INTERNATIONAL JOURNAL OF COMPUTERS COMMUNICATIONS & CONTROL
Online ISSN 1841-9844, ISSN-L 1841-9836, Volume: 17, Issue: 4, Month: August, Year: 2022
Article Number: 4800, https://doi.org/10.15837/ijccc.2022.4.4800

CCC Publications
AGORA UNIVERSITY PRESS

Distributed Adaptive Control for Nonlinear Heterogeneous Multi-agent Systems with Different Dimensions and Time Delay

Bo Qin, Yongqing Fan, Shuo Yang

Bo Qin*
School of Automation, and Xi'an Key Laboratory of Advanced Control and Intelligent Process,
Xi'an University of Posts and Telecommunications, Xi'an, China.
Chang'an West St. Chang'an District, Xi'an, 710121, China
*Corresponding author: qinbo123@xupt.edu.cn

Yongqing Fan
School of Automation, and Xi'an Key Laboratory of Advanced Control and Intelligent Process,
Xi'an University of Posts and Telecommunications, Xi'an, China.
Chang'an West St. Chang'an District, Xi'an, 710121, China
fanyongqing@xupt.edu.cn

Shuo Yang
Beijing Aerospece Institute for Metrology and Measurement Technology,
No. 1, Nandahongmen Road, Fengtai District, Beijing, 100076, China.
tju1105@163.com

Abstract

A distributed neural network adaptive feedback control system is designed for a class of nonlinear multi-agent systems with time delay and nonidentical dimensions. In contrast to previous works on nonlinear heterogeneous multi-agent with the same dimension, particular features are proposed for each agent with different dimensions, and similar parameters are defined, which will be combined parameters of the controller. Second, a novel distributed control based on similarity parameters is proposed using linear matrix inequality (LMI) and Lyapunov stability theory, establishing that all signals in a closed loop system are eventually ultimately bounded. The consistency tracking error steadily decreases to a field with a small number of zeros. Finally, simulated examples with different time delays are utilized to test the effectiveness of the proposed control technique.
Keywords: Multi-agent systems, consensus, neural network, nonlinear, time-delay.

1 Introduction

In the recent decade, multi-agent systems have been widely developed in a range of industries. Examples include unmanned aerial vehicles[1], robot cooperative systems, [2, 3], aerospace systems[4], and so on. As a result, distributed cooperative control has become a topic interesting research topic by many scholars. One of the most important characteristics of this design style is that it is capable of changing the state based on the data provided by each agent and its neighbors. Consistency is one of the most essential subjects, as evidenced by references [5, 6, 7], which indicate consistency algorithm studies that are ground-breaking works on multi-agent system consistency. In [8, 9, 10], authors suggested a number of excellent distributed optimal control research methodologies. The formation control was proposed in [11] , and [12, 13, 14] explored a wide range of issues. The synchronous conditions for multi-agent systems were proposed in [15]. The global asymptotic stability and global robust stability of neural networks with delay were investigated in [16, 17] by using Lyapunov theory and linear matrix inequality. Because neural networks are a broad approximation strategy for existing unknown nonlinear functions or uncertain models in many challenging areas, and a large number of control algorithms for nonlinear multi-agent systems have been proposed in the last few years in [18, 19, 20].

Regrettably, these findings were limited to homogeneous or heterogeneous nonlinear multi-agent systems with the same dimension, so the control approaches for multi-agent systems with different dimensions would be ineffective. It is necessary to seek some unique control technologies to meet the class of nonlinear heterogeneous multi-agent systems with nonidentical dimensions. Up till now, the uniformly bounded stability problem of nonlinear multi-agent systems with various nodes and dimensions is solved in [21, 22, 23], and a novel distributed adaptive control with similar parameters is proposed. In reference [24], the synchronization problem of dynamic networks with varied nodes was investigated. However, none of these previous studies take into account time delays, which could compromise accuracy and render the system unstable.

By above mentioned analyses, in order to solve the consensus of each nonlinear multi-agent system with different dimensions and time delay dependent, we aim to create a novel distributed neural network adaptive control in this research. To cope with unknown nonlinear factors, neural networks are employed to approximate the uncertainties of systems, and then a distributed feedback adaptive with similar parameters is performed by solving the linear matrix inequality (LMI).

The remainder of the paper is structured as follows: Section II provides some background information, a description of the system, and characteristics of similar composite structures and neural network systems. A neural network adaptive control and its stability analysis are described in Section III. A simulated example of the consistency of nonlinear heterogeneous multi-agent systems with time delay is presented in section IV. Section V summarizes the conclusion.

2 The Model of Dynamical Network and Assumptions

A weighted digraph is denotes as $G = \{V, E, A\}$,where $V = [V_1, V_2, ..., V_N]$ represents the non-empty set of each node, edge set E contains one edge (V_j, V_i), adjacency matrix $A = [a_{ij}] \subseteq R^{N\times N}$ represents that node V_j can send the information to node V_i. The value $a_{ij} > 0$, if $(V_j, V_i) \in E$, otherwise $a_{ij} = 0$. In addition, for all $i \in \{1, 2, ..., N\}$, there exists $a_{ii} = 0$. Diagonal matrix $D = diag\{d_{in}\} \in R^{N\times N}$, where $d_{in} = \sum_{j=1}^{N} a_{ji}$ is degree matrix. Laplacian matrix is defined as $L = D - A$. The Kronecker product of matrices A and P is denoted by $A \otimes P$, I represents the appropriate n-dimensional identity matrix. $P > 0(P < 0)$ means P is a positive (negative) matrix, $\|\cdot\|$ represents the 2 norm of vector. A^T and A^{-1} denote the transpose matrix and inverse matrix of A, respectively.

Consider the following dynamical network with nonlinear multiagent system and time delay:

$$\dot{x}_i(t) = A_{i1}x_i(t) + A_{i2}x_i(t - \tau(t)) + B_i[u_i(t) + f_i(x_i)], \tag{1}$$

where $x_i \in R^{n_i}$ is the state of the ith node. $A_i \in R^{n_i\times n_i}$ and $B_i \in R^{n_i\times m_i}$ are some known matrices. u_i denotes the input vector, τ is vary time delay and satisfies $0 \le \tau(t) \le h < \infty$, $f_i(x_i)$ represents the unknown nonlinear function.

