Supplier Selection Model Based on D Numbers and Transformation Function

Leihui Xiong, Xiaoyan Su, Hong Qian

Leihui Xiong

School of Automation Engineering, Shanghai University of Electric Power
Shanghai 200090, China
xionglh@mail.shiep.edu.cn

Xiaoyan Su*

School of Automation Engineering, Shanghai University of Electric Power
Shanghai 200090, China
*Corresponding author: suxiaoyan@shiep.edu.cn.

Hong Qian

School of Automation Engineering, Shanghai University of Electric Power
Shanghai 200090, China
qianhong.sh@163.com

Abstract

Selecting reasonable suppliers can effectively improve the efficiency of enterprise supply chain management. Among them, expert evaluation is an important part of supplier selection problem, but the uncertainty, fuzziness and incompleteness of expert opinions make supplier selection problem difficult to solve. In order to systematically and effectively solve the uncertainty, ambiguity and incompleteness in supplier selection problem, this paper presents a new supplier selection method based on D numbers and transformation function. First, fuzzy preference relation is generated based on the decision matrix of pairwise comparisons given by experts. D numbers which can effectively deal with uncertain information extend fuzzy preference relation (D matrix). Second, the D matrix is converted into a crisp matrix form based on the integration representation of D numbers according to different situations whether or not the information in D matrix is complete. Third, the crisp matrix is converted into judgement matrix by using the transformation functions. Finally, analytic hierarchy process (AHP) method is applied based on the judgment matrix to give a priority weights for decision making. Three numerical examples and application of the supplier selection are used to show the feasibility and effectiveness of the proposed method.

Keywords: D numbers, transformation functions, analytic hierarchy process, fuzzy preference relation.

1 Introduction

Multi-attribute decision making is an important part of decision theory and modern decision science [1]. It has been widely used in investment decision making [2], project evaluation[3], scheme selection[4], factory site selection [5, 6], comprehensive evaluation of economic benefits [7], etc. Therefore, it is of great practical and theoretical significance to study effective and practical multi-attribute decision making methods.

As the market environment becomes more and more complex and the competition among enterprises becomes more and more fierce, the supplier selection in multi-attribute decision making problem attracts more and more attention [8]. Supplier selection and evaluation is an important activity for enterprises to determine their own product suppliers, and it is also the premise to optimize the supplier management system. Enterprise supplier selection evaluation has a set of strict process steps. The enterprise selects suppliers mainly from four aspects: enterprise performance, business capability, quality system and enterprise environment [9]. Different index elements can be obtained by refining these four aspects, and then the evaluation of each index can be established by analyzing these index elements. Finally, AHP and other methods are used to make decisions [10].

Traditional supplier selection process can solve this problem to a certain extent, but with the deepening of the research on supplier selection, scholars find that: due to the complexity of practical problems and the limitation of decision-makers' knowledge and experience, there are many uncertainties and incomplete semantic information in decision makers' determination of attribute values and expression of preference information, which leads to a large amount of uncertain information to be dealt with in supplier selection [11, 12]. In order to improve the supplier selection method, some scholars use fuzzy number theory, grey theory, D-S evidence theory to deal with uncertain information in supplier selection problem.

Rashidi et al.[13] applied fuzzy data envelopment analysis (DEA), and the technique for order of preference by similarity to ideal solution (TOPSIS) to the selection of logistics service providers, and compared and analyzed the two models to demonstrate the effectiveness of these two methods. Chen et al. [14] combined TOPSIS, decision making trial and evaluation laboratory (DEMATEL) and rough fuzzy method in dealing with supplier selection problem. This method uses fuzzy number to represent internal and external uncertainties, and DEMATEL method to reflect the interaction between attributes. Xing et al. [15] proposed a new supplier selection model based on interval two-type trapezoidal fuzzy partial order and Choquet integral, which can effectively deal with the interaction between criteria and consider consensus. In general, although fuzzy number theory can deal with subjective opinions effectively, fuzzy number itself is better at dealing with fuzzy information rather than other uncertain information.

Hekmat et al. [16] used grey principal component analysis (PCA) and DEA to deal with supplier selection in uncertain environment. This method can deal with uncertain information as well as insufficient data information and related situations. Babak et al.[17] ranked suppliers based on TOPSIS and grey theory, and applied this method to the selection of battery suppliers for electric vehicles. In a word, grey theory can express the incompleteness and uncertainty of information to a certain extent, but it is mainly used for fuzzy prediction and cannot express the uncertain information intuitively.

Sureeyatanapas et al. [18] solve the supplier selection problem based on D-S evidence theory and TOPSIS method. In this method, D-S evidence theory is used to construct decision matrix, and then TOPSIS method is used for further fusion. Zhang et al. [19] combined D-S evidence theory and analytic network process (ANP), and this model effectively solved the problem of supplier selection in an uncertain environment and considered the interaction between indicators. Fei et al. [20] solved the supplier selection problem based on D-S evidence theory and an elimination and choice translating reality (ELECTRE), which can better analyze the priority relationship between suppliers while dealing with uncertain information. In general, D-S evidence theory [22] [23] can better process and express uncertain information, so it is widely used in this field. However, D-S evidence theory cannot deal with the incomplete identification framework and the correlation of basic elements.

To sum up, the above methods have certain defects in dealing with uncertain information in supplier selection. Therefore, the method applying D numbers to multi-attribute decision making is proposed [21]. D numbers can effectively deal with the uncertainty and fuzziness of information. At

Table 1: Comparison of methods to deal with uncertain information in supplier selection problem

Method	Feature 1	Feature 2	Feature 3
DEA-TOPSIS method [13]	$\sqrt{}$	$\times$	$\times$
TOPSIS, DEMATEL and rough fuzzy method [14]	$\sqrt{}$	$\times$	$\times$
Interval Type-2 trapezoidal fuzzy numbers method [15]	$\sqrt{}$	$\times$	$\times$
Grey PCA-DEA method [16]	$\sqrt{}$	$\times$	$\times$
TOPSIS and grey theory method [17]	$\sqrt{}$	$\times$	$\times$
D-S evidence theory and TOPSIS method [18]	$\sqrt{}$	$\sqrt{}$	$\times$
D-S evidence theory and ELECTRE method [19]	$\sqrt{}$	$\sqrt{}$	$\times$
D-AHP method [39]	$\sqrt{}$	$\sqrt{}$	$\sqrt{}$
D-ANP method [41]	$\sqrt{}$	$\sqrt{}$	$\sqrt{}$
The proposed method	$\sqrt{}$	$\sqrt{}$	$\sqrt{}$

the same time, D numbers extends D-S evidence theory to a certain extent , which can deal with the situation where the identification framework is incomplete and the basic elements are not independent from each other.

Considering the advantages of D numbers[24, 25, 26, 27, 28, 29, 30, 31] in dealing with uncertain information, D numbers is widely used in various fields [32, 33, 34, 35, 36, 37, 38]. Deng et al. [39, 40] solved the supplier selection problem based on D numbers and analytic hierarchy process (AHP). On the basis of Deng, Fei et al. [41] selected the optimal supplier based on D numbers and network analysis method (ANP). However, the applications of these methods may get counter-intuitive results under some situations.

This paper presents a new supplier selection method based on D numbers and transformation function. Compared with the method of Deng et al. [39], the method in this paper can deal with the situations where the expert opinions are quite uncertain and the decision-making objects are similar. In addition, in the process of consistency evaluation, the method in this paper is more consistent with the actual situation and has a good theoretical basis. (Table 1 shows the comparison of methods for dealing with uncertain information in supplier selection, in which Feature 1 indicates that uncertain information is expressed to a certain extent, Feature 2 indicates that uncertain information can be expressed intuitively, and Feature 3 indicates that it can handle the situation where the identification framework is incomplete and the basic elements are not independent.)

The rest of this paper is organized as follows. In section 2, the preliminaries on AHP, D numbers and transformation functions are briefly introduced. In section 3, a new method to process uncertain information in supplier selection problem is proposed and some examples which can be handled by the method in this paper but cannot be handled by Deng et al.'s method [39] are given. In section 4, an application of the supplier selection is illustrated to show the rationality of this new method. In section 5, conclusions.

