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Decision Tools Regarding Time Constraints Violation in Manufacturing Workshops

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Abstract

This paper is dedicated to the study of constraints violation in manufacturing workshops with time constraints. In such systems, every operation duration is included between minimal and maximal values. P-time Petri nets are used for modeling. A new theorem is introduced, constituting a decision tool about the occurrence of constraints violation at the level of a synchronization transition when various types of time disturbances occur. It shows the robustness properties of a manufacturing system on a range that may include delay and advance disturbances. The theoretical result is illustrated step by step on a given workshop. Two other lemmas are elaborated contributing to the study of the constraints violation problem. The final goal is to generalize the robustness property towards simultaneous occurrence of two delays at two points of the system, each having its own robustness range.

Keywords: manufacturing, maximum time constraints, time disturbance, constraints violation, robustness.

1 Introduction

A system is said critical when its functioning failures may produce catastrophic consequences [5]. A particular class of dysfunction may occur when state duration constraints, being not fulfilled lead to non-acceptable situations. In this case, systems are said time critical. Time critical systems, including minimum and maximum constraints for operation duration, correspond to various industrial processes [2, 20, 24]. From a methodological point of view, time requirements formulations correspond to various kinds of time critical systems [8, 9]. Moreover, synchronization problems in case of crisis use to be critical in transport systems [7]. Maximal bound for an operation execution requires model and approaches [4, 6, 16]. Several works mainly focusing on robust control in the state of the art concerning manufacturing [1, 3, 10, 11, 15, 22]. The presented works keeping the manufacturing area as an illustrative application provides properties that impact a wider field of application.

The two following sections details fundamentals definitions and a classical modeling approach concerning manufacturing production including minimum and maximum operation duration. The third section considers more original assumption and studies time related decisions while mixing delay and advance disturbances in the same system. The section demonstrates the properties and illustrates them on a manufacturing example. The last section discusses the quality of the results highlighting the need of a formalism extension, addressing the analysis of more complex structures like time constraints ensuing from tele-operation in manufacturing and transport.

2 P-time Petri net

Upper bound duration for state durations induces a particular structure of the mathematical problem to be tackled. A dedicated modeling tool, providing useful structural and behavioral properties was presented by [16]. The following definition is used as a basis of a time constrained workshop functional decomposition, which is presented in the next section.

The formal definition of a P-time Petri net is given by a pair $< R; IS >$, where [16]:

- R is a marked Petri net,
- $IS: \ P \longrightarrow (Q^+) \times (Q^+ \cup \{+\infty\})$
  $p_i \longrightarrow IS_i = [a_i, b_i] \ with \ 0 \leq a_i \leq b_i.$

IS_i defines the static interval of staying time of a mark in the place p_i belonging to the set of places P (Q^+ is the set of positive rational numbers). A mark in the place p_i is taken into account in transition validation when it has stayed in p_i at least a duration a_i and no longer than b_i. After the duration b_i the token will be dead.

The synchronization mechanism is the reason of token death. A dead token means a non-compliance with the required specifications. The semantic specificity of P-time Petri nets was studied by Boyer and Roux [4].

Let us denote by:

- T the set of transitions,
- t_i^o (resp. ${}^o t_i$) the output (resp. the input) places of the transition t_i,
- p_i^o (resp. ${}^o p_i$) the output (resp. the input) transitions of the place p_i,
- q_{ie} the expected sojourn time of the token in the place p_i,
- q_i the effective sojourn time of the token in the place p_i,
- $St_{ie}(n)$ the expected n^{nd} firing instant of the transition t_i.
- $St_i(n)$ the effective n^{nd} firing instant of the transition t_i.

3 Functional decomposition

The current section details a functional decomposition, which is used to analytically express the local temporal margins. The functioning assumptions considers that the workshop is running in a repetitive mode, following a mono-periodic cycle. A workshop in repetitive functioning mode is modeled by a Strongly Connected Event Graph (SCEG) [16, 18].

Definition 1. *An Event Graph (EG) is a particular Petri net in which each place has exactly one input transition and one output transition.*

Definition 2. *An EG is a SCEG if and only if it exists an oriented path connecting each node to another.*

Performances of a SCEG running in mono-periodic functioning mode are proved to be the same as when using the K-periodic functioning [16, 18]. Consequently, a mono-periodic functioning is used in order to decrease the complexity of the supervisory problem [6, 18]. In this case, for each transition t, $St_e(n+1) = St_e(n) + \pi_0$ where π_0 is the period of the periodic functioning of the given discrete event system. In this paper, the scheduling task is supposed to be done. Therefore, the SCEG corresponding to the system is provided. Moreover, the setting of transitions firing instants is fixed too. Then, constraints violation will be studied in the following. The problem of time disturbances observability is not considered. It was studied by Jerbi et al. [12, 14].

As the sojourn times in places have not the same functional signification when they are included in the sequential process of a product or when they are associated to a free resource, a decomposition of the P-time Petri net model into four sets is made. The assumption of multi-product job-shops without assembling tasks is used:

- R_U is the set of places representing the used machines,
- R_N corresponds to the set of places representing the free machines which are shared between manufacturing circuits,
- $Trans_C$ is the set of places representing the loaded transport resources,
- $Trans_{NC}$ is the set of places representing the unloaded transport resources (or the interconnected buffers).

Figure 1 shows a P-time Petri net (G) modeling a system composed by two sequential processes GO_1 and GO_2 with two shared machines (M_1, M_2), where:

- $R_U = \{p_2, p_4, p_{11}, p_{13}, p_{15}\}$,
- $R_N = \{p_6, p_7, p_8, p_9\}$,
- $Trans_C = \{p_1, p_3, p_{10}, p_{12}, p_{14}\}$,
- $Trans_{NC} = \{p_5, p_{16}\}$,
- $GO_1 = (t_{12}, p_{10}, t_6, p_{11}, t_7, p_{12}, t_8, p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11})$,
- $GO_2 = (t_5, p_1, t_1, p_2, t_2, p_3, t_3, p_4, t_4)$.