Assumption 1. *Consider the network (1) with N nodes, there exist N matrices, if $F_i \in R^{n_0\times n_i}(F_i \neq$*

$0), K_i \in R^{m_i \times n_i}$ *and a known matrix* $J_i \in R^{1\times p_i}$ *satisfies the following condition*

$$\begin{cases} F_i(A_{i1} + B_i K_{i1}) = (A_{j1} + B_j K_{j1} F_i). \\ F_i(A_{i2} + B_i K_{i2}) = (A_{j2} + B_j K_{j2} F_i). \\ F_i B_i = B_j J_j. \end{cases} \tag{2}$$

Assumption 2. *Based on the good approximation performance of the radial basis function neural network(RBFNN). The RBFNN is used to approximate the unknown nonlinear function. In this paper, the RBFNN is described as the following form:*

$$f_i(x_i) = W_i^T \varphi_i(x_i) + \varepsilon_i(t), \forall x_i \in \Omega, \tag{3}$$

where W_i *means the ideal weight matrix that will be designed automatically,* ε_i *is denoted as approximation error satisfying* $\| \varepsilon_i \| < \bar{\varepsilon}_i$ *, where* $\bar{\varepsilon}_i$ *is a known positive constant.* $\varphi_i(x_i) = [\varphi_{i1}(x_i), \varphi_{i2}(x_i), ..., \varphi_{is_i} (x_i)] \in R^{s_i \times 1}$ *characterizes RBFNN vector, in which* s_i *is the number of basic functions. Define the Gaussian function as follows:*

$$\varphi_i(x_i) = exp(-\frac{\| x_i - \mu_{is_i} \|^2}{2\sigma_{is_i}^2}), \tag{4}$$

where $\mu_{is} = [\mu_{is_1}, \mu_{is_2}, ..., \mu_{is_{ni}}]^T$ *represents the center of RBFNN,* σ_{is_i} *is the width of function.*

Lemma 1. [25] *For any given two vectors* $x, y \in R^{n\times n}$ *and a positive matrix* $H \in R^{n\times n}$ *, the following inequality holds:*

$$2x^T y \le x^T Hx + y^T H^{-1} y. \tag{5}$$

Lemma 2. [26] *The LMI* $\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0$ *is equavalent to one of the following conditions:*

$$1)Q(x) > 0, R(x) - S^T(x)Q^{-1}(x)S(x) > 0; 2)R(x) > 0, Q(x) - S^T(x)R^{-1}(x)S(x) > 0, \tag{6}$$

where $Q(x) = Q^T(x), R(x) = R^T(x)$.

Lemma 3. *There exist positive definite matrices* $S > 0$, $T_{11} > 0$, *non-negative definite matrices* $T_{22} = T_{22}^T \ge 0$, *and any matrices of appropriate dimensions* Y_1, Y_2, T_{12}, *such that the following LMIs are feasible:*

$$\begin{bmatrix} S & Y_1 & Y_2 \\ * & T_{11} & T_{12} \\ * & * & T_{22} \end{bmatrix} \ge 0. \tag{7}$$

The following inequality holds:

$$-\int_{t-\tau}^{t} \dot{z}^T(\alpha) S \dot{z}(\alpha) d\alpha \le \eta^T(t) \Psi \eta(t) + \tau \delta^T(t) T \delta(t), \tag{8}$$

where $\eta^T(t) = \begin{bmatrix} z^T(t) & \dot{z}^T(t) & z^T(t-\tau) \end{bmatrix}$, $\delta^T(t) = \begin{bmatrix} z^T(t) & \dot{z}^T(t) \end{bmatrix}$, $\Psi = \begin{bmatrix} Y_1^T + Y_1 & Y_2 & -Y_1^T \\ * & 0 & -Y_2^T \\ * & * & 0 \end{bmatrix}$, $T = \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix}$.

Proof. For $\forall H_1, H_1 \in R^{n\times n}$, the following inequality holds

$$\begin{aligned} 0 &= 2[z^T(t)H_1^T + \dot{z}^T(t)H_2^T][z(t) - z(t-\tau) - \int_{t-\tau}^{t} \dot{z}(\alpha)d\alpha] \\ &= 2z^T(t)H_1^T z(t) - 2z^T(t)H_1^T z(t-\tau) + 2\dot{z}^T(t)H_2^T z(t) - 2\dot{z}^T(t)H_2^T z(t-\tau) \\ &\quad - 2[z^T(t)H_1^T + \dot{z}^T(t)H_2^T] \int_{t-\tau}^{t} \dot{z}(\alpha)d\alpha. \end{aligned} \tag{9}$$

Let $H = \begin{bmatrix} H_1 & H_2 \end{bmatrix}$, $Y = \begin{bmatrix} Y_1 & Y_2 \end{bmatrix}$, $T = \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix}$, so it knows that

$$-2\delta^T(t)H^T \int_{t-\tau}^{t} \dot{z}(\alpha)d\alpha \leq \int_{t-\tau}^{t} \begin{bmatrix} \dot{z}(\alpha) \\ \delta(t) \end{bmatrix}^T \begin{bmatrix} X & Y-H \\ * & T \end{bmatrix} \begin{bmatrix} \dot{z}(\alpha) \\ \delta(t) \end{bmatrix} d\alpha. \tag{10}$$

The right of the above inequality is equal to

$$\int_{t-\tau}^{t} \dot{z}(\alpha)X\dot{z}(\alpha)d\alpha + 2\int_{t-\tau}^{t} \delta^T(t)(Y^T - H^T)\dot{z}(\alpha)d\alpha + \int_{t-\tau}^{t} \delta^T(t)T\delta(t)d\alpha$$

$$= \int_{t-\tau}^{t} \dot{z}(\alpha)X\dot{z}(\alpha)d\alpha + \eta^T(t) \begin{bmatrix} Y_1^T + Y_1 - H_1^T - H_1 & Y_2 - H_2 & -Y_1^T + H_1^T \\ * & 0 & -Y_1^T + H_2^T \\ * & * & 0 \end{bmatrix} \eta(t) + \tau\delta^T(t)T\delta(t). \tag{11}$$

Taking (10) into (11), inequality (8) can be guaranteed. The proof of Lemma 3 is completed. □

3 The Control Design

In this section, a decentralized controller with similar parameter is proposed for the consensus of multi-agent system with different dimension and time-delay.

The following controller is designed:

$$u_i = K_{i1}x_i(t) + K_{i2}x_i(t-\tau(t)) + \bar{K}_1[F_ix_i(t) - F_jx_j(t)] + \bar{K}_2[F_ix_i(t-\tau(t)) - F_jx_j(t-\tau(t))]$$

$$+ c_i\bar{K}_i\sum_{j=1}^{N} a_{ij}[F_ix_i(t) - F_jx_j(t)] + c_i\bar{K}_2\sum_{j=1}^{N} a_{ij}[F_ix_i(t-\tau(t)) - F_jx_j(t-\tau(t))] - \bar{W}_i^T\varphi_i(x_i), \tag{12}$$

where $\bar{W}_i$ is the estimated value of W_i, $\tilde{W}_i = \bar{W}_i - W_i$ is the estimated error. Here, the adaptive law of $\bar{W}_i$ is designed as follows:

$$\dot{\bar{W}}_i = -\gamma_{wi}\bar{W}_i + \eta_{wi}\varphi_i(x_i)(PB_jJ_j)^Te_i(t). \tag{13}$$

The adaptive law of coupling weight c_i is designed as:

$$\dot{c}_i = -\gamma_{ci}c_i - \beta_{c_{1}i}e_1^T(t)(PB_jJ_j\bar{K}_1)e_1(t) - \beta_{c_{2}i}e_2^T(t)(PB_jJ_j\bar{K}_2)e_2(t). \tag{14}$$

