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Information Volume of Fuzzy Membership Function

J.X. Deng, Y. Deng

Jixiang Deng
Institute of Fundamental and Frontier Science
University of Electronic Science and Technology of China, Chengdu, 610054, China

Yong Deng*
1. Institute of Fundamental and Frontier Science
University of Electronic Science and Technology of China, Chengdu, 610054, China
2. School of Education
Shannxi Normal University, Xi'an, 710062, China
*Corresponding author: dengentropy@uestc.edu.cn; prof.deng@hotmail.com

Abstract

Fuzzy membership function plays an important role in fuzzy set theory. However, how to measure the information volume of fuzzy membership function is still an open issue. The existing methods to determine the uncertainty of fuzzy membership function only measure the first-order information volume, but do not take higher-order information volume into consideration. To address this issue, a new information volume of fuzzy membership function is presented in this paper, which includes the first-order and the higher-order information volume. By continuously separating the hesitancy degree until convergence, the information volume of the fuzzy membership function can be calculated. In addition, when the hesitancy degree of a fuzzy membership function equals to zero, the information volume of this special fuzzy membership function is identical to Shannon entropy. Two typical fuzzy sets, namely classic fuzzy sets and intuitiontistic fuzzy sets, are studied. Several examples are illustrated to show the efficiency of the proposed information volume of fuzzy membership function.

Keywords: fuzzy sets, membership function, information volume, higher-order information volume, entropy.

1 Introduction

In the past decades, plenty of theories have been developed for expressing and dealing with the uncertainty in the uncertain environment, such as probability theory [19], fuzzy set theory [51], Dempster-Shafer evidence theory [6, 28], complex evidence theory [37, 39], Z numbers [53], belief structure [47, 49] and D numbers [23, 24].

Since fuzzy set theory was firstly proposed by Zadeh in 1965 [51], uncertain information processing based on fuzzy set has become a heated field [9, 40]. Lots of researchers have been promoting the development of fuzzy set theory. For instance, Zadeh presented type-2 fuzzy set in 1975 [52], which is a generalization of classic fuzzy set. In 1999, Atanassov extended classic fuzzy set into intuitionistic fuzzy set [2], which introduces hesitancy into fuzzy membership function. Yager proposed Pythagorean fuzzy subset in 2013 [46], which can better express the fuzziness under uncertain circumstances. In addition, because of the advantages of modeling vagueness and imprecision in the real world, fuzzy set theory has been applied in many areas, such as data processing [54], approximate reasoning [48], decision making [10, 14, 15, 30, 44], uncertainty measurement [4], risk analysis [27], failure mode and effects analysis [17], and so on [12].

Given a probability distribution, its corresponding information volume can be measured by Shannon entropy [29]. In evidence theory, mass function is the generalization of probability, whose uncertainty can be measured by Deng entropy [8, 13, 22]. In addition, given a mass function, its corresponding information volume can be calculated [7]. However, how to determine the information volume of fuzzy membership function in fuzzy set theory is still an open issue. Some methods have been proposed to model the uncertainty of fuzzy set theory, such as the uncertainty of the interval type-2 fuzzy set [16], the uncertainty of fuzzy system [21], fuzzy entropy of fuzzy set [5, 25, 31, 50]. Nevertheless, these uncertainty measurements are the first-order information volume of fuzzy set. It is reasonable to take the high-order information volume into consideration [7].

To address the issue mentioned above, an original and novel information volume of fuzzy membership function is proposed in this paper, including the first-order and the higher-order information volume. Since there is extra information volume existing in the hesitancy degree compared with membership function and non-membership function, this extra information volume should be fully utilized to determine the information volume of fuzzy membership function. The major idea of the proposed method is to continuously separate the hesitancy degree until convergence, then the information volume of fuzzy membership function can be calculated.

The rest of this paper is organized as follows. Section **2** briefly reviews some preliminaries. Section **3** proposes a new information volume of fuzzy membership function. Some numerical examples are illustrated in Section **4**. Section **5** makes a brief conclusion.