2 Preliminaries

2.1 Analytic hierarchy process (AHP) [45]

The steps for the AHP are shown below: Firstly, the hierarchical structure of the problem is constructed, and then the pairwise comparison matrix is constructed based on certain criteria.

Definition 1:When the numbers of elements is n, the pairwise comparison matrix is defined as follow.

$$W = \begin{bmatrix} w_{11} \cdots w_{1n} \\ w_{21} \ddots \vdots \\ w_{n1} \cdots w_{nn} \end{bmatrix} \tag{1}$$

Where w_{ij} measures the relationship that i is more important than j. Thirdly, we calculate the eigenvector corresponding to the largest eigenvalue of matrix W. Finally, the matrix W's consistency

Table 2: The value of RI

Dimension	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

is verified by CR which is defined as follow:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{2}$$

$$CR = \frac{CI}{RI} \tag{3}$$

The value of random consistency index (RI) is shown in Table 2.

We can accept the consistency of the pairwise comparison matrix, when CR is less than 0.1. Finally, normalizing the eigenvector to obtain the final weights.

2.2   D numbers [39]

D numbers is an extension of Dempster-Shafer theory, which can deal with the uncertainty information. D numbers have wider application than Dempster-Shafer theory, because D numbers do not have constrain of integral and independent assumption.

Definition 2:Let Ω be a finite nonempty set, D number is a mapping D: $2^{\Omega} \to [0,1]$, which is given as follows:

$$\sum_{B \subseteq \Omega} D(B) \le 1 \quad and \quad D(\phi) = 0 \tag{4}$$

where ϕ is an empty set and B is a subset of Ω. If $\sum_{B \subseteq \Omega} D(B) = 1$ the information is assumed to be complete. If $\sum_{B \subseteq \Omega} D(B) < 1$, which means the information is incomplete. There is an example to show a D numbers. Suppose a system's security needs to be accessed by 10 experts. The "medium" security level is supported by 4 experts. The "high" security level is supported by 3 experts. The remaining experts do not generalize a conclusion because of their limitations in knowledge. We can express this situation using D numbers as follow.

$$D(\{high\}) = 0.3, D(\{medium\}) = 0.4$$

The sum of high and medium level less than one. Because the information is incomplete. Fuzzy numbers and D-S theory cannot describe this well.

Definition 3: Let $D = (\{b_1, v_1\}, \{b_2, v_2\}, \cdots, \{b_i, v_i\}, \cdots \{b_n, v_n\})$ be a D number, the integration representation of D numbers can be calculated as follow:

$$I(D) = \sum_{i=1}^{n} b_i v_i \tag{5}$$

2.3   Transformation functions [42]

Definition 4: Suppose that we have a set of alternatives, $X = \{x_1, \cdots, x_n\}$, and associated with it a reciprocal multiplicative preference relation $A = (a_{ij})$ with $a_{ij} \in [1/9, 9]$. Then, the corresponding reciprocal fuzzy preference relation, $P = (p_{ij})$ with $p_{ij} \in [0,1]$, associated with A is given as follows:

$$p_{ij} = g(a_{ij}) = \frac{1}{2} \times (1 + \log_9 a_{ij}) \tag{6}$$

With such a transformation function g we can relate the research issues obtained for both kinds of preference relations.
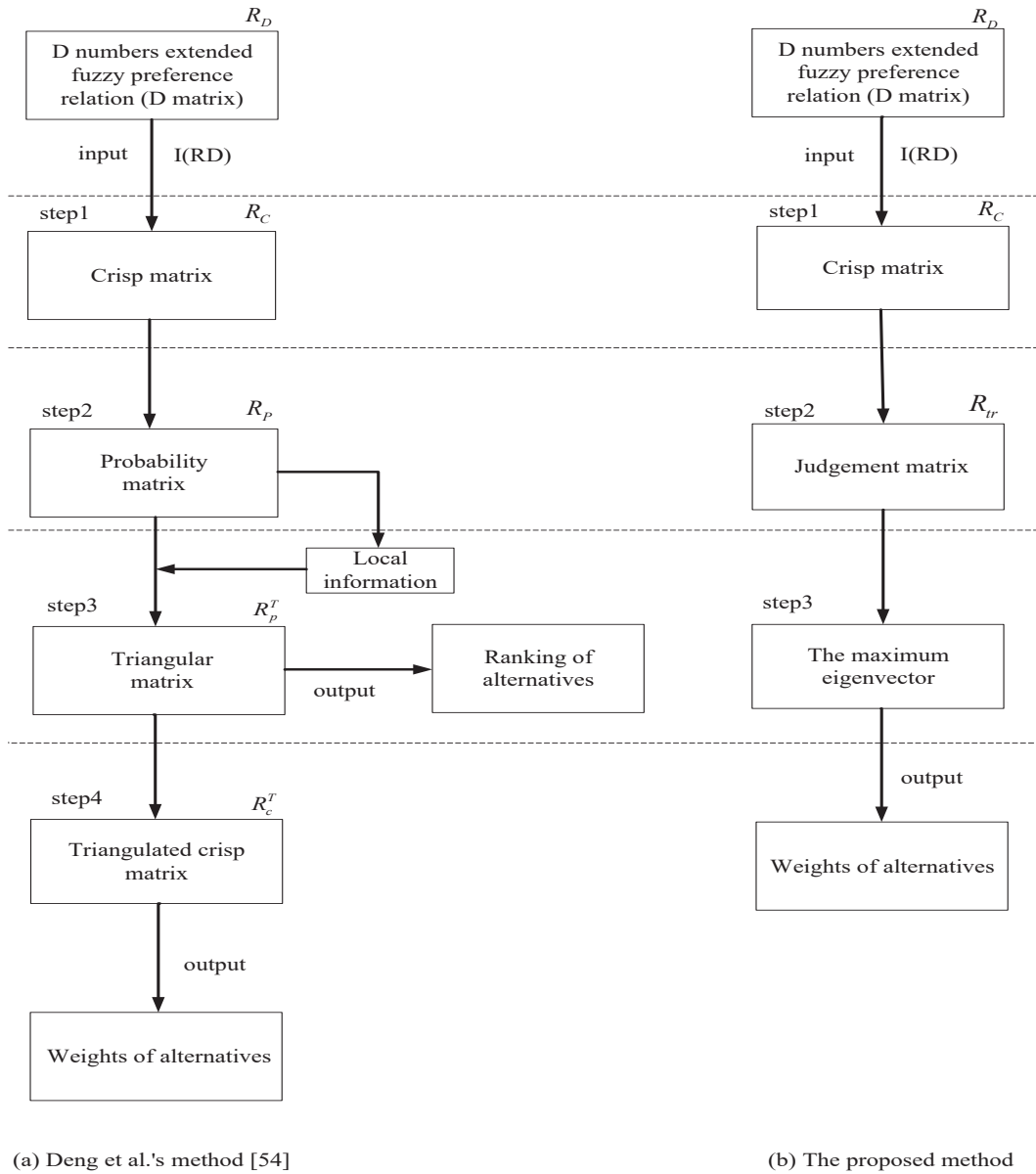
Figure 1: Flow chart of Deng et al.'s method and the proposed method in deriving priority weights of alternatives

3 Proposed method

The main flow chart of the proposed method compared with Deng et al.'s method [39] are shown in Figure 1. More details are illustrated as follows.

3.1 Introduction of Deng et al.'s method [39]

After D numbers extended fuzzy preference relation (D matrix) is constructed, we can gain the crisp matrix by the integration representation of D numbers. After that the main problem is to gain the probability matrix. Two different processes are given in Deng et al.'s method for dealing with complete information and incomplete information to gain the probability matrix. Assume that c_{ij} is the element in the i-th row and j-th column of crisp matrix.

(1) D matrix with complete information

When $c_{ij} + c_{ji} = 1.0$, We believe that c_{ij} has complete information. (i) When the c_{ij} is greater than 0.5, we replace c_{ij} with 1. (ii) When the c_{ij} is lesser than 0.5, we replace c_{ij} with 0.