We define $\bar{x}_i(t) = F_ix_i(t)$, denote $z_{ij}(t) = x_i(t) - x_j(t)$ and $z_{ij}(t-\tau(t)) = x_i(t-\tau(t)) - x_j(t-\tau(t))$, then get $\dot{\bar{x}}_i(t) = F_i\dot{x}_i(t)$. By using controller (12), adaptive laws (13)-(14), the following dynamic equation holds:

$$\dot{\bar{x}}_i(t) = F_i\{A_{i1}x_i(t) + A_{i2}x_i(t-\tau(t)) + B_iK_{i1}x_i(t) + B_iK_{i2}x_i(t-\tau(t)) + B_i\bar{K}_1[F_ix_i(t) - F_jx_j(t)]$$

$$+B_i\bar{K}_2[F_ix_i(t-\tau(t)) - F_jx_j(t-\tau(t))] + c_iB_i\bar{K}_1\sum_{j=1}^{N} a_{ij}[F_ix_i(t) - F_jx_j(t)]$$

$$+c_iB_i\bar{K}_2\sum_{j=1}^{N} a_{ij}[F_ix_i(t-\tau(t)) - F_jx_j(t-\tau(t))] + B_i[f_i(x_i) - \hat{W}_i^T\varphi_i(x_i)]\}$$

$$= F_i[A_{i1} + B_iK_{i1}]x_i(t) + F_i[A_{i2} + B_iK_{i2}]x_i(t-\tau(t)) + F_iB_i\bar{K}_1z_{ij}(t) + F_iB_i\bar{K}_2z_{ij}(t-\tau(t))$$

$$+c_iF_iB_i\bar{K}_1\sum_{j=1}^{N} a_{ij}[F_ix_i(t) - F_jx_j(t)] + c_iF_iB_i\bar{K}_2\sum_{j=1}^{N} a_{ij}[F_ix_i(t-\tau(t)) - F_jx_j(t-\tau(t))]$$

$$+F_iB_i[f_i(x_i)-\bar{W}_i^T\varphi_i(x_i)]. \tag{15}$$

According to Assumption 1, it gets

$$\dot{\bar{x}}_i(t)=(A_{j1}+B_jK_{j1})F_ix_i(t)+(A_{j2}+B_jK_{j2})F_ix_i(t-\tau(t))+B_jJ_j\bar{K}_1z_{ij}(t)+B_jJ_j\bar{K}_2z_{ij}$$

$$(t-\tau(t))+c_iB_jJ_j\bar{K}_1e_i(t)+c_iB_jJ_j\bar{K}_2e_i(t-\tau(t))+B_jJ_j[-\tilde{W}_i^T\varphi_i(x_i)+\varepsilon_i]. \tag{16}$$

At the same time, the jth agent is the tracked system which can be rewritten as:

$$\dot{x}_j(t)=A_{j1}x_j(t)+A_{j2}x_j(t-\tau_i)+B_j[u_j+f_j(x_j)]. \tag{17}$$

Define the consensus tracking error as:

$$\dot{z}_{ij}(t)=\dot{\bar{x}}_i(t)-\dot{\bar{x}}_j(t), \tag{18}$$

then the dynamical of tracking error equation is deduced as:

$$\dot{z}_{ij}(t)=(A_{j1}+B_jK_{j1}+B_jJ_j\bar{K}_1)z_{ij}(t)+(A_{j2}+B_jK_{j2}+B_jJ_j\bar{K}_2)z_{ij}(t-\tau(t))$$

$$+c_iB_jJ_j\bar{K}_1e_i(t)+c_iB_jJ_j\bar{K}_2e_{ij}(t-\tau(t))+B_jJ_j[-\tilde{W}_i^T\varphi_i(x_i,x_j)+\varepsilon_{ij}], \tag{19}$$

for simplicity, (19) can be further transformed as:

$$\dot{z}=[I_N\otimes(A_{j1}+B_jK_{j1}+B_jJ_j\bar{K}_1)]z(t)+cH\otimes(B_jJ_j\bar{K}_1)z(t)+[I_N\otimes(A_{j2}+B_jK_{j2}+B_jJ_j\bar{K}_2)]$$

$$z(t-\tau(t))+cH\otimes(B_jJ_j\bar{K}_2)z(t-\tau(t))+\{I_N\otimes(B_jJ_j)\sum_{i=1}^{N}[-\tilde{W}_{ij}^T\varphi_{ij}(x_i,x_j)+\varepsilon_{ij}]\}, \tag{20}$$

where $z(t)=[z_1^T(t),z_2^T(t),\cdots,z_N^T(t)]^T$ with $z_i^T(t)=[z_{i1}^T(t),z_{i2}^T(t)],\cdots,z_{ij}^T(t)]]^T$, $z(t-\tau(t))=[z_1^T(t-\tau(t)),z_2^T(t-\tau(t)),\cdots,z_N^T(t-\tau(t))]^T$, $c=[c_1^T,c_2^T,\cdots,c_N^T]^T$. Hence (20) is equal to the following form:

$$\dot{z}(t)=[I_N\otimes(A_{j1}+B_jK_{j1}+B_jJ_j\bar{K}_1)]z(t)+[I_N\otimes(A_{j2}+B_jK_{j2}+B_jJ_j\bar{K}_2)]z(t-\tau(t))+[cL\otimes(B_jJ_j\bar{K}_1)]$$

$$z(t)+[cL\otimes(B_jJ_j\bar{K}_2)]z(t-\tau(t))+\{I_N\otimes(B_jJ_j)\sum_{i=1}^{N}B_jJ_j[-\tilde{W}_{ij}^T\varphi(x_i,x_j)+\varepsilon_{ij}]\}, \tag{21}$$

where matrices F_i and K_i are similar parameters that are defined in Assumption 1, $\bar{K}_1$ and $\bar{K}_2$ are two the gain matrices to be designed, which can be obtained by solving the following LMIs:

$$\bar{\phi}=\begin{bmatrix}\bar{\phi}_{11} & \bar{\phi}_{12} & \bar{\phi}_{13}\\ * & \bar{\phi}_{22} & \bar{\phi}_{23}\\ * & * & \bar{\phi}_{33}\end{bmatrix}<0, \tag{22}$$

$$\begin{bmatrix}\bar{S} & \bar{Y}_1 & \bar{Y}_2\\ * & \bar{T}_{11} & \bar{T}_{12}\\ * & * & \bar{T}_{22}\end{bmatrix}\geq 0, \tag{23}$$

in which $\bar{\phi}_{11}=VA_{j1}^T+A_{j1}V+VM_1^T+M_1V+G_1^TN^T+NG_1+\hat{Q}+\hat{R}+\bar{Y}_1^T+\bar{Y}_1+\tau\bar{T}_{11}+\hat{Q}_{11}$, $\bar{\phi}_{12}=\bar{Y}_2+\tau\bar{T}_{12}$, $\bar{\phi}_{13}=A_{j2}V+M_2V+NG_2-\bar{Y}_1^T$, $\bar{\phi}_{22}=\tau\bar{S}+\tau\bar{T}_{22}+\hat{Q}_{22}$, $\bar{\phi}_{23}=-\bar{Y}_2^T$, $\bar{\phi}_{33}=-\hat{Q}++\hat{Q}_{33}$, $\bar{K}_1=G_1V^{-1}$, $\bar{K}_2=G_2V^{-1}$.