The intervals (IS_i) and the expected staying times (q_{ie}) associated to the places (p_i) are: $IS_1 = [30, 50], q_{1e} = 38, IS_2 = [5, 12], q_{2e} = 7, IS_3 = [10, 20], q_{3e} = 15, IS_4 = [5, 20], q_{4e} = 10, IS_5 = [1, +\infty], q_{5e} = 10, IS_6 = [0, +\infty], q_{6e} = 5, IS_7 = [0, +\infty], q_{7e} = 8, IS_8 = [8, +\infty], q_{8e} = 13, IS_9 = [8, +\infty], q_{9e} = 15, IS_{10} = [5, 15], q_{10e} = 12, IS_{11} = [15, 20], q_{11e} = 17, IS_{12} = [3, 7], q_{12e} = 6, IS_{13} = [2, 20], q_{13e} = 5, IS_{14} = [2, 7], q_{14e} = 5, IS_{15} = [15, 20], q_{15e} = 16, IS_{16} = [1, +\infty]$ *and* $q_{16e} = 19$.

The initial expected firing instants of each transition are: $St_{1e}(1) = 15, St_{2e}(1) = 22, St_{3e}(1) = 37, St_{4e}(1) = 7, St_{5e}(1) = 17, St_{6e}(1) = 12, St_{7e}(1) = 29, St_{8e}(1) = 35, St_{9e}(1) = 0, St_{10e}(1) = 5, St_{11e}(1) = 21$ *and* $St_{12e}(1) = 0$. The repetitive functioning mode is characterized by the period $\pi_0 = 40$. See Figure 2.

Definition 3. *A mono-synchronized subpath is a path containing one and only one synchronization transition which is its last node [11].*

Definition 4. *An elementary mono-synchronized subpath is a mono-synchronized subpath beginning with a place p such as ◦p is a synchronization transition [11].*

In Figure 1, there are eight elementary mono-synchronized subpaths constituting a partition of G:

- $Lp_1 = (p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$,

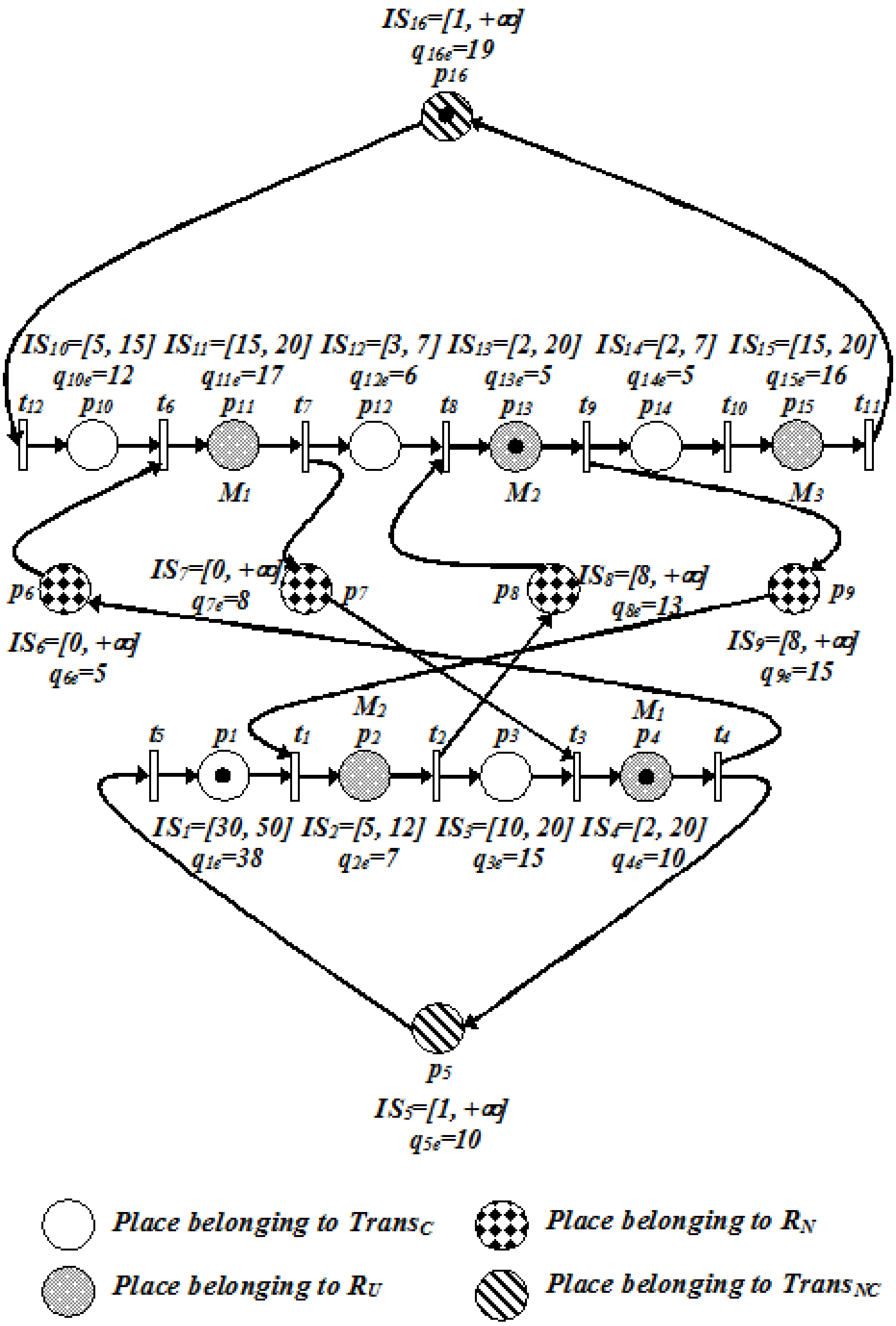

Figure 1: Functional decomposition

- $Lp_2 = (p_{13}, t_9, p_9, t_1)$,
- $Lp_3 = (p_2, t_2, p_3, t_3)$,
- $Lp_4 = (p_2, t_2, p_8, t_8)$,
- $Lp_5 = (p_4, t_4, p_5, t_5, p_1, t_1)$,
- $Lp_6 = (p_4, t_4, p_6, t_6)$,
- $Lp_7 = (p_{11}, t_7, p_7, t_3)$,
- $Lp_8 = (p_{11}, t_7, p_{12}, t_8)$.