(2) D matrix with incomplete information

When $c_{ij} + c_{ji} < 1.0$, We believe that c_{ij} has incomplete information. (i) If $c_{ij} \geq 0.5$ or $c_{ji} \geq 0.5$, the larger one is replaced by 1 and the smaller one is replaced by 0. (ii) If $c_{ij} < 0.5, c_{ji} < 0.5$, we can use Equation (7) and Equation (8) to get elements of the probability matrix.

$$C'_{ij} = 1 - \frac{(0.5 - C_{ij})}{1 - (C_{ij} + C_{ji})} \tag{7}$$

$$C'_{ji} = 1 - \frac{(0.5 - C_{ji})}{1 - (C_{ij} + C_{ji})} \tag{8}$$

where C_{ij} means that the row i and column j of the crisp matrix. C'_{ij} means that the row i and column j of the probability matrix.

And then using triangularization method to convert the probability matrix into the triangular matrix which is used to rank the elements. This method sums up the elements in each row, records the number of rows corresponding to the largest item as i, removes the i-th row and the j-th column of the matrix, and loops until the matrix is empty. The order of deletion is sort of elements. The triangular matrix can be obtained by using this sort. Finally, the interval of weights are calculated by introducing the variable λ which reflects the information of the pairwise comparison.

3.2 An improved D-AHP method

3.2.1 D matrix

D matrix is an extension of fuzzy preference relation. Tanino et al. [43] propose the fuzzy preference relation to deal with ambiguity in expert language. For the multiplicative preference relation, the diagonal elements are the inverse of each other, i.e. $a_{ij} \times a_{ji} = 1$. While the fuzzy preference relation is subjected to an additive reciprocal, i.e. $r_{ij} + r_{ji} = 1$.

Suppose $A = \{A_1, A_2, ..., A_n\}$ is a set of alternatives. A fuzzy set $A \times A$, which is calculated by a membership function ($\mu_R : A \times A \to [0, 1]$), represent a fuzzy preference relation R as follows.

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} A_1 \quad A_2 \quad \cdots \quad A_n \\ \left[\begin{array}{cccc} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{array} \right] \end{array} \tag{9}$$

where $r_{ij} = \mu_R (A_i, A_j), \forall i, j \in \{1, 2, \ldots, n\}$. r_{ij} represents the degree of preference for alternative A_i over alternative A_j. The five scenarios for r_{ij} are as follows: (1)When $r_{ij} = \mu_R (A_i, A_j) = \mu_R (A_i, A_j) = 0$, it means that A_j is absolutely preferred to A_i; (2)When $r_{ij} = \mu_R (A_i, A_j) \in (0, 0.5)$, it means that A_j is preferred to A_i to some degree; (3)When $r_{ij} = \mu_R (A_i, A_j) = 0.5$, it means that

indifference between A_i and A_j; (4)When $r_{ij} = \mu_R(A_i, A_j) \in (0.5, 1)$, it means that A_i is preferred to A_j to some degree; (5)When $r_{ij} = \mu_R(A_i, A_j) = 1$, it means that A_i is absolutely preferred to A_j. At the same time, R must satisfy three constraints: (1) $r_{ij} \geq 0$; (2) $r_{ij} + r_{ji} = 1, \forall i, j \in \{1, 2, \ldots, n\}$; (3) $r_{ii} = 0.5, \forall i, j \in \{1, 2, \ldots, n\}$.

With this method, linguistic values can be a way of generating preference information. But the fuzzy preference relation still has some drawbacks. Firstly, there will be inconsistencies in the fuzzy preference. How to measure the consistency of the fuzzy preference is still an open question. Secondly, the fuzzy preference can't use in some situations. For example, there are two schemes A_1, A_2 and 10 experts give their idea about them.

Situation 1: 8 experts give the conclusion that $r_{12} = 0.7$. The other 2 experts give the conclusion that $r_{12} = 0.6$.

Situation 2: 6 experts give the conclusion that $r_{12} = 0.8$. Because of the lack of relevant knowledge, the other 4 experts did not give suggestions.

In both cases, the use of the fuzzy preference alone does not adequately represent the information. To solve the above shortcomings, this paper combine the fuzzy preference relation with D number and replaces the r_{ij} in R with the D number, then the resulting new matrix is called D matrix. We can transform the fuzzy preference relation R (see Equation (9)) into D matrix R_D (see Equation (10)).

$$
R_D = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array}
\begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \end{array} \\
\left[\begin{array}{cccc}
D_{11} & D_{12} & \cdots & D_{1n} \\
D_{21} & D_{22} & \cdots & D_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
D_{n1} & D_{n2} & \cdots & D_{nn}
\end{array} \right] \end{array}
\tag{10}
$$

where R_D satisfies the following constraints:
$(1) D_{ij} = \left\{ \left(b_1^{ij}, v_1^{ij} \right), \left(b_2^{ij}, v_2^{ij} \right), \ldots, \left(b_m^{ij}, v_m^{ij} \right) \right\}, \forall i, j \in \{1, 2, \ldots, n\}$
$(2) D_{ji} = \left\{ \left(1 - b_1^{ij}, v_1^{ij} \right), \left(1 - b_2^{ij}, v_2^{ij} \right), \ldots, \left(1 - b_m^{ij}, v_m^{ij} \right) \right\}, \forall i, j \in \{1, 2, \ldots, n\}$
$(3) b_k^{ij} \in [0, 1], \forall k \in \{1, 2, \ldots, m\}$

Viewed from another perspective, the fuzzy preference is a special case of D matrix. When $m = 1, v_1^{ij} = 1, \forall i, j \in \{1, 2, \ldots, n\}$, b_1^{ij} is the same thing as r_{ij}.

Situation 1 and Situation 2 can be expressed in terms of D numbers as follows.
Situation 1:

$$
R_{D_1} = \begin{array}{c} A_1 \\ A_2 \end{array}
\begin{array}{c} \begin{array}{cc} A_1 & \qquad\qquad A_2 \end{array} \\
\left[\begin{array}{cc}
\{(0.5, 1.0)\} & \{(0.7, 0.8), (0.6, 0.2)\} \\
\{(0.3, 0.8), (0.4, 0.2)\} & \{(0.5, 1.0)\}
\end{array} \right] \end{array}
\tag{11}
$$

Situation 2:

$$
R_{D_2} = \begin{array}{c} A_1 \\ A_2 \end{array}
\begin{array}{c} \begin{array}{cc} A_1 & A_2 \end{array} \\
\left[\begin{array}{cc}
\{(0.5, 1.0)\} & \{(0.8, 0.6)\} \\
\{(0.2, 0.6)\} & \{(0.5, 1.0)\}
\end{array} \right] \end{array}
\tag{12}
$$

3.2.2 Priority weights of alternatives based on D matrix

In the previous section we defined the D matrix and explained the advantages of the D matrix. The generation of priority weight is the key step of the MADM and we can calculate priority weight by fuzzy preference relation. But how to get it by matrix D. The proposed method will discuss the problem in two cases.

Case 1: D matrix with complete information   Assume all the experts participate in the evaluation, and we get a D matrix with complete information on all the elements. That is to say, in Equation (10) $v_1^{ij} + v_2^{ij} + \ldots + v_m^{ij} = 1, \forall i, j \in \{1, 2, \ldots, n\}$. For example, there is a D matrix with complete

information. (see Equation (13)). Take the following steps to get the priority weights.

$$R_D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} \{(0.5, 1.0)\} & \{(0.2, 1.0)\} & \{(0.7, 0.8), (0.8, 0.2)\} \\ \{(0.8, 1.0)\} & \{(0.5, 1.0)\} & \{(0.9, 1.0)\} \\ \{(0.3, 0.8), (0.2, 0.2)\} & \{(0.1, 1.0)\} & \{(0.5, 1.0)\} \end{array} \right] \end{array} \tag{13}$$

Step1. Use the integration representation of D numbers (see Definition 3) to calculate the crisp matrix. For example, we can calculate $D_{13} = \{(0.7, 0.8), (0.8, 0.2)\}$ which is the element in Equation (13), according to Equation (5).

$$I(D_{13}) = 0.7 \times 0.8 + 0.8 \times 0.2 = 0.72$$

And then, we can get the crisp matrix R_c.

$$R_c = I(R_D) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} 0.5000 & 0.2000 & 0.7200 \\ 0.8000 & 0.5000 & 0.9000 \\ 0.2800 & 0.1000 & 0.5000 \end{array} \right] \end{array} \tag{14}$$

Step 2. Convert a crisp matrix to a judgement matrix. For instance, we can calculate the element $R_c(1, 3) = 0.72$ of the crisp matrix Equation (14), according to Equation (6).