Theorem 1. *Suppose Assumption 1-2 are satisfied, if there exist some positive definite matrices* $V>0$, $\hat{Q}>0$, $\hat{R}>0$, $\hat{Q}_{11}>0$, $\hat{Q}_{22}>0$, $\hat{Q}_{33}>0$, $\bar{S}>0$, *and proper dimension matrices* M_1, M_1, G_1, G_2, $\bar{Y}_1$, $\bar{Y}_2$, $\bar{T}_{11}$, $\bar{T}_{12}$, $\bar{T}_{22}$ *such that (22) and (23) can be satisfied, then the multi-agent system (1) in a connected graph can be guaranteed to consensus by applying the feedback neural network control (12) with adaptive laws (13) and (14), and all the signals in closed-loop system are bounded.*

Proof: The following candidate Lyapunov function is considered:

$$V(t) = z^T(t)(I_N \otimes P)z(t) + \int_{t-\tau}^{t} z^T(\alpha)(I_N \otimes Q)z(\alpha)d\alpha + \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{z}^T(\alpha)(I_N \otimes S)\dot{z}(\alpha)d\alpha d\beta + \frac{1}{2}\beta_c^{-1}c^T c + \frac{1}{2}\eta_w^{-1}tr(\tilde{W}^T\tilde{W}). \quad (24)$$

Denoting $\bar{A}_1 = A_{j1} + B_jK_{j1} + B_jJ_j\bar{K}_1$ and $\bar{A}_2 = A_{j2} + B_jK_{j2} + B_jJ_j\bar{K}_2$, By using (13) and (14), then it has

$$\begin{aligned}\dot{V}(t) &= \dot{z}^T(t)(I_N \otimes P)z(t) + z^T(t)(I_N \otimes P)\dot{z}(t) + \dot{z}^T(t)(I_N \otimes Q)z(t) - z^T(t-\tau)(I_N \otimes Q)z(t-\tau)\\
&\quad +\tau\dot{z}^T(t)(I_N \otimes S)\dot{z}(t) - \int_{t-\tau}^{t}\dot{z}^T(\alpha)(I_N \otimes S)\dot{z}(\alpha)d\alpha - \sum_{i=1}^{N}\frac{\gamma_{ci}}{\beta_{ci}}c_i^2\\
&\quad +2z^T(t)(I_N \otimes P)[I_N \otimes (B_jJ_j)]\sum_{i=1}^{N}[-\tilde{W}_{ij}^T\varphi_{ij}(x_i,x_j) + \varepsilon_{ij}] + \frac{1}{\eta_w}tr(\tilde{W}^T\dot{\tilde{W}})\\
&= z^T(t)[I_N \otimes (\bar{A}_1^TP + P\bar{A}_1)]z(t) + z^T(t)[I_N \otimes (\bar{A}_2^TP + P\bar{A}_2)]z(t-\tau) - 2z^T(t)[cL \otimes (PBJ\bar{K}_1)]z(t)\\
&\quad -2z^T(t)[cL \otimes (PBJ\bar{K}_2)]z(t-\tau) + 2z^T(t)[I \otimes (PBJ)]\sum_{i=1}^{N}[-\tilde{W}_{ij}^T(t)\varphi_{ij}(x_i,x_j) + \varepsilon_{ij}(t)]\\
&\quad +z^T(t)(I_N \otimes Q)z(t) - z^T(t-\tau)(I_N \otimes Q)z(t-\tau) + \tau\dot{z}^T(t)(I \otimes S)\dot{z}(t)\\
&\quad -\int_{t-\tau}^{t}\dot{z}^T(\alpha)(I \otimes S)\dot{z}(\alpha)d\alpha - \sum_{i=1}^{N}\frac{\gamma_{wi}}{\eta_{wi}}\tilde{W}_{ij}^T\bar{W}_{ij} - \sum_{i=1}^{N}\frac{\gamma_{ci}}{\beta_{ci}}c_i^2. \quad (25)\end{aligned}$$

According to Lemma 3, it becomes

$$\begin{aligned}\dot{V}(t) &\le z^T(t)[I_N \otimes (\bar{A}_1^TP + P\bar{A}_1)]z(t) + z^T(t)[I_N \otimes (P\bar{A}_2 + \bar{A}_2^TP)]z(t-\tau)\\
&\quad -2z^T(t)[cL \otimes (PBJ\bar{K}_1)]z(t) - 2z^T(t)[cL \otimes (PBJ\bar{K}_2)]z(t-\tau)\\
&\quad +2z^T(t)[I_N \otimes (PBJ)]\sum_{i=1}^{N}[-\tilde{W}_{ij}^T(t)\varphi_{ij}(x_i,x_j) + \varepsilon_{ij}(t)] + z^T(t)(I_N \otimes Q)z(t)\\
&\quad -z^T(t-\tau)(I_N \otimes Q)z(t-\tau) + \tau\dot{z}^T(t)(I_N \otimes S)\dot{z}(t) - \int_{t-\tau}^{t}\dot{z}^T(\alpha)(I_N \otimes S)\dot{z}(\alpha)d\alpha\\
&\quad + \eta^T(t)(I_N \otimes \Psi)\eta(t) + \tau z^T(t)(I_N \otimes T)z(t) - \sum_{i=1}^{N}\frac{\gamma_{wi}}{\eta_{wi}}\tilde{W}_{ij}^T\bar{W}_{ij} - \sum_{i=1}^{N}\frac{\gamma_{ci}}{\beta_{ci}}c_i^2. \quad (26)\end{aligned}$$

Denoting $\sum_{i=1}^{n}\varepsilon_{ij} = \bar{\varepsilon}_j$, because the following two inequalities hold

$$2z^T(t)[I_N \otimes (PB_jJ_j)\bar{\varepsilon}_j] \le z^T(t)(I_N \otimes R)z(t) + \bar{\varepsilon}_j^T[I_N \otimes (J_j^TB_j^TP^TR^{-1}PB_jJ_j)]\bar{\varepsilon}_j. \quad (27)$$

$$-\frac{\gamma_{wi}}{\eta_{wi}}\tilde{W}_{ij}^T\bar{W}_{ij} \le -\frac{\gamma_{wi}}{\eta_{wi}}\tilde{W}_{ij}^T\tilde{W}_{ij} + \frac{\gamma_{wi}}{2\eta_{wi}}\tilde{W}_{ij}^T\tilde{W}_{ij} + \frac{\gamma_{wi}}{2\eta_{wi}}W_{ij}^TW_{ij} = -\frac{\gamma_{wi}}{2\eta_{wi}}\tilde{W}_{ij}^T\tilde{W}_{ij} + \frac{\gamma_{wi}}{2\eta_{wi}}W_{ij}^TW_{ij}. \quad (28)$$

Combining with (14) and (15), the following inequality is obtained:

$$\dot{V} \le \eta^T(t)\phi\eta(t) - \frac{\gamma_w}{2\eta_w}tr(\tilde{W}^T\tilde{W}) + \frac{\gamma_w}{2\eta_w}tr(W^TW) - \frac{\gamma_c}{2\beta_c}c^Tc + \bar{\varepsilon}_j^T[I_N \otimes (J_j^TB_j^TP^TR^{-1}PB_jJ_j)]\bar{\varepsilon}_j, \quad (29)$$

where $\phi = \begin{bmatrix}\phi_{11} & \phi_{12} & \phi_{13}\\ * & \phi_{22} & \phi_{23}\\ * & * & \phi_{33}\end{bmatrix}$ with $\phi_{11} = \bar{A}_1^TP + P\bar{A}_1 + Q + R + Y_1^T + Y_1 + \tau T_{11}$, $\phi_{12} = Y_2 + \tau T_{12}$, $\phi_{13} = P\bar{A}_2 - Y_1^T$, $\phi_{22} = \tau(S + T_{22})$, $\phi_{23} = -Y_2^T$, $\phi_{33} = -Q$.