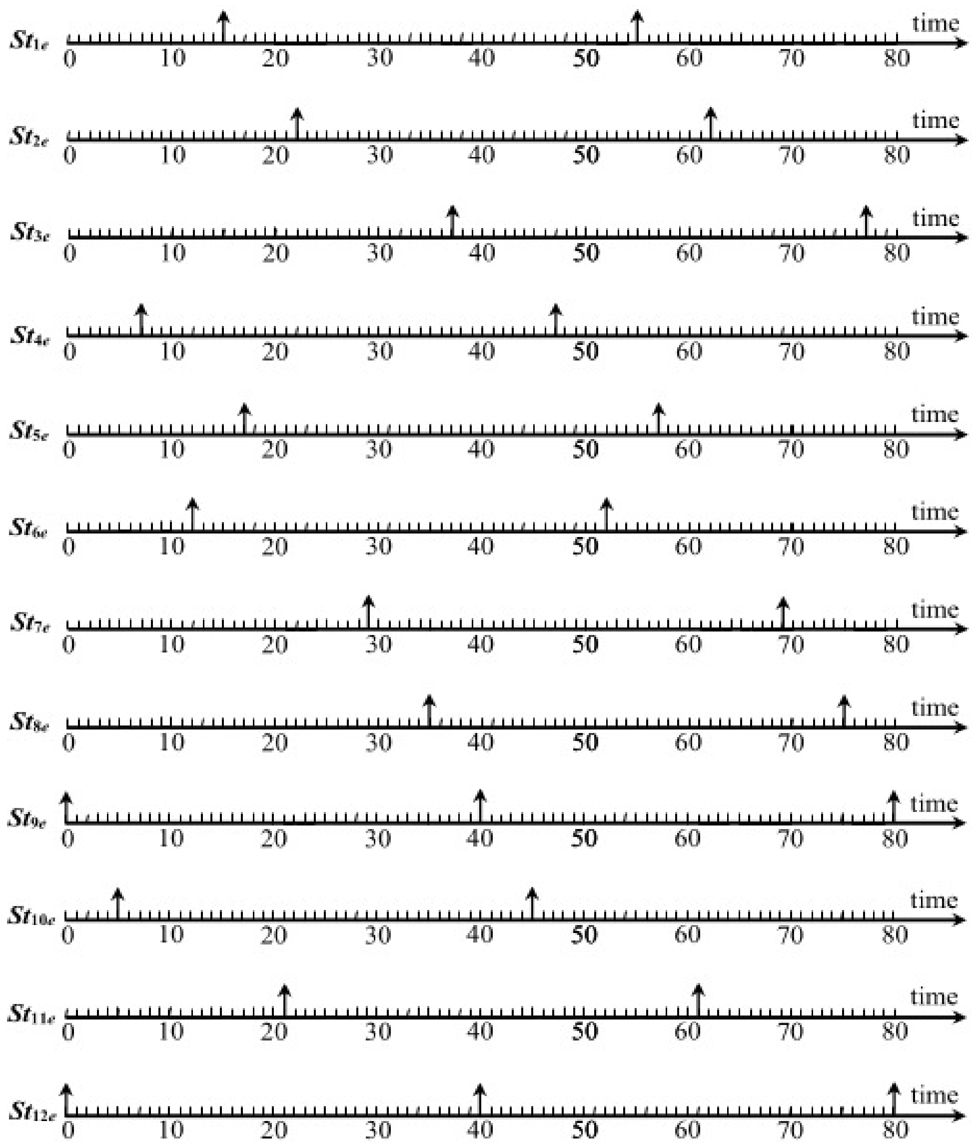

Figure 2: Expected firing instants

4 Decision tools for constraints violation

The following section provide a set of properties, which are useful for decision making. Depending on the nature of the time disturbance, various kinds of policy may be used, like shifting firing instants of a set of controllable transitions, trying to perform a one line scheduling because the current scheduling cannot be maintained without constraints violations or doing nothing because the non-occurrence of time constraints violation can be proved. The main results of this section provide proofs that the current scheduling can be preserved without constraints violation, providing various types of robustness proofs.

4.1 Theorem

Definition 5. *Let us consider a Discrete Event System (DES) and* G *the associated Petri Net model. Let us call* $B(G)$ *the behavior of* G *corresponding to the trajectory of states successively reached. Let* $C(B(G))$ *be the schedule of conditions established on the system behavior* $B(G)$*.* $C(B(G))$ *is materialized by a series of constraints which must be checked by* $B(G)$*. A non-respect of* $B(G)$ *corresponds to a violation of* $C(B(G))$ *[11].*

Definition 6. *It is said that a DES has a passive robustness on* $[\Delta_{min}, \Delta_{max}]$ *in a node* n *if the* *occurrence of a disturbance $\delta \in [\Delta_{min}, \Delta_{max}]$ at the node n does not involve a violation of $C(B(G))$ [11].*

Theorem 1. *Let us consider:*

- ts *a synchronization transition,*
- ${}^o ts = \left\{p_i, p_j / (p_i^o = p_j^o = ts) \wedge (p_i \in Trans_C) \wedge (p_j \in R_N)\right\}$,
- Res_{p_i} *the residue in* p_i *of a time disturbance,*
- Res_{p_j} *the residue in* p_j *of a time disturbance.*

The temporal residues Res_{p_i} and Res_{p_j} do not generate constraints violation in ${}^o ts$ if and only if: $Res_{p_j} - Res_{p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$.

Proof. Let us consider the synchronization transition ts of Figure 3.