$$R_{tr}(1, 3) = 9^{-1+2 \times 0.72} = 2.6295$$

Consequently, a judgement matrix R_{tr} is derived as follow.

$$R_{tr} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} 1.0000 & 0.2676 & 2.6295 \\ 3.7372 & 1.0000 & 5.7995 \\ 0.3803 & 0.1724 & 1.0000 \end{array} \right] \end{array} \tag{15}$$

Step 3. Calculate the maximum eigenvector and eigenvalue of the judgement matrix. The eigenvalues and eigenvectors can express the information of the matrix to the greatest extent. According to the above steps, the judgement matrix we get is similar to the paired comparison matrix in AHP, both of which are multiplicative preference matrices. The elements in the judgement matrix are on an interval of $[1/9, 9]$, which is same as the pairwise comparison matrix in AHP method. So we use the same treatment as AHP for the judgement matrix. According to formula $R_{tr}x = \lambda_{max}x$ and after normalization, we can get the final weights of alternatives A_1, A_2 and A_3 in D matrix shown in Equation (13).

$$\omega_1 : \omega_2 : \omega_3 = 0.2180 : 0.6832 : 0.0988$$

The ranking of alternatives is $A_2 \succ A_1 \succ A_3$ (The '$\succ$' sign means 'better than'). Similarly, consistency indicators CR can be obtained. In this example, $CR = 0.0155 < 0.1$, so the consistency of the D matrix in Equation (13) is acceptable.

Case 2: D matrix with incomplete information Assume part of experts participate in the evaluation, and we get a D matrix with incomplete information on some of the elements. That is to say, in Equation (10), $v_1^{ij} + v_2^{ij} + \ldots + v_m^{ij} < 1, \forall i, j \in \{1, 2, \ldots, n\}$. For example, there is a D matrix with incomplete information. (see Equation (16)). Take the following steps to get the priority weights.

$$R_D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} \{(0.5, 1.0)\} & \{(0.2, 1.0)\} & \{(0.7, 0.8)\} \\ \{(0.8, 1.0)\} & \{(0.5, 1.0)\} & \{(0.9, 1.0)\} \\ \{(0.3, 0.8)\} & \{(0.1, 1.0)\} & \{(0.5, 1.0)\} \end{array} \right] \end{array} \tag{16}$$

Step1. In this case, elements with complete information and incomplete information is processed separately.

(1) D_{ij} in D matrix with complete information

If $\sum\limits_{k=1}^{m} v_k^{ij} = 1$, then the integration representation of D numbers is used to calculate the row i and column j element in the crisp matrix.

(2) D_{ij} in D matrix with incomplete information

If $\sum\limits_{k=1}^{m} v_k^{ij} < 1$, a new processing method is proposed as follow.

$$I^{'}(D_{ij}) = \sum_{m=1}^{n} b_m v_m + 0.5 \times (1 - \sum_{m=1}^{n} v_m) \tag{17}$$

where $I^{'}(D_{ij})$ is the row i and column j element in the crisp matrix. When the pros and cons of two events are not clear, people are more likely to assume that they are similar. In fuzzy preference relation, 0.5 is used to express this situation when alternatives are assumed to be equal with respect to a certain criterion. Thus, this method applies 0.5 to represent the preference assignment under the condition of incomplete information.

For example, in Equation (16), $D_{13} = \{(0.7, 0.8)\}$ and $D_{31} = \{(0.3, 0.8)\}$ are elements with incomplete information. According to Equation (17)

$$I^{'}(D_{13}) = 0.7 \times 0.8 + 0.5 \times (1 - 0.8) = 0.66$$

$$I^{'}(D_{31}) = 0.3 \times 0.8 + 0.5 \times (1 - 0.8) = 0.34$$

After dealing with the above two cases, then we can get the results of the crisp matrix R_c.

$$R_c = I(R_D) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \begin{bmatrix} 0.5000 & 0.2000 & 0.6600 \\ 0.8000 & 0.5000 & 0.9000 \\ 0.3400 & 0.1000 & 0.5000 \end{bmatrix} \tag{18}$$

Step 2. Convert a crisp matrix to a judgement matrix. Just like Case 1, for instance, we can calculate the element $R_{c13} = 0.66$ of the crisp matrix Equation (18), according to Equation (6).

$$R_{tr13} = 9^{-1+2\times 0.66} = 2.0200$$

Consequently, a judgement matrix R_{tr} is derived as follow.

$$R_{tr} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \begin{bmatrix} 1.0000 & 0.2676 & 2.0200 \\ 3.7372 & 1.0000 & 5.7995 \\ 0.4950 & 0.1724 & 1.0000 \end{bmatrix} \tag{19}$$

Step 3. Calculate the maximum eigenvector and eigenvalue of the judgement matrix. The treatment is the same as Case 1. According to formula $R_{tr}x = \lambda_{max}x$ and after normalization, we can get the final weights of alternatives A_1, A_2 and A_3 in D matrix shown in Equation (16).

$$\omega_1 : \omega_2 : \omega_3 = 0.2015 : 0.6896 : 0.1089$$

The ranking of alternatives is $A_2 \succ A_1 \succ A_3$ (The '$\succ$' sign means 'better than'). Similarly, consistency indicators CR can be obtained. As $CR = 0.0038 < 0.1$, the consistency of the D matrix of Equation (16) is acceptable.

3.2.3 Summary of Case 1 and Case 2

Case 1 and Case 2 represent the cases of D matrix with complete information and D matrix with incomplete information, respectively. Here we express the pairwise comparison opinions of experts in the form of a D matrix, that is, our input is a D matrix. It can be concluded from the above that D-matrix is an extension of fuzzy preference relation, which can deal with uncertain and incomplete information more effectively. The process can be concluded as follows (shown in Figure 1). Firstly, convert D matrix to a crisp matrix according to two different situations: when the information is complete, the integration representation of D numbers is directly used; when the information is incomplete, a new method is proposed. Secondly, convert the crisp matrix to the judgement matrix based on the transformation functions to get a good preparation for using AHP. Finally, AHP is used to deal with the judgement matrix. The eigenvector corresponding to the largest eigenvalue of the judgement matrix is calculated to get the weights. The CR is also used to measure the consistency.

4 Numerical examples

In this part, three examples are illustrated to show effectiveness and superiority of the proposed method compared with the Deng et al.'s method [39]. Assume that the following three examples are expert opinions in different situations. Here we directly use the D matrix to express this information. Deng et.al's method and the proposed method are applied to get the priority weights of alternatives, respectively.

4.1 Example 1

$$
R_D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} \{(0.5000, 1)\} & \{(0.4990, 1)\} & \{(0.5010, 1)\} \\ \{(0.5010, 1)\} & \{(0.5000, 1)\} & \{(0.4990, 1)\} \\ \{(0.4990, 1)\} & \{(0.5010, 1)\} & \{(0.5000, 1)\} \end{array} \right] \end{array} \tag{20}
$$

In the first example, the elements of the D matrix are complete information. The pairwise comparison values are all close to 0.5, indicating that A_1, A_2, and A_3 are approximately equally important.

4.1.1 Deng et al.'s method

Firstly, dealing with the D matrix by the integration representation of D numbers. Then, the crisp matrix R_c is obtained in Equation (21).

$$
R_c = I(R_D) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} 0.5000 & 0.4990 & 0.5010 \\ 0.5010 & 0.5000 & 0.4990 \\ 0.4990 & 0.5010 & 0.5000 \end{array} \right] \end{array} \tag{21}
$$

Secondly, construct the probability matrix. The elements in the D numbers are all information complete. So there are only two choices of 0 or 1 of the elements in the probability matrix R_p. The probability matrix can be obtained as follow in Equation (22).

$$
R_p = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} 0.0000 & 0.0000 & 1.0000 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 \end{array} \right] \end{array} \tag{22}
$$

Finally, the triangularization method is used to rank the alternatives. However, based on the probability matrix the sum of each row is equal, the subsequent steps cannot be carried out.

4.1.2   The proposed method

Step 1 The information of the D matrix is all complete. So the process just like the Deng's method. The crisp matrix is same as the Equation (21).