Let $\phi < -\Omega$, in which $\Omega = diag\{\bar{Q}_1, \bar{Q}_2, \bar{Q}_3\}$, multiplying $diag\{P^{-1}, P^{-1}, P^{-1}\}$ on both side of this inequality, define $P^{-1} = V$, then one has

$$\tilde{\phi} = \begin{bmatrix} \tilde{\phi}_{11} & \tilde{\phi}_{12} & \tilde{\phi}_{13} \\ * & \tilde{\phi}_{22} & \tilde{\phi}_{23} \\ * & * & \tilde{\phi}_{33} \end{bmatrix} < 0, \tag{30}$$

where $\tilde{\phi}_{11} = V\bar{A}_1^T + \bar{A}_1 V + VQV + VRV + VY_1^T V + VY_1 V + \tau V T_{11} V + V\bar{Q}_1 V$, $\tilde{\phi}_{12} = VY_2V + \tau VT_{12}V$, $\tilde{\phi}_{13} = \bar{A}_2 V - VY_1^T V$, $\tilde{\phi}_{22} = \tau(VSV + VT_{22}V) + V\bar{Q}_2 V$, $\tilde{\phi}_{23} = -VY_2^T V$, $\tilde{\phi}_{33} = -VQV + V\bar{Q}_3 V$.

Now, we let $B_j\bar{K}_{j1} = M_1$, $B_jJ_j = N$, $\bar{K}_1 V = G_1$, $VY_1V = \bar{Y}_1$, $VY_2V = \bar{Y}_2$, $VT_{11}V = \bar{T}_{11}$, $VQV = \hat{Q}$, $VRV = \hat{R}$, $VSV = \bar{S}$, $B_j\bar{K}_{j2} = M_2$, $\bar{K}_2 V = G_2$, then (29) is equal to (22).

Similarly, multiplying $diag\{P^{-1}, P^{-1}, P^{-1}\}$ on the right and left of (7), so it becomes inequality (23).

Based on the inequality (30), let $\delta = \frac{1}{2}tr(W^TW) + \bar{\varepsilon}_j^T[I_N \otimes (J_j^T B_j^T P^T R^{-1} P B_j J_j)]\bar{\varepsilon}_j$, (29) becomes the following result:

$$\dot{V} \leq -\zeta V(t) + \delta, \tag{31}$$

inequality (31) means that all the signals in closed-loop multi-agent system are bounded.

4 Experimental Results and Analysis

In this section, a simulation example is given to prove the effectiveness of the proposed control method.We consider an multi-agent systems with eight subsystem, which consisting of a leader labeled 1 and seven followers labeled 2, 3, 4, 5, 6, 7, 8, that are shown as Figure 1.

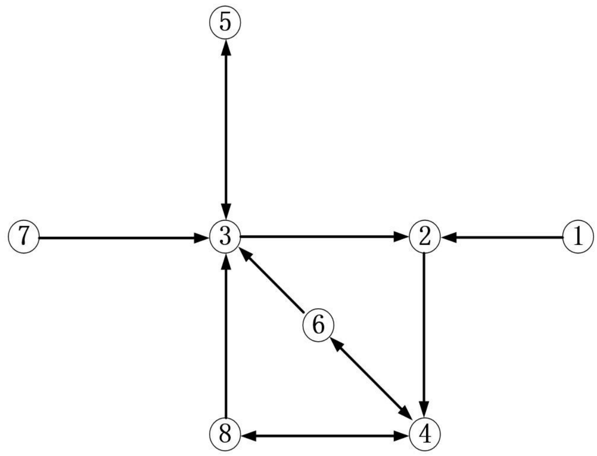

Figure 1: network topology with eight agents

The adjacency matrix is:

$$\begin{bmatrix} 0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&1&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&1 \\ 0&0&1&0&0&0&0&0 \\ 0&0&1&1&0&0&0&0 \\ 0&0&1&0&0&1&0&1 \\ 0&0&1&1&1&0&0&0 \end{bmatrix}. \tag{32}$$

In this multi-agent system, the matrices are provided with different dimensions as: $A_{11} = \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix}$,

$$A_{21} = \begin{bmatrix} 0.1 & -0.3 & -0.4 \\ 0.2 & 0.1 & 0.5 \\ 0 & 0 & -0.5 \end{bmatrix}, A_{31} = \begin{bmatrix} 0.1 & -0.3 & -0.4 & 0.1 \\ 0.2 & 0.1 & 0.5 & 0.3 \\ 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & -0.3 \end{bmatrix},$$

$$A_{41} = \begin{bmatrix} 0.1 & -0.3 & -0.4 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.5 & 0.3 & -0.1 \\ 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & -0.3 & 0 \\ 0 & 0 & 0 & 0 & -0.2 \end{bmatrix}, A_{51} = \begin{bmatrix} 0.1 & -0.3 & -0.4 & 0.1 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.5 & 0.3 & -0.1 & 0.2 \\ 0 & 0 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix},$$

$$A_{61} = \begin{bmatrix} 0.1 & -0.3 & -0.4 & 0.1 & 0.2 & 0.2 & -0.2 \\ 0.2 & 0.1 & 0.5 & 0.3 & -0.1 & 0.2 & -0.5 \\ 0 & 0 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.3 \end{bmatrix},$$

$$A_{71} = \begin{bmatrix} 0.1 & -0.3 & -0.4 & 0.1 & 0.2 & 0.2 & -0.2 & 0.1 \\ 0.2 & 0.1 & 0.5 & 0.3 & -0.1 & 0.2 & -0.5 & 0.5 \\ 0 & 0 & -0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4 \end{bmatrix},$$

$$A_{81} = \begin{bmatrix} 0.1 & -0.3 & -0.4 & 0.1 & 0.2 & 0.2 & -0.2 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.5 & 0.3 & -0.1 & 0.2 & -0.5 & 0.5 & 0.4 \\ 0 & 0 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 \end{bmatrix}. A_{11} = \begin{bmatrix} 1 & 0.5 \\ -0.5 & 0 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, A_{22} = \begin{bmatrix} 0.2 & -0.1 & -0.4 \\ 0.3 & 0.2 & 0.5 \\ 0 & 0 & -0.3 \end{bmatrix}, A_{32} = \begin{bmatrix} 0.2 & -0.1 & -0.4 & -0.1 \\ 0.3 & 0.2 & 0.5 & 0.2 \\ 0 & 0 & -0.3 & 0 \\ 0 & 0 & 0 & -0.4 \end{bmatrix},$$

