and then combine the underlying probability distributions with their corresponding possibilities.

3.1 Uncertainty measure of discrete Z-numbers

For a discrete Z-number $Z = (A, B)$, $A =< u, \mu_A(u) >$, $B =< v, \mu_B(v) >$, the underlying probability distribution $P_X^v(u)$ of reliability v satisfy the following three conditions:

(1) $\sum_{i=1}^{n} P_X^v(u_i) = 1$,

(2) $\sum_{i=1}^{n} P_X^v(u_i)\mu_A(u_i) = v$,

(3) $\sum_{i=1}^{n} u_i P_X^v(u_i) = \frac{\sum_{i=1}^{n} u_i\mu_A(u_i)}{\sum_{i=1}^{n} \mu_A(u_i)}$,

where $\{u_1, u_2, \cdots, u_n\}$ is the base values of fuzzy number A, $v \in \{v_1, v_2, \cdots, v_m\}$ is a base value of fuzzy number B.

Most of time, we may obtain more than one underlying probability distribution according to three conditions. The maximum entropy method [12] is used to select one with the maximum Shannon entropy. Then the calculation of underlying probability distribution is an optimization problem:

$$max: \quad H(P_X^v) = -\sum_{k=1}^{n} P_X^v \log(P_X^v), \tag{9}$$

subject to:

$$\sum_{i=1}^{n} P_X^v(u_i) = 1,$$

$$\sum_{i=1}^{n} P_X^v(u_i)\mu_A(u_i) = v,$$

$$\sum_{i=1}^{n} u_i P_X^v(u_i) = \frac{\sum_{i=1}^{n} u_i\mu_A(u_i)}{\sum_{i=1}^{n} \mu_A(u_i)}$$

We obtained all underlying probability distributions of reliabilities $\{v_1, v_2, \cdots, v_m\}$, the uncertainty of Z is calculated by

$$H_Z(Z) = \sum_{i=1}^{m} H(P_X^{v_i})\mu_B(v_i) \tag{10}$$

3.2 Uncertainty measure of continuous Z-numbers

For a continuous Z-number $Z = (A, B)$, $A =< u, \mu_A(u) >$, $B =< v, \mu_B(v) >$, we assume the underlying probability density function is a normal distribution:

$$P_X^v(u) = N(\alpha, \beta^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(u-\alpha)^2}{2\beta^2}), \tag{11}$$

subject to:

$$\int P_X^v(u)du = 1,$$

$$\int P_X^v(u)\mu_A(u)du = v,$$

$$\int uP_X^v(u)du = \alpha = \frac{\int u\mu_A(u)du}{\int \mu_A(u)du}$$

According to Equation (11), the obtained underlying probability is a function about variable u and v. More precisely, its β is a function about variable v. Then calculate the uncertainty of this normal probability density function by:

$$\begin{aligned} H(P_X^v(u)) &= -\int P_X^v(u)log(P_X^v(u))du \\ &= \frac{1}{2}(\log(2\pi\beta^2) + 1) \end{aligned} \tag{12}$$

where β is a function about variable v.

The uncertainty of Z is

$$\begin{aligned} H_Z(Z) &= \int H(P_X^v(u))\mu_B(v)dv \\ &= \int \frac{1}{2}(\log(2\pi\beta^2) + 1)\mu_B(v)dv \end{aligned} \tag{13}$$

4 Numerical example

In this section, some numerical examples are used to show the calculation process and the advantages of the proposed method. In which, Example 1 and Example 2 show the uncertainty calculation processes of discrete Z-numbers and continuous Z-numbers in detail. Example 3, Example 4 and Example 5 verify the proposed measurement satisfies the three intuitions and compare the proposed method and the measurement in [16].

Example 1. *Given a discrete Z-number $Z = (A, B)$, the membership functions of A and B are*

$$A = \frac{\mu_A}{u} = \frac{0}{1} + \frac{0.3}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{1}{5} + \frac{0.7}{6} + \frac{0.5}{7} + \frac{0.3}{8} + \frac{0}{9},$$

$$B = \frac{\mu_B}{v} = \frac{0}{0.1} + \frac{0.3}{0.2} + \frac{0.5}{0.3} + \frac{0.7}{0.4} + \frac{1}{0.5} + \frac{0.7}{0.6} + \frac{0.5}{0.7} + \frac{0.3}{0.8} + \frac{0}{0.9}.$$