Step 2 According to Equation (6), the judgement matrix can be obtained. For instance, $R_{tr13}=9^{-1+2\times I(D_{13})} = 0.9956$. The judgement matrix is as follow in Equation (23).

$$R_{tr} = \begin{matrix} & \begin{matrix} A_1 & A_2 & A_3 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} & \begin{bmatrix} 1.0000 & 0.9956 & 1.0044 \\ 1.0044 & 1.0000 & 0.9956 \\ 0.9956 & 1.0044 & 1.0000 \end{bmatrix} \end{matrix} \tag{23}$$

Step 3 According to the $R_{tr}x = \lambda x$, the largest eigenvalue of the judgement matrix is calculated. Normalized the eigenvector of the largest eigenvalue and then we can get the final weights. The final weights of the three alternatives are 0.3333 0.3333 0.3333 and $CR < 0.1$.

4.1.3   Discussion of Example 1

In this example, the alternatives are quite similar, and it can be seen that the D number matrix has good consistency. The correct result can be obtained by the proposed method, and CR is less than 0.1. However, Deng et al.'s method cannot be applied since the sum of each row in the probability matrix R_p is equal, so that the following interval cannot be processed, and the consistency index cannot be obtained.

4.2   Example 2

$$R_D = \begin{matrix} & \begin{matrix} A_1 & A_2 & A_3 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} & \begin{bmatrix} \{(0.5, 1.0)\} & \{(0.1, 0.5)\} & \{(0.0, 0.4)\} \\ \{(0.9, 0.5)\} & \{(0.5, 1.0)\} & \{(0.4, 0.2)\} \\ \{(1.0, 0.4)\} & \{(0.6, 0.2)\} & \{(0.5, 1.0)\} \end{bmatrix} \end{matrix} \tag{24}$$

In the second example, from the first column of the D-matrix, the pairwise comparisons of A_2 and A_1, A_3 and A_1, are both greater than 0.5. And the pairwise comparison of A_3 and A_1 is larger than the pairwise comparison of A_2 and A_1. This shows that A_1 is the worst, A_3 and A_2 are both superior to A_1, and A_3 is superior to A_1 to a greater extent than A2. Meanwhile, the value of the pairwise comparison between A_3 and A_2 is greater than 0.5, and the value of the pairwise comparison between A_2 and A_3 is less than 0.5, which further indicates that A_3 is superior to A_2. So given the D matrix, the order from best to worst should be A_3, A_2, A_1.

4.2.1   Deng et al.'s method

Firstly, same as the first step of Example 1. The Equation (5) is used to obtain the crisp matrix R_c as follows.

$$R_c = I(R_D) = \begin{matrix} & \begin{matrix} A_1 & A_2 & A_3 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} & \begin{bmatrix} 0.5000 & 0.0050 & 1.0000 \\ 0.4500 & 0.5000 & 0.0800 \\ 0.4000 & 0.1200 & 0.5000 \end{bmatrix} \end{matrix} \tag{25}$$

Secondly, in this D matrix, some elements are information complete and other elements are information incomplete. The probability matrix(R_p) can be obtained in Equation (26).

$$R_p = \begin{matrix} & \begin{matrix} A_1 & A_2 & A_3 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} & \begin{bmatrix} 0.0000 & 0.1000 & 0.1700 \\ 0.9000 & 0.0000 & 0.4750 \\ 0.8300 & 0.5250 & 0.0000 \end{bmatrix} \end{matrix} \tag{26}$$

Finally, the sum of the first row of the R_p matrix is 0.27, the sum of the second row is 1.375, and the sum of the third row is 1.355, so A_2 is ranked first, A_3 is second, and A_1 is third.

4.2.2 The proposed method

Step 1 Because of incomplete information, processing is divided into two categories. If the information is complete, the operation shown as Equation (5) is implemented. If the information is incomplete, the Equation (17) is used. The crisp matrix is derived.

$$R_c = I(R_D) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} 0.5000 & 0.3000 & 0.3000 \\ 0.7000 & 0.5000 & 0.4800 \\ 0.7000 & 0.5200 & 0.5000 \end{array} \right] \end{array} \tag{27}$$

Step 2 The operator of the crisp matrix is shown in Equation (6). We can get the judgement matrix as is shown in Equation (28).

$$R_{tr} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccc} A_1 & A_2 & A_3 \\ \left[\begin{array}{ccc} 1.0000 & 0.4152 & 0.4152 \\ 2.4082 & 1.0000 & 0.9159 \\ 2.4082 & 1.0918 & 1.0000 \end{array} \right] \end{array} \tag{28}$$

Step 3 Same as the third step of the method proposed in this article in example 1. The weights can be obtained. The CR can be calculate by Equation (3). Finally, the weights of three alternatives A_1, A_2, A_3 are: 0.1718 0.3989 0.4292, and $CR = 0.0013 < 0.1$. Sort the weight of three alternatives from the largest to the smallest, and the ranking from the best to the worst can be A_3, A_2, A_1.

4.2.3 Discussion of Example 2

In this example, as discussed in the first paragraph in Section 4.2, the reasonable ranking of the alternatives should be $A_3 \succ A_2 \succ A_1$. The weights of the alternatives by the proposed method are 0.1718 0.3989 0.4292, which is consistent with the conclusion made in Section 4.2. However, using Deng et al.'s method, the ranking of the alternatives is $A_2 \succ A_3 \succ A_1$, which is inconsistent with the actual situation. The reason for this result is that Deng et al's method directly uses the distance ratio to divide the reliability of the unknown part into the information we know from the beginning, which lacks solid physical foundation.

4.3 Example 3

$$R_D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ \left[\begin{array}{cccc} \{(0.5, 1.0)\} & \{(0.1, 1.0)\} & \{(0.6, 0.8)\} & \{(0.3, 0.6)\} \\ \{(0.9, 1.0)\} & \{(0.5, 1.0)\} & \{(0.8, 1.0)\} & \{(0.6, 1.0)\} \\ \{(0.4, 0.8)\} & \{(0.2, 1.0)\} & \{(0.5, 1.0)\} & \{(0.9, 1.0)\} \\ \{(0.7, 0.6)\} & \{(0.4, 1.0)\} & \{(0.1, 1.0)\} & \{(0.5, 1.0)\} \end{array} \right] \end{array} \tag{29}$$

In the third example, the value of the pairwise comparison of A_4 and A_1 is greater than the value of the pairwise comparison of A_3 and A_1 and both are greater than 0.5, indicating that A_4 is superior to A_3. However, the pairwise comparison value of A_3 and A_4 is greater than 0.5, indicating that A_3 is superior to A_4. That is to say, D matrix has obvious inconsistency.

4.3.1 Deng et al.'s method

The processing method is the same as the Deng et al.'s method in Example 2. We can get the crisp matrix and the probability matrix as are shown in Equation (30) and Equation (31). Further I.D. can be obtained, according to the following calculation.

$$I.D. = \frac{0.1 + 0.8}{4(4-1)/2} = 0.15$$

$$R_c = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ \left[\begin{array}{cccc} 0.5000 & 0.1000 & 0.4800 & 0.1800 \\ 0.9000 & 0.5000 & 0.8000 & 0.6000 \\ 0.3200 & 0.2000 & 0.5000 & 0.9000 \\ 0.4200 & 0.4000 & 0.1000 & 0.5000 \end{array} \right] \end{array} \tag{30}$$

$$R_c = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ \left[\begin{array}{cccc} 0.0000 & 0.0000 & 0.9000 & 0.2000 \\ 1.0000 & 0.0000 & 1.0000 & 1.0000 \\ 0.1000 & 0.0000 & 0.0000 & 1.0000 \\ 0.8000 & 0.0000 & 0.0000 & 0.0000 \end{array} \right] \end{array} \tag{31}$$

4.3.2   The proposed method

The processing method is the same as the method proposed in this article in Example 2. We can get the crisp matrix and the judgement matrix in Equation (32) and Equation (33). We can get CR according to the Equation (3).

$$R_c = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ \left[\begin{array}{cccc} 0.5000 & 0.1000 & 0.5800 & 0.6000 \\ 0.9000 & 0.5000 & 0.8000 & 0.6000 \\ 0.4200 & 0.2000 & 0.5000 & 0.9000 \\ 0.4000 & 0.4000 & 0.1000 & 0.5000 \end{array} \right] \end{array} \tag{32}$$

$$R_{tr} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ \left[\begin{array}{cccc} 1.0000 & 0.1724 & 1.4228 & 0.5902 \\ 5.7995 & 1.0000 & 3.7372 & 1.5519 \\ 0.7036 & 0.2676 & 1.0000 & 5.7995 \\ 1.6944 & 0.6444 & 0.1724 & 1.0000 \end{array} \right] \end{array} \tag{33}$$

Finally, the weights of alternatives A_1, A_2, A_3, A_4 are: 0.1300 0.4746 0.2549 0.1406 and $CR = 0.3952 > 0.1$.