$$A_{42} = \begin{bmatrix} 0.2 & -0.1 & -0.4 & -0.1 & 0.2 \\ 0.3 & 0.2 & 0.5 & 0.2 & -0.1 \\ 0 & 0 & -0.3 & 0 & 0 \\ 0 & 0 & 0 & -0.4 & 0 \\ 0 & 0 & 0 & 0 & -0.1 \end{bmatrix}, A_{52} = \begin{bmatrix} 0.2 & -0.1 & -0.4 & -0.1 & 0.2 & 0.3 \\ 0.3 & 0.2 & 0.5 & 0.2 & -0.1 & -0.3 \\ 0 & 0 & -0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2 \end{bmatrix},$$

$$A_{62} = \begin{bmatrix} 0.2 & -0.1 & -0.4 & -0.1 & 0.2 & 0.3 & -0.1 \\ 0.3 & 0.2 & 0.5 & 0.2 & -0.1 & -0.3 & 0.2 \\ 0 & 0 & -0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix},$$

$$A_{72} = \begin{bmatrix} 0.2 & -0.1 & -0.4 & -0.1 & 0.2 & 0.3 & -0.1 & 0.5 \\ 0.3 & 0.2 & 0.5 & 0.2 & -0.1 & -0.3 & 0.2 & 0.1 \\ 0 & 0 & -0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3 \end{bmatrix},$$

$$A_{82} = \begin{bmatrix} 0.2 & -0.1 & -0.4 & -0.1 & 0.2 & 0.3 & -0.1 & 0.5 & 0.3 \\ 0.3 & 0.2 & 0.5 & 0.2 & -0.1 & -0.3 & 0.2 & 0.1 & -0.1 \\ 0 & 0 & -0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T, B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T, B_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^T, B_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$B_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T, B_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, B_7 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$B_8 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, F_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, F_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$F_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, F_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, F_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$F_7 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, F_8 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, K_{11} = \begin{bmatrix} -6 & -3 \\ -1 & 0 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} -4.1 & -1.7 & 0.4 \\ -2.2 & -2.1 & -0.5 \end{bmatrix}, K_{31} = \begin{bmatrix} -4.1 & -1.7 & 0.4 & -0.1 \\ -2.2 & -2.1 & -0.5 & -0.3 \end{bmatrix},$$

$$K_{41} = \begin{bmatrix} -4.1 & -1.7 & 0.4 & -0.1 & -0.2 \\ -2.2 & -2.1 & -0.5 & -0.3 & 0.1 \end{bmatrix}, K_{51} = \begin{bmatrix} -4.1 & -1.7 & 0.4 & -0.1 & -0.2 & -0.2 \\ -2.2 & -2.1 & -0.5 & -0.3 & 0.1 & -0.2 \end{bmatrix},$$

$$K_{61} = \begin{bmatrix} -4.1 & -1.7 & 0.4 & -0.1 & -0.2 & -0.2 & 0.2 \\ -2.2 & -2.1 & -0.5 & -0.3 & 0.1 & -0.2 & 0.5 \end{bmatrix},$$

$$K_{71} = \begin{bmatrix} -4.1 & -1.7 & 0.4 & -0.1 & -0.2 & -0.2 & 0.2 & -0.1 \\ -2.2 & -2.1 & -0.5 & -0.3 & 0.1 & -0.2 & 0.5 & -0.5 \end{bmatrix},$$

$$K_{81} = \begin{bmatrix} -4.1 & -1.7 & 0.4 & -0.1 & -0.2 & -0.2 & 0.2 & -0.1 & 0.2 \\ -2.2 & -2.1 & -0.5 & -0.3 & 0.1 & -0.2 & 0.5 & -0.5 & -0.4 \end{bmatrix}. \ K_{12} = \begin{bmatrix} -2 & 1 \\ -2 & -2 \end{bmatrix},$$

$$K_{22} = \begin{bmatrix} -1.2 & 2.1 & 0.4 \\ -1.3 & -0.2 & -0.5 \end{bmatrix}, K_{32} = \begin{bmatrix} -1.2 & 2.1 & 0.4 & 0.1 \\ -1.3 & -0.2 & -0.5 & -0.2 \end{bmatrix},$$

$$K_{42} = \begin{bmatrix} -1.2 & 2.1 & 0.4 & 0.1 & -0.2 \\ -1.3 & -0.2 & -0.5 & -0.2 & 0.1 \end{bmatrix},$$

$$K_{52} = \begin{bmatrix} -1.2 & 2.1 & 0.4 & 0.1 & -0.2 & -0.3 \\ -1.3 & -0.2 & -0.5 & -0.2 & 0.1 & 0.3 \end{bmatrix},$$

$$K_{62} = \begin{bmatrix} -1.2 & 2.1 & 0.4 & 0.1 & -0.2 & -0.3 & 0.1 \\ -1.3 & -0.2 & -0.5 & -0.2 & 0.1 & 0.3 & -0.2 \end{bmatrix},$$

$$K_{72} = \begin{bmatrix} -1.2 & 2.1 & 0.4 & 0.1 & -0.2 & -0.3 & 0.1 & -0.5 \\ -1.3 & -0.2 & -0.5 & -0.2 & 0.1 & 0.3 & -0.2 & -0.1 \end{bmatrix},$$

$K_{82} = \begin{bmatrix} -1.2 & 2.1 & 0.4 & 0.1 & -0.2 & -0.3 & 0.1 & -0.5 & -0.3 \\ -1.3 & -0.2 & -0.5 & -0.2 & 0.1 & 0.3 & -0.2 & -0.1 & 0.1 \end{bmatrix}$. By solving the LMIs (19) and

(20), time delay, the positive matrix and control gain matrices are obtained respectively as: $\tau = 0.2$
$P = \begin{bmatrix} 0.2399 & -0.1842 \\ -0.1842 & 0.6083 \end{bmatrix}$, $\bar{K}_1 = \begin{bmatrix} -0.4473 & 5.7975 \\ 4.4031 & -7.4369 \end{bmatrix}$, $\bar{K}_2 = \begin{bmatrix} 0.2569 & 0.4542 \\ 0.0320 & 0.7431 \end{bmatrix}$.

The nonlinear functions are chosen as:

$$\begin{cases} f(x_1) = [-x_{11}sin(x_{12}) + x_{11}sin(x_{11})cos(x_{12}); x_{11}sin(x_{12})]. \\ f(x_2) = [-x_{21}sin(x_{22}) + x_{22}sin(x_{23})cos(x_{22}); x_{21}sin(x_{22})]. \\ f(x_3) = [-x_{31}sin(x_{32}) + x_{32}sin(x_{33})cos(x_{32}); x_{31}sin(x_{34})]. \\ f(x_4) = [-x_{41}sin(x_{42}) + x_{43}sin(x_{41})cos(x_{44}); x_{41}sin(x_{45})]. \\ f(x_5) = [-x_{51}sin(x_{52}) + x_{53}sin(x_{54})cos(x_{52}); x_{55}sin(x_{56})]. \\ f(x_6) = [-x_{61}sin(x_{62}) + x_{63}sin(x_{64})cos(x_{65}); x_{66}sin(x_{67})]. \\ f(x_7) = [-x_{71}sin(x_{72})cos(x_{74}) + x_{73}sin(x_{75})cos(x_{76}); x_{77}sin(x_{78})]. \\ f(x_8) = [-x_{81}sin(x_{82})cos(x_{83}) + x_{84}sin(x_{85})cos(x_{86}); x_{87}sin(x_{88})cos(x_{89})]. \end{cases} \quad (33)$$