Probability distributions corresponding to different reliability value for Z and their entropy are list as Table 1. The uncertainty of Z is

v \ $p_X(u)$	$u_1 = 1$	$u_2 = 2$	$u_3 = 3$	$u_4 = 4$	$u_5 = 5$	$u_6 = 6$	$u_7 = 7$	$u_8 = 8$	$u_9 = 9$	Entropy
$v_1 = 0.1$	0.3724	0.0838	0.0310	0.0115	0.0026	0.0115	0.0310	0.0838	0.3724	1.4849
$v_2 = 0.2$	0.2759	0.1156	0.0647	0.0362	0.0152	0.0362	0.0647	0.1156	0.2759	1.8678
$v_3 = 0.3$	0.1978	0.1240	0.0908	0.0665	0.0417	0.0665	0.0908	0.1240	0.1978	2.0875
$v_4 = 0.4$	0.1348	0.1175	0.1072	0.0978	0.0853	0.0978	0.1072	0.1175	0.1348	2.1871
$v_5 = 0.5$	0.0851	0.1008	0.1129	0.1264	0.1497	0.1264	0.1129	0.1008	0.0851	2.1815
$v_6 = 0.6$	0.0478	0.0776	0.1071	0.1478	0.2395	0.1478	0.1071	0.0776	0.0478	2.0733
$v_7 = 0.7$	0.0223	0.0514	0.0896	0.1564	0.3605	0.1564	0.0896	0.0514	0.0223	1.8552
$v_8 = 0.8$	0.0073	0.0263	0.0617	0.1447	0.5199	0.1447	0.0617	0.0263	0.0073	1.5067
$v_9 = 0.9$	0.0010	0.0073	0.0271	0.1011	0.7270	0.1011	0.0271	0.0073	0.0010	0.9765

Table 1: Probability distributions corresponding to different reliability value for Z.

$$\begin{aligned} H_Z(Z) &= 1.4849 \times 0 + 1.8678 \times 0.3 + 2.0875 \times 0.5 + 2.1871 \times 0.7 + 2.1815 \times 1 \\ &+ 2.0733 \times 0.7 + 1.8552 \times 0.5 + 1.5067 \times 0.3 + 0.9765 \times 0 \\ &= 8.1475 \end{aligned}$$

Example 2. *Given a continuous Z-number $A = (A, B)$, where A and B are two triangular fuzzy number $A = (2, 5, 8)$, $B = (0.2, 0.5, 0.8)$. The membership functions of A and B are:*

$$\mu_A(u) = \begin{cases} \frac{u-2}{3} & 2 < u \le 5, \\ \frac{8-u}{3} & 5 < u \le 8, \\ 0, & otherwise. \end{cases}$$

$$\mu_B(v) = \begin{cases} \frac{10u-2}{3} & 0.2 < u \le 0.5, \\ \frac{8-10u}{3} & 0.5 < u \le 0.8, \\ 0, & otherwise. \end{cases}$$

According to Equation (11), The analytical solutions of underlying probability density functions is very hard to obtain in practice. Thus, we use the approximate method. First set a precision $pre \in [0.0001, 0.1]$, we assume the underlying probability density function is same in interval $[v, v + pre]$. Then calculate the underlying probability density functions at points $\cdots, v, v + pre, v + 2pre, \cdots$. The results as shown in Table 2. The precision is $pre = 0.1$. The uncertainty of Z is

	$v_1 = 0.2$	$v_2 = 0.3$	$v_3 = 0.4$	$v_4 = 0.5$	$v_5 = 0.6$	$v_6 = 0.7$	$v_7 = 0.8$
β	3.0079	2.6319	2.2559	1.8799	1.5039	1.1279	0.7519
$H(p_X)$	2.5201	2.3866	2.2324	2.0501	1.8270	1.5392	1.1337

Table 2: The value of β and entropy under different reliability.

$$\begin{aligned} H_Z(Z) &= \sum_{i=1}^{m-1} \int_{v_i}^{v_i+pre} \frac{H(p_X^{v_i}) + H(p_X^{v_i+pre})}{2} \mu_B(v)dv \\ &= 0.0408 + 0.1154 + 0.1784 + 0.1615 + 0.0842 + 0.0223 \\ &= 0.6026 \end{aligned}$$