4.3.3   Discussion of Example 3

From the first column of the D number matrix given in Example 3, it can be seen that A_3 is worse than A_1 with a greater probability, and A_4 is better than A_1 with a greater probability. However, A_3 can be obtained A_4 is less than A_3 with a particularly large probability, which is obviously in conflict. In other words, the elements (evaluations given by experts) of this D number matrix is inconsistent. Using the proposed method, CR is close to 0.4 and far greater than 0.1, which correctly illustrates this point. However, using Deng et al.'s method, the consistency index is 0.15 which shows that the D number matrix is consistent. This is obviously inappropriate.

4.4   Discussion and comparison

Deng et al.'s method proposed an interesting method, i.e., the AHP method extended by D numbers preference relation, which can deal with the complex decision problem under uncertain environment [39]. However, some situations cannot be well handled in Deng et al.'s method. Three examples in this section shows the certain situations. From these examples, some conclusions can be made. (1) The proposed method can handle situations that cannot be handled by Deng et al.'s method, as is shown in example 1. (2) For the processing of incomplete information, this article considers that the incomplete information is because some of the information is unknown, and the unknown part should be expressed as the same importance of the two alternatives to be pairwise compared, that is, they cannot be distinguished. In the proposed method, 0.5 is multiplied by the degree of uncertainty. This method of expression is more in line with thinking habits and has a clearer physical meaning. Deng et
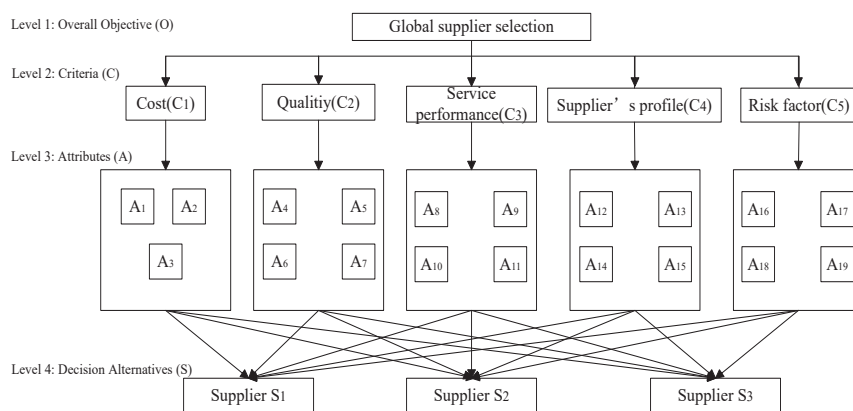
Figure 2: The hierarchical structure for the supplier selection

al.'s method redistributes information according to distance which may not work out well under certain situations. Meanwhile, information is artificially added during the redistribution process. Example 2 shows this point. (3) In the evaluation of consistency of D matrix, the consistency ration CR in AHP can be directly adopted in the proposed method since a bridge (the transformation functions) is built between the crisp matrix of D matrix and the judgement matrix, which has a relatively sufficient theoretical basis. The consistency evaluation process in Deng et al.'s method may draw intuitive results. In example 3, the consistency indicator obtained is unreasonable and shows this point. (4)As can be seen in Figure 1, the proposed method is more simple and convenient to use. In summary, the proposed method has the merit of dealing with uncertain information, and can also improve the flexibility, efficiency and performance compared with Deng et al.'s method.

5   Case study

In this section, the problem of selecting the best supplier [44] is adopted to illustrate the use and effectiveness of the proposed method. The hierarchical structure of the problem is shown in Figure 2 [44]. As shown in the figure, there are four levels. The overall objective is placed at the first level. The second level is the criteria. The third level is the attributes. The final level is the decision alternatives.

The second level is consisted of five components. It contains the product's cost(C_1), product's quality(C_2), service performance (C_3), supplier's profile(C_4) and factor of the risk(C_5). Each item of the third level corresponds to each item of the second level. The define of attribute is as follow. A_1: Product's total price; A_2: Prices of delivery; A_3: Tariff and custom duties; A_4: Rejection rate of the product; A_5: Increased lead time; A_6: Quality assessment; A_7: Remedy for quality problems; A_8: Delivery schedule; A_9: Technological and support; A_{10}: Response to changes; A_{11}: Ease of communication; A_{12}: Financial status; A_{13}: Customer base; A_{14}: Performance history; A_{15}: Production facility and capacity; A_{16}: Geographical location; A_{17}: Political stability; A_{18}: Political stability; A_{19}: Terrorism. More details are given in [44].

For this problem, we can construct the D matrix at each level based on expert evaluation. For example, We can construct D matrix between the overall objective and criteria in Table.3 [39]. According to the method proposed in the paper, we can get the priority weights of criteria. W_{Deng} and $W_{Proposed}$ on the last two columns of Table 3 respectively represent the relative weights obtained by Deng et al.'s method and the relative weights obtained by the proposed method. It can be seen that the weights derived by Deng et al's method and the proposed method are consistent.

In a similar way, we can get the D matrix of the attributes corresponding to the criteria, and then calculate the priority weights of the attributes corresponding to the criteria as shown from column 2 to column 6 of Table 4. By integrating the weights of the criteria level and attributes level, we can get the contribution ratio and ranking of attributes relative to the overall objective. The results are shown on column 7 of Table 4, and you can see that the most important attribute is A_1. It can be
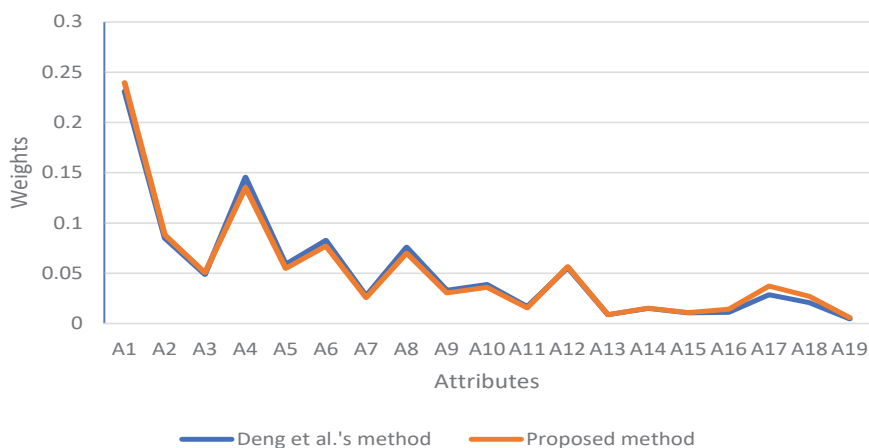
Figure 3: Compare Deng et al.'s method with the proposed method to produce the weights of attributes with respect to the overall objective

Table 3: D numbers preference relation of criteria with respect to the overall objective.

O	C_1	C_2	C_3	C_4	C_5	W_{Deng}	$W_{Proposed}$
C_1	$\{(0.5,1.0)\}$	$\{(0.6,1.0)\}$	$\{(0.65,0.6)\}$	$\{(0.85,1.0)\}$	$\{(0.9,1.0)\}$	0.3650	0.3786
C_2	$\{(0.4,1.0)\}$	$\{(0.5,1.0)\}$	$\{(0.8,1.0)\}$	$\{(0.7,1.0)\}$	$\{(0.7,1.0)\}$	0.3150	0.2929
C_3	$\{(0.35,0.6)\}$	$\{(0.2,1.0)\}$	$\{(0.5,1.0)\}$	$\{(0.65,1.0)\}$	$\{(0.6,1.0)\}$	0.1650	0.1520
C_4	$\{(0.15,1.0)\}$	$\{(0.3,1.0)\}$	$\{(0.35,1.0)\}$	$\{(0.5,1.0)\}$	$\{(0.55,1.0)\}$	0.0900	0.0917
C_5	$\{(0.1,1.0)\}$	$\{(0.3,1.0)\}$	$\{(0.4,1.0)\}$	$\{(0.45,1.0)\}$	$\{(0.5,1.0)\}$	0.0650	0.0849

seen from Figure 3 that the weights of attributes with respect to the overall objective obtained by Deng et al.'s method and proposed method is not significantly different.