The initial values of the states in the multi-agent system are chosen as: $x_1(0) = \begin{bmatrix} 0.2 & 0.18 \end{bmatrix}$, $x_2(0) = \begin{bmatrix} 0.3 & -0.16 & 0.1 \end{bmatrix}$, $x_3(0) = \begin{bmatrix} 0.19 & 0.24 & 0.15 & 0.2 \end{bmatrix}$, $x_4(0) = \begin{bmatrix} 0.11 & 0.29 & -0.1 & 0.16 & 0.24 \end{bmatrix}$, $x_5(0) = \begin{bmatrix} -0.9 & 0.6 & 0.21 & -0.11 & -0.13 & 0.22 \end{bmatrix}$, $x_6(0) = \begin{bmatrix} 0.25 & 0.2 & 0.1 & 0.24 & 0.2 & 0.11 & 0.2 \end{bmatrix}$, $x_7(0) = \begin{bmatrix} 0.18 & 0.21 & -0.1 & 0.16 & 0.21 & 0.1 & 0.14 & -0.11 \end{bmatrix}$, $x_8(0) = \begin{bmatrix} 0.24 & 0.2 & 0.11 & 0.1 & 0.1 & 0.14 & 0.21 & -0.1 & -0.12 \end{bmatrix}$. The initial values of adaptive parameters $\bar{W}_i(t)$ are given as:
$\bar{W}_1(0) = \begin{bmatrix} 0.49 & 0.45 & 0.41 & 0.46 & 0.39 \end{bmatrix}^T$, $\bar{W}_2(0) = \begin{bmatrix} 0.9 & 0.22 & 0.10 & 0.10 & 0.13 \end{bmatrix}^T$,
$\bar{W}_3(0) = \begin{bmatrix} 0.10 & 0.27 & 0.31 & 0.11 & 0.21 \end{bmatrix}^T$, $\bar{W}_4(0) = \begin{bmatrix} 0.19 & 0.15 & 0.17 & 0.14 & 0.21 \end{bmatrix}^T$,
$\bar{W}_5(0) = \begin{bmatrix} 0.15 & 0.12 & 0.25 & 0.21 & 0.31 \end{bmatrix}^T$, $\bar{W}_6(0) = \begin{bmatrix} 0.6 & 0.3 & 0.7 & 0.2 & 0.9 \end{bmatrix}^T$,
$\bar{W}_7(0) = \begin{bmatrix} 0.17 & 0.19 & 0.15 & 0.17 & 0.14 \end{bmatrix}^T$, $\bar{W}_8(0) = \begin{bmatrix} 0.4 & 0.5 & 0.7 & 0.3 & 0.2 \end{bmatrix}^T$. The simulation results are illustrated in Fig.2-4.

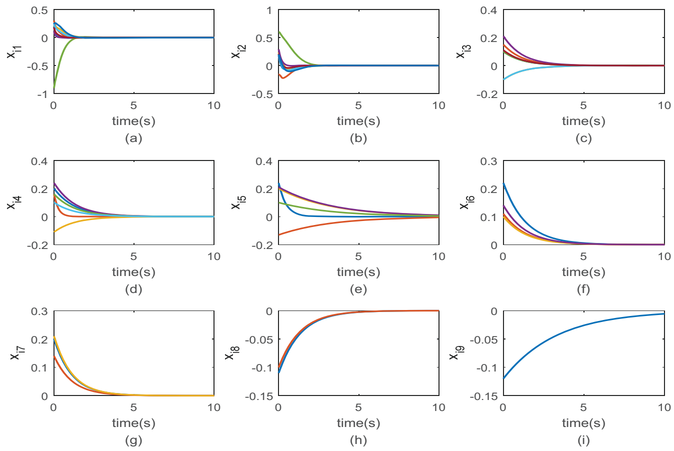

Figure 2: Trajectiories of the state of $x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}, x_{i7}, x_{i8}, x_{i9}$

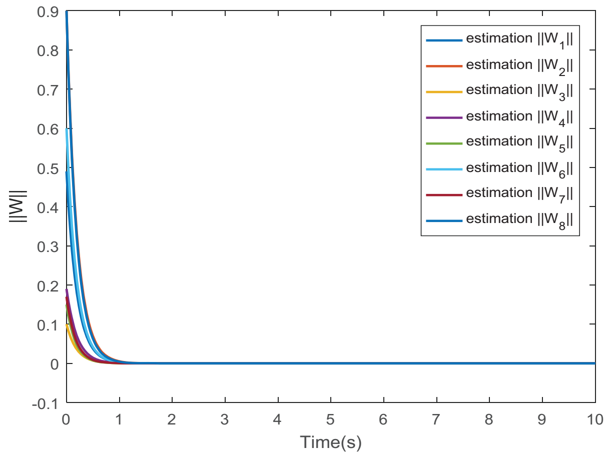

Figure 3: Time response of the adaptive estimation parameters $\bar{W}_i$

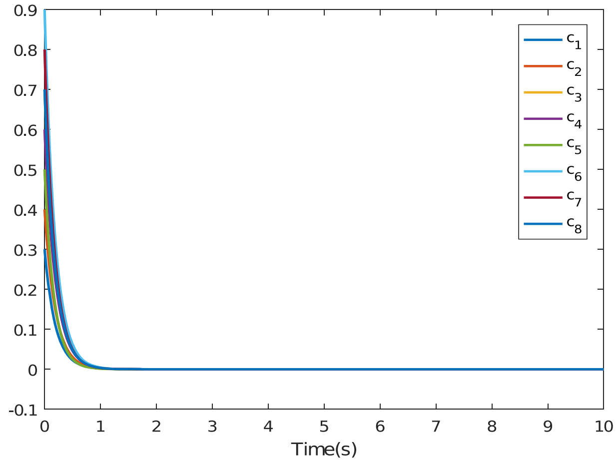

Figure 4: Time response of coupling laws c_i

The states of multi-agents with diverse dimensions and time delays can attain uniform stability using the developed distributed feedback neural network adaptive control, as shown in subfigures (a)-(i) in Fig. 2. Fig. 3 and Fig. 4 show the time responses of RBFNN estimate parameters and coupling weights, which are guaranteed to be semi-globally uniformly ultimately bounded.

5 Conclusion

This research focus on the consensus control of nonlinear heterogeneous multi-agent systems with time delays and nonidentical dimensions. A distributed neural network adaptive control based on similar parameters and time delay is built, with the goal of achieving the consensus of all the agents, and the neural network systems are used to approximate the unknown nonlinear functions in the subsystems. Using the suggested control approach, the states of each follower system can track the state of the leader system. The control technique can be used to control both homogeneous multi-agent systems of the same dimension and agents of different dimensions.

Funding

This work is supported by the National Natural Science Foundation of China under Grant (62003262, 61903298), National Natural Science Foundation of Shaanxi under Grant (2022JM-389, 2019JQ-341),Shaanxi Provincial Department of Science and Technology key project in the field of industry (2018ZDXM-GY-039), Shaanxi Provincial Scientific and Technological Activities for Overseas Staff Preferential Projects under Grant 35.