Example 3. *The first intuition is: when the membership function of A is sharper, the uncertainty of Z should be smaller. In Example 1, when the membership degrees of u_2 and u_8 reduce to 0, and then the membership degrees of u_3 and u_7 reduce to 0, and then the membership degrees of u_4 and u_6 reduce to 0. The graphical representations of changes are shown in Figure 1. The changes of the uncertainty of Z computed by the proposed approach and the measurement in [16] are shown in Figure 2.*

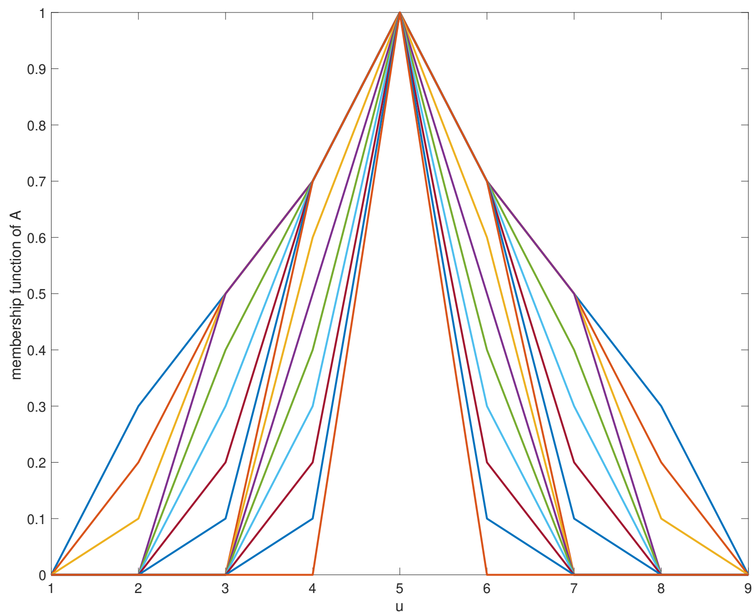

Figure 1: The change of membership function of fuzzy number A.

As shown in Figure 2, the results of the two methods are consistent with our intuition.

Example 4. *The second intuition is: when the membership function of B is sharper, the uncertainty of Z should be smaller. In Example 1, when the membership degrees of v_2 and v_8 reduce to 0, and then the membership degrees of v_3 and v_7 reduce to 0, and then the membership degrees of v_4 and v_6 reduce to 0. The graphical representations of changes are shown in Figure 3. The changes of the uncertainty of Z computed by the proposed approach and the measurement in [16] are shown in Figure 4.*

As shown in Figure 4, the uncertainty calculated by method in [16] increases slowly as the second fuzzy number becomes sharper. The uncertainty calculated by the proposed measurement reduce sharply. it is consistent with our intuition.

Example 5. *When the first fuzzy number is more reliable, the uncertainty of the Z-number is smaller. Given a discrete Z-number $Z_1 = (A_1, B_1)$, where $A_1 = A$, the membership function B_1 is*

$$B_1 = \frac{\mu_B}{v} = \frac{0}{0.2} + \frac{0.3}{0.3} + \frac{0.5}{0.4} + \frac{0.7}{0.5} + \frac{1}{0.6} + \frac{0.7}{0.7} + \frac{0.5}{0.8} + \frac{0.3}{0.9} + \frac{0}{1}.$$

$$H_Z(Z_1) = 7.6651 < H_Z(Z).$$

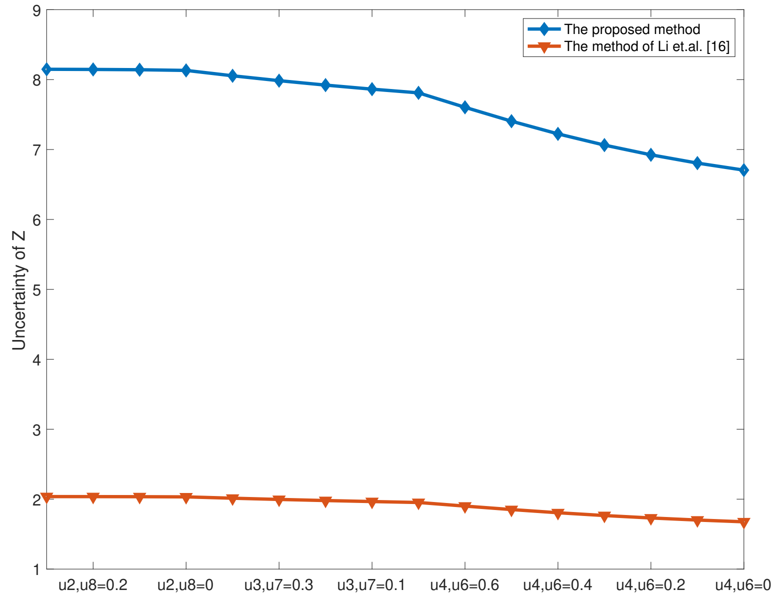

Figure 2: The change of uncertainty of Z as A changes.