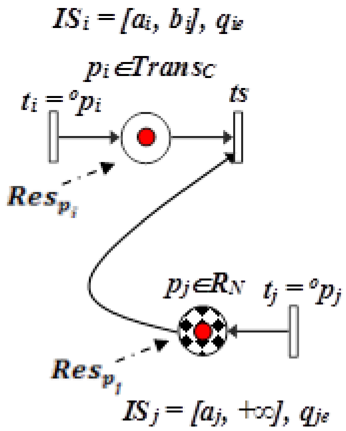

Figure 3: Study of constraints violation at the synchronization transition ts

Direction 1, hypothesis: the temporal residues do not generate constraints violation in ${}^o ts$. Without disturbance:

$$Sts_e(n) = St_{ie}(n) + q_{ie} = St_{je}(n) + q_{je}$$

$$Sts(n) = St_{ie}(n) + q_i + Res_{p_i}$$

$$Sts(n) = St_{je}(n) + q_j + Res_{p_j}$$

$$St_{ie}(n) + q_i + Res_{p_i} = St_{je}(n) + q_j + Res_{p_j}$$

$$(St_{ie}(n) + q_{ie}) + (q_i - q_{ie} + Res_{p_i}) = (St_j(n) + q_{je}) + (q_j - q_{je} + Res_{p_j})$$

$$Res_{p_j} - Res_{p_i} = (q_i - q_{ie}) - (q_j - q_{je})$$

If there is no violation of constraints, then:

$$q_i \leq b_i$$

$$q_j \geq a_j$$

$$q_i - q_j \leq b_i - a_j$$

$$Res_{p_j} - Res_{p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$$

Conversely, hypothesis: $Res_{p_j} - Res_{p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$.

- If both residues have the same sign, three cases are distinguished:

- $Res_{p_j} - Res_{p_i} < 0$,
- $0 \leq Res_{p_j} - Res_{p_i} \leq (q_{je} - a_j)$,
- $(q_{je} - a_j) < Res_{p_j} - Res_{p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$.

Case 1: $Res_{p_j} - Res_{p_i} < 0$.

If both residues are positive, $Res_{p_j} < Res_{p_i}$ means that the token in p_j keeps waiting until the sojourn time in p_i is $q_i = q_{ie}$. This wait does not involve a token death in p_j since this place represents a free machine ($b_j = +\infty$).

If both residues are negative, $Res_{p_j} < Res_{p_i}$ means that the temporal advance in p_j is strictly less than that in p_i. This advance does not cause a token death in p_j since this place represents a free machine. The token in p_i sojourns as it is expected $q_i = q_{ie}$ since the token in p_j is already available.

Case 2: $0 \leq Res_{p_j} - Res_{p_i} \leq (q_{je} - a_j)$. One has:

$$St_i = St_{ie} + Res_{p_i}$$

$$St_j = St_{je} + Res_{p_j}$$

$$Sts_e = St_{ie} + q_{ie} = St_{je} + q_{je}$$

$$St_{ie} - St_{je} = q_{je} - q_{ie}$$

$$Sts = St_i + q_i = St_{ie} + Res_{p_i} + q_i$$

Let us show that the firing of the transition ts can take place when $q_i = q_{ie}$. In other words, let us prove when $q_i = q_{ie}$ the token in the place p_j is available ($q_j \geq a_j$). We have:

$$Sts = St_{ie} + Res_{p_i} + q_{ie} \qquad (q_i = q_{ie})$$

$$q_j = Sts - St_j = Sts - St_{je} - Res_{p_j}$$

$$q_j = St_{ie} + Res_{p_i} + q_{ie} - St_{je} - Res_{p_j}$$

$$q_j = (St_{ie} - St_{je}) + q_{ie} + (Res_{p_i} - Res_{p_j})$$

$$q_j = (q_{je} - q_{ie}) + q_{ie} - (Res_{p_j} - Res_{p_i})$$

$$q_j \geq q_{je} - (q_{je} - a_j)$$

$$q_j \geq a_j$$

Therefore, the transition ts is fired when $q_i = q_{ie}$ and there is no violation of constraints.

Case 3: $(q_{je} - a_j) < Res_{p_j} - Res_{p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$.

Let us show if the firing of the transition ts occurs at $q_j = a_j$, the effective sojourn time of the token in the place p_i satisfies: $q_{ie} < q_i \leq b_i$. One obtains:

$$Sts = St_j + q_j = St_{je} + Res_{p_j} + q_j$$

$$Sts = St_{je} + Res_{p_j} + a_j \qquad (q_j = a_j)$$

$$q_i = Sts - St_i = Sts - St_{ie} - Res_{p_i}$$

$$q_i = St_{je} + Res_{p_j} + a_j - St_{ie} - Res_{p_i}$$

$$q_i = (St_{je} - St_{ie}) + a_j + (Res_{p_j} - Res_{p_i})$$

$$q_i = (q_{ie} - q_{je}) + a_j + (Res_{p_j} - Res_{p_i})$$

$$(q_{je} - a_j) + (q_{ie} - q_{je}) + a_j < q_i \leq (q_{ie} - q_{je}) + a_j + (b_i - q_{ie}) - (a_j - q_{je})$$

$$q_{ie} < q_i \leq b_i$$

Hence, there is no violation of constraints since the transition ts is fired when $q_j = a_j$ and $q_{ie} < q_i \leq b_i$.

Interpretation. Let us take the worst case: $Res_{p_j} - Res_{p_i} = (b_i - q_{ie}) + (q_{je} - a_j)$. If both residues are positive, the token in p_j arrives late compared to that in p_i. Part of this delay is compensated by $(q_{je} - a_j)$. Instead of staying q_{ie}, the token in p_j sojourns only $q_j = a_j$. Indeed, the token in the place p_j, representing a waiting machine, is available as soon as $q_j = a_j$, which makes it possible to reject part of the delay disturbance equal to $(q_{je} - a_j)$. Consequently, it remains a delay difference equal to $(b_i - q_{ie})$. In the place p_i, the token is blocked for the time $(b_i - q_{ie})$ until the token in p_j is available. The transition ts is fired when $q_i = b_i$ and $q_j = a_j$ which is the admissible limit case. Thus, there is no token death.