Author contributions

The authors contributed equally to this work.

Conflict of interest

The authors declare no conflict of interest.

References

[1] Seo, J. and Kim, Y. and Kim, S. and Tsourdos, A. (2017). Collision Avoidance Strategies for Unmanned Aerial Vehicles in Formation Flight, *IEEE Transactions on Aerospace and Electronic Systems*, 53(6), 2718-2734, 2017.

[2] Zhang, B. and Shang, W. and Cong, S. and Li, Z. (2021). Coordinated Dynamic Control in the Task Space for Redundantly Actuated Cable-Driven Parallel Robots, *IEEE/ASME Transactions on Mechatronics*, 26(5), 2396-2407, 2021.

[3] Sieber, D. and Hirche, S. (2019). Human-Guided Multirobot Cooperative Manipulation, *IEEE/ASME Transactions on Control Systems Technology*, 27(4), 1492-1509, 2019.

[4] Sharf, M. and Zelazo, D. (2019). Analysis and Synthesis of MIMO Multi-Agent Systems Using Network Optimization, *arXiv e-prints*, 64(11), 4512-4524, 2019.

[5] Reynolds, C. W. and Flocks, H. (1987). And Schools: A Distributed Behavioral Model *ACM SIGGRAPH Computer Graphics*, 21(4), 25-34, 1987.

[6] Vicsek, T. and Czirok, A. and Ben-Jacob, E. and Cohen, I. and Sochet, O. (1995). Novel Type of Phase Transition in a System of Self-Driven Particles, *Physical Review Letters*, 75(6), 1226-1229, 1995.

[7] Jadbabaie, A. and Jie, L. and Morse, A. S. (2003). Coordination of groups of mobile autonomous agents using nearest neighbor rules, *IEEE Transactions on Automatic Control*, 48(6), 988-1001, 2003.

[8] Liu, D. and Wang, D. and Li, H. (2014). Decentralized Stabilization for a Class of Continuous-Time Nonlinear Interconnected Systems Using Online Learning Optimal Control Approach, *IEEE Transactions on Neural Networks and Learning Systems*, 25(2), 418-428, 2014.

[9] Han, X. and Mandal, S. and Pattipati, K. R. and Kleinman, D. L. and Mishra, M. (2017). An Optimization-Based Distributed Planning Algorithm: A Blackboard-Based Collaborative Framework, *IEEE Transactions on Systems Man and Cybernetics Systems*, 44(6), 673-686, 2017.

[10] Zhang, H. and Feng, T. and Yang, G. H. and Liang, H. (2010). Distributed Cooperative Optimal Control for Multiagent Systems on Directed Graphs: An Inverse Optimal Approach, *IEEE Transactions on Cybernetics*, 45(7), 1315-1326, 2015.

[11] Lin, J. and Hwang, K. and Wang, Y. (2014). A Simple Scheme for Formation Control Based on Weighted Behavior Learning, *IEEE Transactions on Neural Networks and Learning Systems*, 25(6), 1033-1044, 2014.

[12] Loia, V. and Vaccaro, A. (2014). Decentralized Economic Dispatch in Smart Grids by Self-Organizing Dynamic Agents, *IEEE Transactions on Systems Man and Cybernetics Systems*, 44(4), 397-408, 2014.

[13] Perez, I. J. and Cabrerizo, F. J. and Alonso, S. and Herrera-Viedma, E. (2017). A New Consensus Model for Group Decision Making Problems With Non-Homogeneous Experts, *IEEE Transactions on Systems Man and Cybernetics Systems*, 44(4), 494-498, 2017.

[14] Owliya, M. and Saadat, M. and Jules, G. G. and Goharian, M. and Anane, R. (2013). Agent-Based Interaction Protocols and Topologies for Manufacturing Task Allocation, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 43(1), 38-52, 2013.

[15] Duan, Z. and Wang, Q. and Feng, Z. and Chen, G. (2008). Estimation of Delay on Synchronization Stability in a Class of Complex Systems with Coupling Delays, *Taiwanese Journal of Mathematics*, 12(8), 2141-2154, 2008.

[16] Cao, Y. J. and Wu, Q. H. (1996). A note on stability of analog neural networks with time delays, *IEEE Transactions on Neural Networks*, 7(6), 1533-1535, 1996.

[17] Chen, T. (2001). Global convergence of delayed dynamical systems, *IEEE Transactions on Neural Networks*, 12(6), 1532-1536, 2001.

[18] Peng, Z. and Wang, D. and Zhang H. and Sun G. (2014). Distributed Neural Network Control for Adaptive Synchronization of Uncertain Dynamical Multiagent Systems, *IEEE Transactions on Neural Networks and Learning Systems*, 25(8), 1508-1519, 2014.

[19] Howard A. (2006). Experiments with a Large Heterogeneous Mobile Robot Team: Exploration, Mapping, Deployment and Detection, *International Journal of Robotics Research*, 25(5), 431-447, 2006.

[20] Chung, S. J. and Slotine, J. (2009). Cooperative Robot Control and Concurrent Synchronization of Lagrangian Systems, *IEEE Transactions on Robotics*, 25(3), 686-700, 2009.

[21] Fan, Y. and Xiao, T. and Li, Z. (2020). Distributed Fuzzy Adaptive Control for Heterogeneous Nonlinear Multiagent Systems with Similar Composite Structure, *Complexity*, https://doi: 10.1155/2020/4081904, 2020.

[22] Qin, B. and Fan, Y. and Xiao, T. and Li, Z. (2021). Distributed type-2 fuzzy adaptive control for heterogeneous nonlinear multiagent systems, *Asian Journal of Control*, https://doi:org/10.1002/asjc.2566, 2021.

[23] Zhao, J. and Hill, D. J. and Tao, L. (2011). Synchronization of Dynamical Networks With Nonidentical Nodes: Criteria and Control, *IEEE Transactions on Circuits and Systems I: Regular Papers*, 58(3), 584-594, 2011.

[24] Lv, Y. and Li, Z. and Duan, Z. and Feng, G. (2015). Novel Distributed Robust Adaptive Consensus Protocols for Linear Multi-agent Systems with Directed Graphs and External Disturbances, *International Journal of Control*, 90(2), 137-147, 2015.

[25] Tong, S. and Wang, T. and Li, Y. (2014). Fuzzy Adaptive Actuator Failure Compensation Control of Uncertain Stochastic Nonlinear Systems With Unmodeled Dynamics, *IEEE Transactions on Fuzzy Systems*, 22(3), 563-574, 2014.

[26] Last, E. (2014). Linear Matrix Inequalities in System and Control Theory, *Proceedings of the IEEE*, 86(12), 2473-2474, 1994.

Cite this paper as:
Qin, B.; Fan, Y.; Yang, S. (2022). (2022). Distributed Adaptive Control for Nonlinear Heterogeneous Multi-agent Systems with Different Dimensions and Time Delay, *International Journal of Computers Communications & Control*, 17(4), 4800, 2022.
https://doi.org/10.15837/ijccc.2022.4.4800