2 Preliminaries

Several preliminaries are briefly introduced in this section, including fuzzy sets, intuitionistic fuzzy sets and Shannon entropy.

2.1 Fuzzy sets

Let X be a universe of discourse. A fuzzy set A based on X can be characterized by the set of pairs which is defined as [51]:

Definition 2.1: *Fuzzy sets*

$$A = \{\langle x, \mu(x)\rangle \,|x \in X\} \tag{1}$$

where $\mu(x) : X \to [0, 1]$ is the membership function (MF) of A, which describes the membership degree of each element x to the fuzzy set A. The closer $\mu(x)$ is to 1, the more likely x belongs to A.

2.2 Intuitionistic fuzzy sets

Given a universe of discourse X, an intuitionistic fuzzy set A is defined as follows [2]:

Definition 2.2: *Intuitionistic fuzzy sets*

$$A = \{\langle x, \mu(x), v(x)\rangle \,|x \in X\} \tag{2}$$

where $\mu(x) : X \to [0, 1]$ and $v(x) : X \to [0, 1]$ are the membership function (MF) and the non-membership function (non-MF) of A, respectively. $\mu(x)$ describes the membership degree of x to *the set A, and $v(x)$ describes the non-membership degree of x to the set A. For every $x \in X$, $0 \le \mu(x) + v(x) \le 1$. The hesitancy degree is defined as:*

$$\pi(x) = 1 - \mu(x) - v(x) \tag{3}$$

which represents the hesitancy degree of each element $x \in X$.

Besides, intuitionistic fuzzy sets can be represented as mass function, and a lot of works under evidence theory are presented [36, 42, 43, 45].

2.3 Shannon entropy

Entropy function plays an important role in measuring the uncertainty of a system [3, 26, 32, 55] In the field of probability theory, Shannon entropy [29] is often used to measure the uncertainty of a probability distribution. Consider a probability distribution P defined on the set $\Theta = \{H_1, H_2, H_3, \cdots, H_N\}$.
Definition 2.3: *Shannon entropy*

Shannon entropy $H_s(P)$ is defined as follows:

$$H_s(P) = -\sum_{\theta\in\Theta} P(\theta)\log P(\theta). \tag{4}$$

where $\sum_{\theta\in\Theta} P(\theta) = 1$ and $P(\theta) \in [0, 1]$.

3 Information volume of fuzzy membership function

In this section, firstly, the first-order information volume of fuzzy membership function is proposed. Then, the higher-order information volume of fuzzy membership function is presented.

3.1 First-order information volume of fuzzy membership function

In the course of information processing in fuzzy environment, it is important to identify the uncertainty of the data. Taking intuitionistic fuzzy sets for example, intuitively, the uncertainty of fuzzy membership function $\langle x_1, 0.4, 0.4\rangle$ is larger than that of $\langle x_2, 0.9, 0.1\rangle$. In fact, this uncertainty of fuzzy membership function is actually the first-order information volume. Hence, the first-order information volume of fuzzy membership function is proposed in this subsection.
Definition 3.1: *First-order information volume of fuzzy membership function*

Let the universe discourse be X, and an intuitionistic fuzzy set be $\{\langle x, \mu, v\rangle | x \in X\}$ where μ is membership function, v is non-membership function and π is hesitancy degree. The first-order information volume of fuzzy membership function $H_1(\mu, v)$ is defined as:

$$H_1(\mu, v) = -\mu\log\mu - v\log v - \pi\log\frac{\pi}{C} \tag{5}$$

where $\pi = 1 - \mu - v$, and C is the cardinality of fuzzy sets defined as:

$$C = \begin{cases} 2 \ (classic\ fuzzy\ sets) \\ 3 \ (intuitionistic\ fuzzy\ sets) \end{cases} \tag{6}$$

The hesitancy degree π can be considered as 'hesitant' which is a hybrid proposition of 'support' and 'oppose'. Hence, π contains more information volume than μ and v. It is reasonable that π should be divided by the cardinality of the fuzzy sets.