If both residues are negative, the token in p_i arrives in advance compared to that in p_j. Part of this advance is compensated by the delay $(b_i - q_{ie})$. Instead of staying q_{ie}, the token in p_i sojourns $q_i = b_i$. The token in the place p_j is available as soon as $q_j = a_j$ which offers an advance $(a_j - q_{je})$ making it possible to avoid a token death in p_i. Then, the transition ts is fired when $q_i = b_i$ and $q_j = a_j$.

- If one of the residues is positive and the other is negative, we distinguish two cases: $Res_{p_j} - Res_{p_i} < 0$ or $0 \leq Res_{p_j} - Res_{p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$.

Case 1: $Res_{p_j} - Res_{p_i} < 0$.

Necessarily, we have a delay in the place p_i and an advance in the place p_j. The token in p_j waits for the one in p_i until $q_i = q_{ie}$. This does not cause any constraints violation since $p_j \in R_N$.

Case 2: $0 \leq Res_{p_j} - Res_{p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$.

Forcibly, we have an advance in the place p_i and a delay in the place p_j. This amounts to the study of the case where the residue in p_i is zero and the residue in p_j is a delay equal to $(Res_{p_j} - Res_{p_i})$. This is a similar case to the occurrence of two delay residues, already studied. Since $0 \leq Res_{p_j} - Res_{p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$, there is no violation of constraints. □

4.2 Example

Let us take the synchronization t_6 of Figure 4. It comes: ${}^o t_6 = \{p_{10} \in Trans_C, p_6 \in R_N\}$ and $(b_{10} - q_{10e}) + (q_{6e} - a_6) = (15 - 12) + (5 - 0) = 8$.

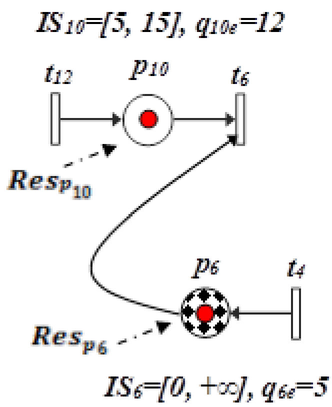

Figure 4: Study of constraints violation at the synchronization transition t_6

- Case of delay residues.

Let us suppose that the transition t_4 is fired with a delay equal to 13 and the transition t_{12} with a delay equal to 5. In other words, $Res_{p_6} = 13$ and $Res_{p_{10}} = 5$ $(Res_{p_6} - Res_{p_{10}} = 8)$. We have:

$$St_{6e}(2) = St_{4e}(2) + q_{6e} = St_{12e}(2) + q_{10e} = 52$$

$$St_4(2) = St_{4e}(2) + Res_{p_6} = 60$$

$$St_{12}(2) = St_{12e}(2) + Res_{p_{10}} = 45$$

Since $a_6 = 0$, the token is available as soon as it enters the place p_6. The maximum sojourn time in the place p_{10} is $q_{10} = b_{10} = 15$. In order not to have a token death in p_{10}, the transition t_6 must be fired as: $St_6(2) = St_{12}(2) + b_{10} = 45 + 15 = 60$, which coincides well with the firing instant of the transition t_4 and the availability of the token in the place p_6. From Figure 5, it is clear that a residue difference strictly greater than 8 automatically generates a token death in p_{10}. There is no longer an overlap in the token availability intervals (static intervals) relating to the two places p_6 and p_{10}.