In the same way, we can construct the D matrix of suppliers corresponding to attributes. From column 3 to column 5 of Table 5 shows the priority weight of suppliers with respect to each attribute. The final weight of suppliers with respect to the overall objective is obtained at the bottom of Table 5.

Figure 4 shows the final result of using Deng et al.'s method and the proposed method to deal with the problem, it can be seen that the suppliers generated by the two methods have the same ranking: $S_1 \succ S_3 \succ S_2$. The result is consistent with the expert's final assessment. Furthermore, the results obtained by the method in this paper are more accurate, and the difference of priority weights between two suppliers is larger, which indicates that the division of different suppliers is more obvious. The reason for this may be concluded that the proposed method deal with the uncertain information more reasonably, especially with the incomplete information cases.

6 Conclusion

In this study, a new supplier selection method based on D numbers and transformation function is proposed. Firstly, D numbers theory is used to express the uncertain information in preference relation and form a D matrix; Secondly, the D matrix is converted into a crisp matrix form according to different situations whether or not the information in D matrix is complete. Thirdly, the crisp matrix are converted into judgement matrix by using the transformation functions. Finally, AHP is used to select the best supplier. The proposed method has the following advantages: (1) Due to the characteristics of D numbers itself, the method in this paper is more advantageous in the expression of uncertain information. (2) Be able to deal with a variety of situations, including situations with large uncertainties in expert opinions or situations with similar decision-making objects. (3) The consistency evaluation index is consistent with the actual situation and has a good theoretical basis.

In the future research, the integration rules of D numbers should be further improved to make full use of incomplete information rather than simple discount processing. In addition, the method in this

Table 4: The weights and ranking of attributes with respect to the overall objective based on proposed method.

Criteria	C_1 0.3786	C_2 0.2929	C_3 0.1520	C_4 0.0917	C_5 0.0849	Weight	Ranking
Attributes							
A_1	0.6330					0.2397	1
A_2	0.2330					0.0882	3
A_3	0.1340					0.0507	8
A_4		0.4620				0.1353	2
A_5		0.1870				0.0548	7
A_6		0.2630				0.0770	4
A_7		0.0880				0.0258	13
A_8			0.4610			0.0701	5
A_9			0.2010			0.0306	11
A_{10}			0.2360			0.0359	10
A_{11}			0.1020			0.0155	14
A_{12}				0.6180		0.0567	6
A_{13}				0.0970		0.0089	18
A_{14}				0.1670		0.0153	15
A_{15}				0.1180		0.0108	17
A_{16}					0.1700	0.0144	16
A_{17}					0.4400	0.0374	9
A_{18}					0.3200	0.0272	12
A_{19}					0.0700	0.0059	19

Table 5: The priority weights and ranking of suppliers with respect to the overall objective based on the proposed method.

| Attributes		Suppliers		
A_i		S_1	S_2	S_3
A_1	0.2397	0.4670	0.1660	0.3670
A_2	0.1353	0.4270	0.1960	0.3770
A_3	0.0882	0.6170	0.1670	0.2160
A_4	0.0770	0.5340	0.2330	0.2330
A_5	0.0701	0.5410	0.1180	0.3410
A_6	0.0567	0.2830	0.5330	0.1840
A_7	0.0548	0.5930	0.1640	0.2430
A_8	0.0507	0.4170	0.1160	0.4670
A_9	0.0374	0.6170	0.0660	0.3170
A_{10}	0.0359	0.4330	0.4330	0.1340
A_{11}	0.0306	0.6730	0.0140	0.3130
A_{12}	0.0272	0.3330	0.3330	0.3340
A_{13}	0.0258	0.6000	0.2000	0.2000
A_{14}	0.0155	0.3170	0.5170	0.1660
A_{15}	0.0153	0.3240	0.0230	0.6530
A_{16}	0.0144	0.6000	0.3000	0.1000
A_{17}	0.0108	0.4330	0.2340	0.3330
A_{18}	0.0089	0.2930	0.5130	0.1940
A_{19}	0.0059	0.5870	0.1060	0.3070
Proposed method		0.4858	0.2054	0.3089
Supplier's ranking		1	3	2
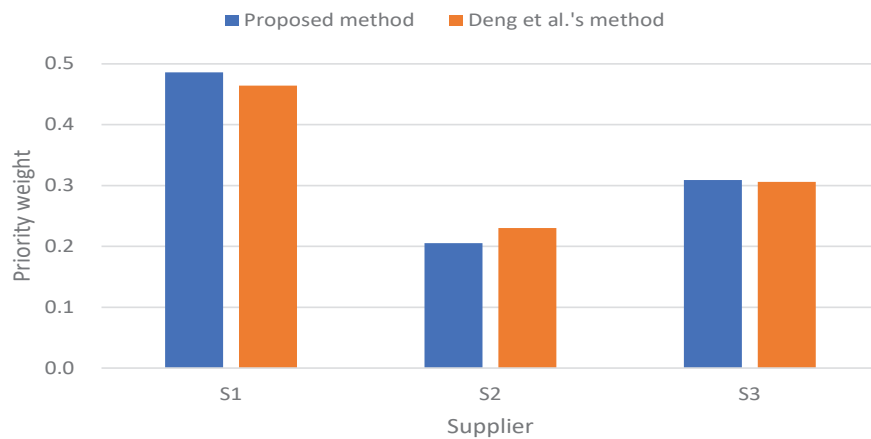
Figure 4: Compare Deng et al.'s method with the proposed method to produce the final priority weights

paper did not consider the interaction between indicators, so ANP and other methods were considered for improvement.

Acknowledgments

Author contributions

The authors contributed equally to this work.

Conflict of interest

The authors declare no conflict of interest.

References

[1] Pelissari, R.; Khan, S.; Ben-Amor, S.(2021). Application of Multi-Criteria Decision-Making Methods in Sustainable Manufacturing Management: A Systematic Literature Review and Analysis of the Prospects, *International Journal of Information Technology & Decision Making*, 2(21),493–515, 2021.

[2] Zheng, Y.; Zheng, J.(2021). A novel portfolio optimization model via combining multi-objective optimization and multi-attribute decision making, *Applied Intelligence*, 52(5),5684—5695, 2021.

[3] Fu, S.; Xiao, Y. Z.; Zhou, H. J.; Liu, S. Z.(2021). Venture capital project selection based on interval number grey target decision model, *Soft Computing*, 25(6),4865–4874, 2021.

[4] Deng X.; Kong Z.(2021). Humanitarian rescue scheme selection under the COVID-19 crisis in China: Based on group decision-making method, *Symmetry*, 13(4),668, 2021.

[5] Wu Y.; Deng Z.; Tao Y.(2021). Site selection decision framework for photovoltaic hydrogen production project using BWM-CRITIC-MABAC: A case study in Zhangjiakou, *Journal of Cleaner Production*, 324, 2021.

[6] Emeksiz C, Yüksel A.(2022). A suitable site selection for sustainable bioenergy production facility by using hybrid multi-criteria decision making approach, case study: Turkey, *Fuel*, 315, 2022.

[7] Konneh K.V., Masrur H., Othman M.L.(2021). Multi-Attribute decision-Making approach for a cost-Effective and sustainable energy system considering weight assignment analysis, *Sustainability*, 13(10),5615,2021.

[8] Bai C.; Kusi-Sarpong S.; Badri A.H.(2019). Social sustainable supplier evaluation and selection: a group decision-support approach, *International Journal of Production Research*, 57(22), 7046–7067, 2019.

[9] Schramm V.B.; Cabral L.; Schramm F.(2020). Approaches for supporting sustainable supplier selection-A literature review, *Journal of Cleaner Production*, 273, 2020.

[10] Hruška R.; Průša P.; Babić D.(2014). The use of AHP method for selection of supplier, *Transport*, 29(2), 195–203, 2014.