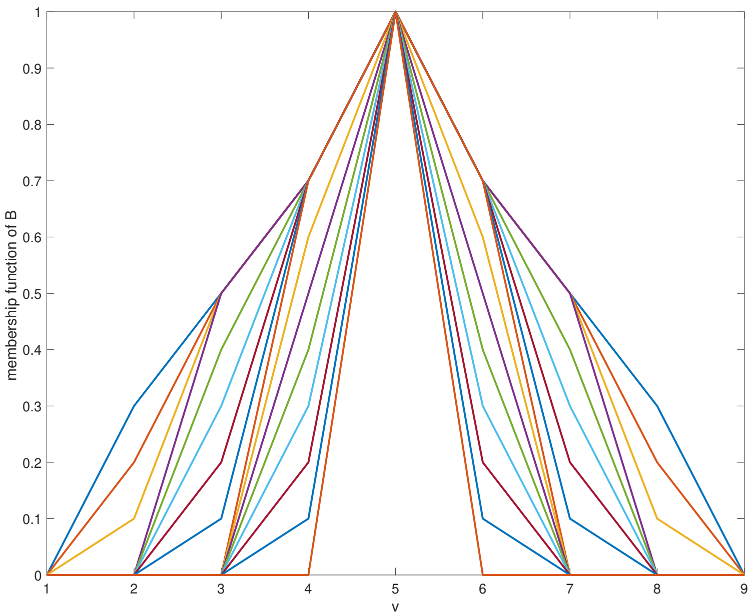

Figure 3: The change of membership function of fuzzy number B.

5 Application

In this section, the weights of criteria in a decision-making problem are determined by the proposed uncertainty measurement.

There are three different options of vehicle for journey, car (a_1), taxi (a_2) and train (a_3). The set of alternatives is denoted as $A = \{a_1, a_2, a_3\}$. Three criteria, expense (c_1), time consumption (c_2) and comfort (c_3) are took into account. The set of criteria is denoted as $C = \{c_1, c_2, c_3\}$. The Z-information decision matrix is shown Table 3.

The uncertainty of Z-numbers in decision matrix are displayed in Table 4 (the precision is 0.01).

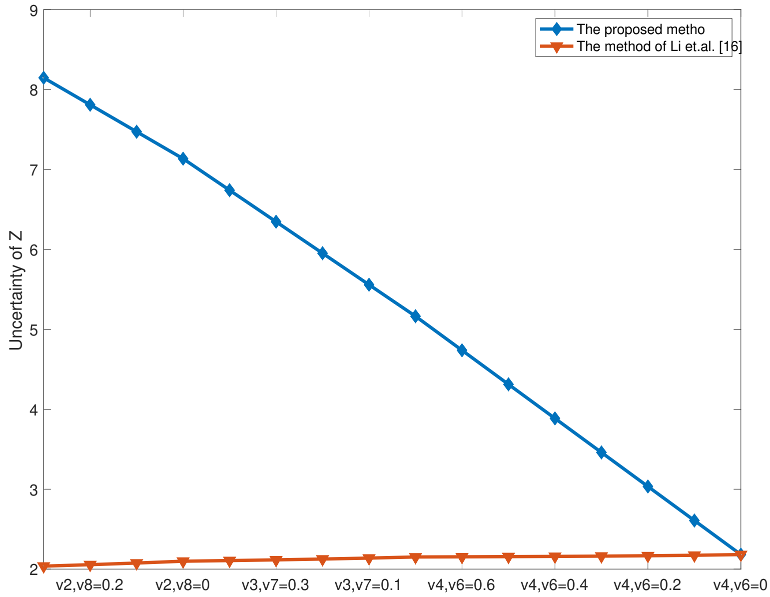

Figure 4: The change of uncertainty of Z as B changes.

	c_1	c_2	c_3
a_1	$((8, 10, 12), (0.83, 1, 1))$	$((70, 100, 120), (0, 0.17, 0.33))$	$((3, 5, 7), (0.33, 0.5, 0.67))$
a_2	$((20, 24, 25), (0, 0, 0.17))$	$((40, 70, 100), ((0.33, 0.5, 0.67))$	$((5, 8, 11), (0.67, 0.83, 1))$
a_3	$((14, 15, 16), (0.83, 1, 1))$	$((60, 80, 100), (0.67, 0.83, 1))$	$((1, 4, 7), (0.67, 0.83, 1))$

Table 3: The decision matrix with Z-information.