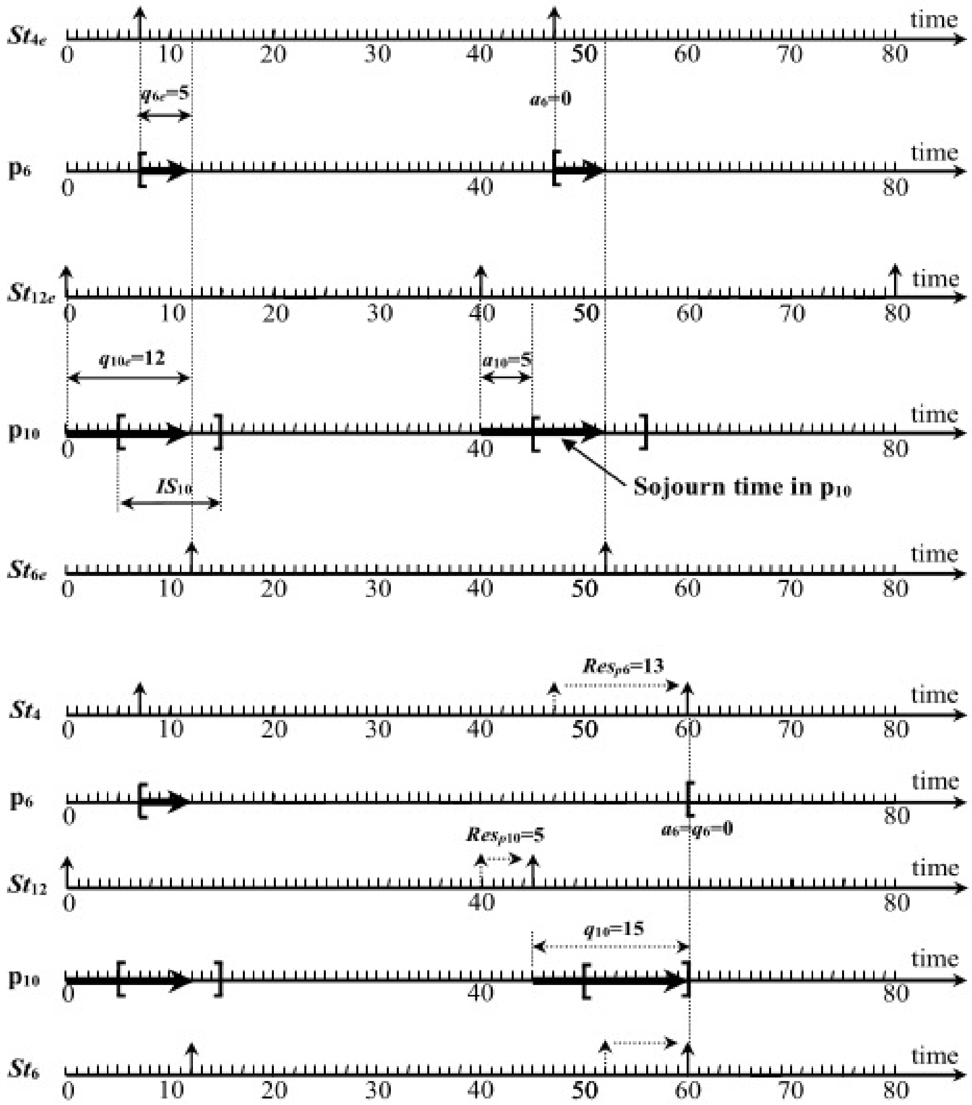

Figure 5: Case of delay residues

- Case of advance residues.

Let us assume that the transition t_4 is fired with an advance equal to -6 and the transition t_{12} with an advance equal to -14, which means $Res_{p_6} = -6$ *and* $Res_{p_{10}} = -14$ ($Res_{p_6} - Res_{p_{10}} = 8$). We get:

$$St_4(2) = St_{4e}(2) + Res_{p_6} = 47 - 6 = 41$$

$$St_{12}(2) = St_{12e}(2) + Res_{p_{10}} = 40 - 14 = 26.$$

The token in p_6 is available from the instant $St_4(2) = 41$. To avoid a token death in p_{10}, the transition t_6 must be fired as: $St_6(2) = St_{12}(2) + b_{10} = 26 + 15 = 41$, which coincides well with the

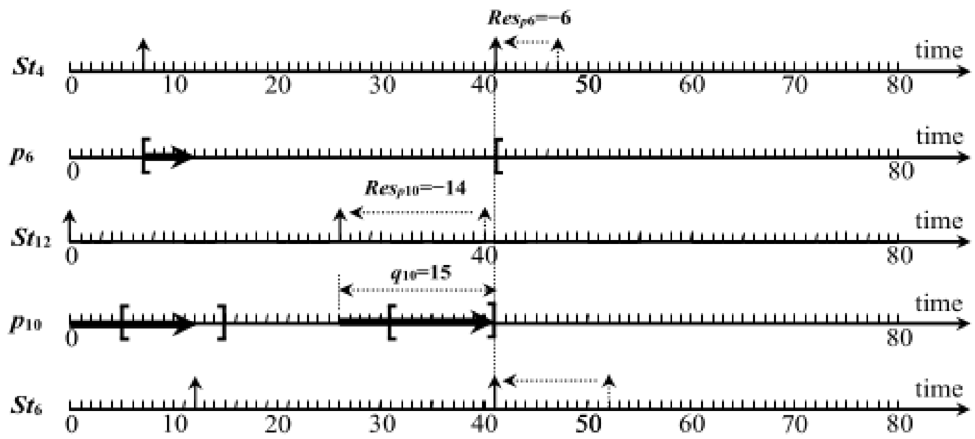

Figure 6: Case of advance residues

firing instant of the transition t_4 and the availability of the token in place p_6. From Figure 6, it is also clear that a residue difference strictly greater than 8 automatically generates a token death in p_{10}.

- Case of delay and advance residues.

If the transition t_4 is fired with a delay equal to 5 and the transition t_{12} with an advance equal to -3, which means $Res_{p_6} = 5$ *and* $Res_{p_{10}} = -3$ $(Res_{p_6} - Res_{p_{10}} = 8)$, then:

$$St_4(2) = St_{4e}(2) + Res_{p_6} = 47 + 5 = 52$$

$$St_{12}(2) = St_{12e}(2) + Res_{p_{10}} = 40 - 3 = 37$$

The token in p_6 is available from the instant $St_4(2) = 52$. To avoid a token death in p_{10}, the transition t_6 must be fired as: $St_6(2) = St_{12}(2) + b_{10} = 37 + 15 = 52$, which coincides well with the firing instant of the transition t_4 and the availability of the token in place p_6. From Figure 7, it is also clear that a residue difference strictly greater than 8 automatically generates a token death in p_{10}.