[11] Liu M.; Wang X.; Li Y.(2022). Service supplier selection under fuzzy and stochastic uncertain environments, *Journal of Intelligent & Fuzzy Systems*, 3(42),1301–1315,2022.

[12] Pelissari R.; Oliveira M.C.; Abackerli A.J.(2021). Techniques to model uncertain input data of multi-criteria decision-making problems: a literature review, *International Transactions in Operational Research*, 28(2): 523–559,2021.

[13] Rashidi, K.; Cullinane, K. (2018). A Comparison of Fuzzy DEA and Fuzzy TOPSIS in Sustainable Supplier Selection: Implications for Sourcing Strategy, *Expert Systems with Applications*, 121, 2018.

[14] Chen, Z.; Ming, X.; Zhou, T.; Chang, Y. (2020). Sustainable supplier selection for smart supply chain considering internal and external uncertainty: An integrated rough-fuzzy approach, *Applied Soft Computing*, 87, 2020.

[15] Xing, Y.;Cao M.; Liu Y.; Zhou M. ; Jian Wu (2022). A Choquet integral based interval Type-2 trapezoidal fuzzy multiple attribute group decision making for Sustainable Supplier Selection, *Computers & Industrial Engineering*, 165, 2022.

[16] Hekmat, S.; Amiri, M.; Madraki, G.;Shi, Y.(2021). Strategic Supplier Selection in Payment Industry: A Multi-Criteria Solution for Insufficient and Interrelated Data Sources, *International Journal of Information Technology & Decision Making*, 6(20),1711–1745, 2021.

[17] Babak A. ; Meysam R. ; Madjid T.(2021). An integrated information fusion and grey multi-criteria decision-making framework for sustainable supplier selection, *International Journal of Systems Science: Operations & Logistics*, 4(8),348–370, 2021.

[18] Sureeyatanapas, P.;Waleekhajornlert, N.; Arunyanart, S.; Niyamosoth, T.(2020). Resilient supplier selection in electronic components procurement: An integration of evidence theory and rule-based transformation into TOPSIS to tackle uncertain and incomplete information, *Symmetry*, 7(12),1109, 2020.

[19] Zhang, X.; Deng, Y.; Chan, F.; Adamatzky, A.; Mahadevan, S.(2016). Supplier selection based on evidence theory and analytic network process, *Proceedings of the institution of Mechanical Engineers, Part B: Journal of Engineering manufacture*, 3(230),562–573, 2016.

[20] Fei, L.; Xia, J.;Feng, Y.; Liu, L.(2019). An ELECTRE-based multiple criteria decision making method for supplier selection using Dempster-Shafer theory, *IEEE Access*, 7,2019.

[21] Liu, PD.; Zhang, XH.(2019). A novel approach to multi-criteria group decision-making problems based on linguistic D numbers, *Computational & Applied Mathematics*, 39(2), 132, 2020.

[22] Dempster, A. P. (1967). Upper and Lower Probabilities Induced by a Multivalued Mapping, *The Annals of Mathematical Statistics*, 38(2), 325 – 339, 1967.

[23] Shafer, G. (1976). *A mathematical theory of evidence*, Princeton university Press, 1976.

[24] Guan, X.; Liu, HQ.; Yi, X.; Zhao, J.(2018). The improved combination rule of D numbers and its application in radiation source identification, *Mathematical Problems in Engineering*, 15(4), 15501477198, 2018.

[25] Deng, XY.; Jiang, W.(2019). A total uncertainty measure for D numbers based on belief intervals, *International Journal of Intelligent Systems*, 34(12), 3302–3316, 2019.

[26] Xia, J; Feng, YQ.; Liu, LN.; Liu, DJ.; Fei, LG.(2019). On entropy function and reliability indicator for D numbers, *Applied Intelligence*, 49(9), 3248–3266, 2019.

[27] Li, XH.; Chen, XH.(2018). D-intuitionistic hesitant fuzzy sets and their application in multiple attribute decision making, *Cognitive Computation*, 10(3), 496–505, 2018.

[28] Zhang, J.; Zhong, DH.; Zhao, MQ.; Yu, J.; Lv, F.(2019). An optimization model for construction stage and zone plans of rockfill dams based on the enhanced whale optimization algorithm, *Energies*, 12(3), 466, 2019.

[29] Mo, HM.(2021). A SWOT method to evaluate safety risks in life cycle of wind turbine extended by D number theory, *Journal of Intelligent & Fuzzy Systems*, 40(3), 4439–4452, 2021.

[30] Mo, HM.(2020). A New Evaluation Methodology for Quality Goals Extended by D Number Theory and FAHP, *Journal of Intelligent & Fuzzy Systems*, 11(4), 206, 2020.

[31] Mo, HM.(2020). An Emergency Decision-Making Method for Probabilistic Linguistic Term Sets Extended by D Number Theory, *Symmetry*, 12(3), 380, 2020.

[32] Deng, XY.; Jiang, W.(2019). Evaluating green supply chain management practices under fuzzy environment: a novel method based on D number theory, *International Journal of Fuzzy Systems*, 21(5), 1389–1402, 2019.

[33] Deng, XY.; Jiang, W.(2019). D number theory based game-theoretic framework in adversarial decision making under a fuzzy environment, *International Journal of Approximate Reasoning*, 106(1), 194–213, 2019.

[34] Mi, XJ.; Tian, Y.; Kang, BY.(2021). A hybrid multi-criteria decision making approach for assessing health-care waste management technologies based on soft likelihood function and D-numbers, *Applied Intelligence*, 15(2), 1–20, 2021.

[35] Mousavi-Nasab, S.H.; Sotoudeh-Anvari, A.(2020). An extension of best-worst method with D numbers: Application in evaluation of renewable energy resources, *Sustainable Energy Technologies and Assessments*, 40,2020.

[36] Seiti, H.; Hafezalkotob, A.; Herrera-Viedma, E.(2020). A novel linguistic approach for multi-granular information fusion and decision-making using risk-based linguistic D numbers, *Information Sciences*, 530, 43–65, 2020.

[37] Tian, Y.; Mi, XJ.; Liu, LL.; Kang, BY.(2020). A New Soft Likelihood Function Based on D Numbers in Handling Uncertain Information, *International Journal of Fuzzy Systems*, 22(7), 31–34, 2020.

[38] Xiao, FY. (2019). A Multiple-Criteria Decision-Making Method Based on D Numbers and Belief Entropy, *International Journal of Fuzzy Systems*, 21(4), 1144–1153, 2019.

[39] Deng, XY.; Hu, Y.; Deng, Y.; Sankaran M.(2014). Supplier selection using AHP methodology extended by D numbers, *Expert Systems with Application*, 41(1), 156–167, 2014.

[40] Deng, XY.; Deng, Y.(2019). D-AHP method with different credibility of information, *Soft Computing*, 23(2), 683–691, 2019.

[41] Fei, L.(2020). D-ANP: a multiple criteria decision making method for supplier selection, *Applied Intelligence*, 8(50),2537–2554,2020.

[42] Herrera Viedma, E.; Herrera, F.; Chiclana, F.; Luque, M.(2004). Some issues on consistency of fuzzy preference relations, *European journal of operational research*, 154(1), 98–109, 2004.

[43] Tanino, T.(1984). Fuzzy preference orderings in group decision making, *Fuzzy sets and systems*, 12(2), 117–131, 1984.

[44] Felix, T.S.C.; Niraj, K.(2007). Global supplier development considering risk factors using fuzzy extended AHP-based approach, *Omega: The international journal of management science*, 35(4), 417–431, 2007.

[45] Saaty, T.L.(1982). The analytic hierarchy process: A new approach to deal with fuzziness in architecture, *Architectural Science Review*, 25(3), 64–69, 1982.

C	O	P	E

Member since 2012
JM08090

This journal is a member of, and subscribes to the principles of,
the Committee on Publication Ethics (COPE).
https://publicationethics.org/members/international-journal-computers-communications-and-control

Cite this paper as:

Xiong, L.; Su, X.; Qian, H. (2022). Supplier Selection Model Based on D Numbers and Transformation Function, *International Journal of Computers Communications & Control*, 17(5), 4468, 2022.
https://doi.org/10.15837/ijccc.2022.5.4468