The sum of uncertainty of criteria 'Price' is

$$H_{sum}(c_1) = 0.1378 + 0.2491 - 0.1967 = -0.0854.$$

The sum of the uncertainty of criteria 'Journey time' is

$$H_{sum}(c_2) = 0.7719 + 0.7408 + 0.4514 = 1.9641.$$

The sum of the uncertainty of criteria 'Comfort' is

$$H_{sum}(c_3) = 0.2788 + 0.1389 + 0.1389 = 0.5566.$$

AS there are negative numbers, we add a constant:

$$H_{nor}(c_j) = H_{sum}(c_1) + 0.5, j = 1, 2, 3.$$

Then the weight of criteria 'Price' is

$$w_{c_1} = \frac{1/(-0.0854 + 0.5)}{1/(-0.0854 + 0.5) + 1/(1.9641 + 0.5) + 1/(0.5566 + 0.5)} = 0.6408.$$

	c_1	c_2	c_3
a_1	-0.1378	0.7719	0.2788
a_2	0.2491	0.7408	0.1389
a_3	-0.1967	0.4514	0.1389

Table 4: The uncertainty of Z-numbers.

	c_1	c_2	c_3
a_1	$((0.6667, 0.8, 1), (0.83, 1, 1))$	$((0.3333, 0.4, 0.5714), (0, 0.17, 0.33))$	$((0.2727, 0.4545, 0.6363), (0.33, 0.5, 0.67))$
a_2	$((0.32, 0.3333, 0.4), (0, 0, 0.17))$	$((0.4, 0.5714, 1), ((0.33, 0.5, 0.67))$	$((0.4545, 0.7272, 1), (0.67, 0.83, 1))$
a_3	$((0.5, 0.5333, 0.5714), (0.83, 1, 1))$	$((0.4, 0.5, 0.6666), (0.67, 0.83, 1))$	$((0.0909, 0.3636, 0.6363), (0.67, 0.83, 1))$

Table 5: The normalized decision matrix.

	c_1	c_2	c_3
a_1	0.7985	0.1775	0.3214
a_2	0.0835	0.4646	0.5311
a_3	0.5195	0.4767	0.2655

Table 6: The crisp numbers of Z-numbers in decision matrix.

The weight of criteria 'Journey time' is

$$w_{c_1} = \frac{1/(1.9641 + 0.5)}{1/(-0.0854 + 0.5) + 1/(1.9641 + 0.5) + 1/(0.5566 + 0.5)} = 0.1078.$$

The weight of criteria 'Comfort' is

$$w_{c_1} = \frac{1/(0.5566 + 0.5)}{1/(-0.0854 + 0.5) + 1/(1.9641 + 0.5) + 1/(0.5566 + 0.5)} = 0.2514.$$

The scales of criteria are different. In order to compare these Z-numbers, the first fuzzy numbers must be normolized. The results are summarized in Table 5.

Then we used the converting methods to converse the Z-numbers to crisp numbers [15], the decision matrix with crisp numbers is shown in Table 6. The evalution value of alternative a_1 is:

$$\Re(a_1) = 0.7985 * 0.6408 + 0.1775 * 0.1078 + 0.3214 * 0.2514 = 0.6116.$$

The evalution value of alternative a_2 is:

$$\Re(a_2) = 0.0835 * 0.6408 + 0.4646 * 0.1078 + 0.5311 * 0.2514 = 0.2371.$$

And the evalution value of alternative a_2 is:

$$\Re(a_3) = 0.5195 * 0.6408 + 0.4767 * 0.1078 + 0.2655 * 0.2514 = 0.4510.$$

The ranking of alternatives is $a_1 > a_3 > a_2$. The best option of journey is a_1, car.

6 Conclusion

Z-number is a new framework to address uncertain and partial reliable information. This paper modifies the uncertainty measure of discrete and continuous Z-numbers, overcomes the shortcoming of the previous method. The proposed measurement satisfies all intuitions about the uncertainty of Z-numbers. Finally, some numerical examples and an application illustrate the advantages and effectiveness of the proposed method.

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