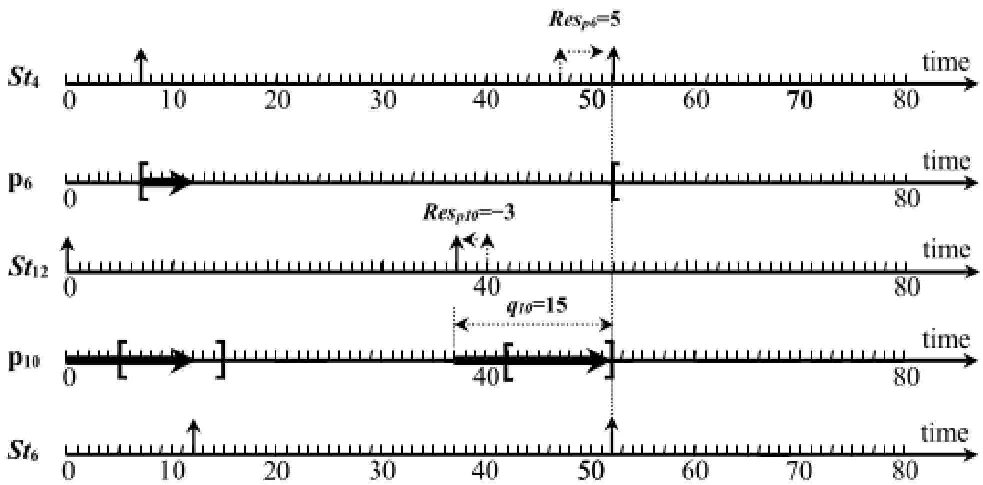

Figure 7: Case of delay and advance residues

4.3 Lemmas

Lemma 2. *Let us consider:*

- *ts a synchronization transition,*
- $^{o}ts = \{p_i, p_j/(p_i \in Trans_C) \wedge (p_j \in R_N)\}$,
- $(p_k \in P) \wedge (^{o}p_k = ts)$,
- $(t_i \in T) \wedge (t_i =^{o} p_i)$,
- $(t_j \in T) \wedge (t_j =^{o} p_j)$,
- *δ_1 (resp. δ_2) a delay time disturbance at the input of the place p_i (resp. p_j),*
- *Res_{1p_k} (resp. Res_{2p_k}) the residue in p_k of the disturbance δ_1 (resp. δ_2) when it occurs alone,*
- *Res_{p_k} the residue in p_k of the disturbances δ_1 and δ_2 when they occur simultaneously.*

If the occurrence of δ_2 alone does not cause a token death in ^{o}ts then the simultaneous occurrence of δ_1 and δ_2 does not cause a token death in ^{o}ts and $Res_{p_k} = max(Res_{1p_k}, Res_{2p_k})$.

Proof. Let us consider the synchronization transition ts of Figure 8.

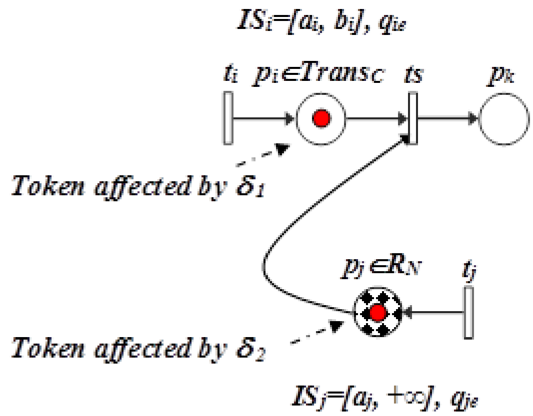

Figure 8: Study of constraints violation when two delay time disturbances occur simultaneously

The occurrence of δ_1 alone does not cause a token death in ^{o}ts since the place $p_j \in R_N$ represents a free machine ($b_j = +\infty$). The disturbance δ_1 is totally transmitted in p_k : $Res_{p_k} = \delta_1$. The place p_j allows to compensate or reject part of the delay disturbance equal to $(q_{je} - a_j)$. When the disturbance δ_2 occurs alone, it comes: $Res_{2p_k} = max(0, \delta_2 - (q_{je} - a_j))$.

Case 1: $\delta_1 \geq \delta_2$.

The token in p_j keeps waiting until the sojourn time in p_i is equal to q_{ie} without any violation of constraints and we have: $Res_{p_k} = Res_{1p_k} = \delta_1 = max(Res_{1p_k}, Res_{2p_k})$.

Case 2: $\delta_2 \geq \delta_1 + (q_{je} - a_j\ )$.

By hypothesis, the occurrence of δ_2 alone does not cause a token death in ^{o}ts. According to the theorem 1, it comes:

$$\delta_2 \leq (b_i - q_{ie}) + (q_{je} - a_j)$$

$$\delta_1 + (q_{je} - a_j) \leq \delta_2 \leq (b_i - q_{ie}) + (q_{je} - a_j)$$

$$(q_{je} - a_j) \leq \delta_2 - \delta_1 \leq (b_i - q_{ie}) + (q_{je} - a_j) - \delta_1$$

$$(q_{je} - a_j) \leq \delta_2 - \delta_1 \leq (b_i - q_{ie}) + (q_{je} - a_j)$$

According to the theorem 1, there is no token death. This corresponds to the third case of the theorem proof in the case where the disturbances have the same sign. The token in the place p_i keeps waiting ($q_i \geq q_{ie}$) until the token in the place p_j is available: $q_j = a_j$. Then, the firing of the transition

ts takes place when $q_j = a_j$ and the sojourn time of the token in the place p_i satisfies: $q_{ie} \leq q_i \leq b_i$. One gets:

$$Sts = St_{je} + \delta_2 + a_j \qquad (q_j = a_j)$$

$$Res_{p_k} = Sts - Sts_e = \delta_2 + a_j + (St_{je} - Sts_e) = \delta_2 - (q_{je} - a_j)$$

$$Res_{p_k} = \delta_2 - (q_{je} - a_j) = Res_{2p_k} = max(Res_{1p_k}, Res_{2p_k}) \quad (\delta_2 - (q_{je} - a_j) \geq \delta_1)$$

Case 3: $\delta_1 < \delta_2 < \delta_1 + (q_{je} - a_j)$. One has:

$$0 < \delta_2 - \delta_1 < (q_{je} - a_j)$$

$$0 < \delta_2 - \delta_1 < (b_i - q_{ie}) + (q_{je} - a_j)$$

According to the theorem 1, there is no token death. This corresponds to the second case of the theorem proof in the case where the disturbances have the same sign. The token in the place p_j remains waiting until $q_i = q_{ie}$. Accordingly, one obtains:

$$Sts = St_{ie} + \delta_1 + q_{ie} \qquad (q_i = q_{ie})$$

$$Res_{p_k} = Sts - Sts_e = \delta_1 + q_{ie} + (St_{ie} - Sts_e\ ) = \delta_1 + q_{ie} - q_{ie} = \delta_1$$

$$Res_{p_k} = \delta_1 = Res_{1p_k} = max(Res_{1p_k}, Res_{2p_k}) \quad (\delta_1 > \delta_2 - (q_{je} - a_j))$$

□

Lemma 3. *Let:*

- δ_1 *and* δ_2 *two delay time disturbances respectively at the transitions* t_1 *and* t_2*,*
- $[\Delta_{1min}, \Delta_{1max}]$ *and* $[\Delta_{2min}, \Delta_{2max}]$ *the passive robustness intervals respectively at* t_1 *and* t_2*.*

If $\delta_1 \in [\Delta_{1min}, \Delta_{1max}]$ *and* $\delta_2 \in [\Delta_{2min}, \Delta_{2max}]$ *then the simultaneous occurrence of* δ_1 *and* δ_2 *does not involve any constraints violation.*

Proof. Let us denote by:

- ts a synchronization transition,
- ${}^o ts = \{p_i, p_j / (p_i \in Trans_C) \wedge (p_j \in R_N)\}$,
- Res_{1p_i} (resp. Res_{1p_j}) the residue in p_i (resp. p_j) of the disturbance δ_1 when it occurs alone,
- Res_{2p_i} (resp. Res_{2p_j}) the residue in p_i (resp. p_j) of the disturbance δ_2 when it occurs alone,
- Res_{p_i} (resp. Res_{p_j}) the residue in p_i (resp. p_j) of the disturbances δ_1 and δ_2 when they occur simultaneously.

Lemma 2 gives: $Res_{p_i} = max(Res_{1p_i}, Res_{2p_i})$ and $Res_{p_j} = max(Res_{1p_j}, Res_{2p_j})$. Four cases are possible, which are:

- $(Res_{p_i}, Res_{p_j}) = (Res_{1p_i}, Res_{1p_j})$,
- $(Res_{p_i}, Res_{p_j}) = (Res_{1p_i}, Res_{2p_j})$,
- $(Res_{p_i}, Res_{p_j}) = (Res_{2p_i}, Res_{1p_j})$,
- $(Res_{p_i}, Res_{p_j}) = (Res_{2p_i}, Res_{2p_j})$.

Case 1: $(Res_{p_i}, Res_{p_j}) = (Res_{1p_i}, Res_{1p_j})$.
Res_{p_i} and Res_{p_j} are the residues of δ_1 only. There is no token death since $\delta_1 \in [\Delta_{1min}, \Delta_{1max}]$.
Case 2: $(Res_{p_i}, Res_{p_j}) = (Res_{1p_i}, Res_{2p_j})$.
Knowing $\delta_2 \in [\Delta_{2min}, \Delta_{2max}]$, according to the theorem 1 we have:

$$Res_{2p_j} - Res_{2p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$$

$$Res_{p_i} = max(Res_{1p_i}, Res_{2p_i}) = Res_{1p_i} \longrightarrow Res_{1p_i} \geq Res_{2p_i}$$

$$\longrightarrow Res_{2p_j} - Res_{1p_i} \leq Res_{2p_j} - Res_{2p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$$

According to the theorem 1, there is no constraints violation.
Case 3: $(Res_{p_i}, Res_{p_j}) = (Res_{2p_i}, Res_{1p_j})$.
Knowing $\delta_1 \in [\Delta_{1min}, \Delta_{1max}]$, according to the theorem 1 we have:

$$Res_{1p_j} - Res_{1p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$$

$$Res_{p_i} = max(Res_{1p_i}, Res_{2p_i}) = Res_{2p_i} \longrightarrow Res_{2p_i} \geq Res_{1p_i}$$

$$\longrightarrow Res_{1p_j} - Res_{2p_i} \leq Res_{1p_j} - Res_{1p_i} \leq (b_i - q_{ie}) + (q_{je} - a_j)$$

According to the theorem 1, there is no constraints violation.
Case 4: $(Res_{p_i}, Res_{p_j}) = (Res_{2p_i}, Res_{2p_j})$.
Res_{p_i} and Res_{p_j} are the residues of δ_2 only. There is no constraints violation since $\delta_2 \in [\Delta_{2min}, \Delta_{2max}]$.
□

Finally, this section presented an original kind of robustness towards a disturbance, which may be an advance or a delay. Some formal definitions are provided in order to build a proved way of computing the corresponding robustness values. Two lemmas provide first results towards concurrent time disturbances occurring on the same time. The last section of this paper stands the quality of the results and discuss the limit of the proposed approach, targeting new industrial applications.

5 Conclusion

The paper has presented the theoretical background in order to build an original contribution to the state of the art in the area of manufacturing processes where every operation time has minimal and maximal values. The proposed study considers the effect of delays and advances all together in the same manufacturing process. The first step of the construction presents the classical modeling tool dedicated to duration constrained discrete event systems: the P-time Petri nets. Then, they are mapped on the dedicated historical functional decomposition of a manufacturing process. Using this framework provided by the state of the art, the fourth section presents new behavioral properties. It presented the mixed analysis of disturbance residues at the level of synchronization transitions. After building some proofs and providing illustrations and interpretations, two final properties are formalized. The first one is the robustness properties of a manufacturing system on a range that may include delay and advance. The second one is the generalization of the robustness property towards simultaneous occurrence of two delay time disturbances at two points of the system each having its own robustness range.

Theoretical results of this paper are useful for alarm triggering at a plant supervision level. Moreover, improving the quality of alarm filtering like presented in [13, 20], by enriching the class of disturbances from which the system robustness can be proved, is clearly a step forward. It is an evidence that increasing the range of the robustness properties will decrease the quantity of false alarms. The presented new results of this paper consider the possibility of a mix between advances and delays in running manufacturing processes, by getting deeper in the scenario understanding of a constraint violation clearly allows a better diagnosis.

However, the historical functional decomposition only considers a local knowledge, but does not integrate a possible distant knowledge that is consulted through a communication network producing transmission delays. Taking advantages of distributed information centers is a modern proposition used in the digital twin concepts and in the big data scientific area [17, 23]. A new functional decomposition will have to introduce new classes in order to address the specific needs required by industry 4.0, separating concepts of knowledge being immediately available and knowledge that have to be processed a given amount of time before being available. This new decomposition has to take into account that in particular nodes of the system, the process and resources are synchronized to respect minimum and maximum operations time. The enriched concept of the new functional decomposition may allow addressing targeted industrial applications where the time is particularly critical, like tele-operated trains [19].

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