The definition of the cardinality C is explained as follows. In classic fuzzy sets, μ and $\pi = 1-\mu$ can be viewed as these two propositions: 'support' and 'hesitant', so that the cardinality of classic fuzzy sets is 2. In intuitionistic fuzzy sets, μ, v and π can be respectively seen as these three propositions: 'support', 'oppose' and 'hesitant', so that the cardinality of intuitionistic fuzzy sets is 3.

Actually, the classic fuzzy set is a special case of the intuitionistic fuzzy set, which means that the classic fuzzy set satisfies $v \equiv 0$ and $\pi = 1 - \mu$. Hence, the first-order information volume of classic fuzzy membership function is that:

$$H_1(\mu, v) = -\mu \log \mu - (1-\mu) \log \frac{1-\mu}{2} \tag{7}$$

3.2 Higher-order information volume of the fuzzy membership function

The real world is very complex and complicated [11, 18, 20]. It's more reasonable to explore a complex thing from different aspects [38]. For example, negation is paid attention recently since it can provides new view from negative side [1, 41]. Another typical modeling is with complex networks [33, 34, 35]. About the information volume, the issue of the first-order information volume is that it divides the hesitancy degree π only once, which cannot make fully use of the information volume of π. Because π can be seen as the proposition of 'hesitant', which is a hybrid proposition of 'support' and 'oppose', π contains extra information volume compared with μ and v. This extra information volume should be fully utilized by continuously separating the hesitancy degree until convergence. Inspired by the idea mentioned above, the higher-order information volume of fuzzy membership function is presented in this subsection.

Definition 3.2: *Higher-order information volume of fuzzy membership function*

Let the universe discourse be X, and an intuitionistic fuzzy set be $\{\langle x, \mu, v\rangle | x \in X\}$ where μ is membership function, v is non-membership function and π is the hesitancy degree. The higher-order information volume of fuzzy membership function $H_\infty(\mu, v)$ is defined as follows:

$$H_\infty(\mu, v) = -\sum_{i=1}^{\infty} \mu_i \log \mu_i - \sum_{i=1}^{\infty} v_i \log v_i - \mathop{lim}_{i \to \infty} \pi_i \log \frac{\pi_i}{C} \tag{8}$$

where C is the cardinality of the fuzzy sets defined as:

$$C = \begin{cases} 2 \ (classic\ fuzzy\ sets) \\ 3 \ (intuitionistic\ fuzzy\ sets) \end{cases} \tag{9}$$

$\mu_1 = \mu$, $v_1 = v$ and $\pi_1 = \pi = 1 - \mu - v$ denote the initial states of membership function, non-membership function and hesitancy degree. i denotes the times of separation. μ_i, v_i and π_i $(i > 1)$ are the terms derived by separating π_{i-1} which are defined as:

$$\begin{cases} \mu_i = k_1 \pi_{i-1} \\ v_i = k_2 \pi_{i-1} \\ \pi_i = k_3 \pi_{i-1} \end{cases} \tag{10}$$

where $k_1 = \mu$, $k_2 = v$ and $k_3 = \pi = 1 - \mu - v$ are the separating proportions.

The reason for the definition of the separating proportion k_1, k_2 and k_3 is that, μ, v and π can be seen as the prior information of fuzzy sets, so that the hesitancy degree can be separated based on these prior information.

Besides, it is a convention that the expression $0 \log 0$ is defined as 0. Hence, when the hesitancy degree $\pi_1 = 0$, the higher-order information volume of fuzzy membership function $H_\infty(\mu, v)$ can be written as follows:

$$H_\infty(\mu, v) = -\mu_1 \log \mu_1 - v_1 \log v_1 \tag{11}$$

Actually, the classic fuzzy set is a special case of the intuitionistic fuzzy set, which means that the classic fuzzy set satisfies $v_i \equiv 0$ and $\pi_i = 1 - \mu_i$. Thus, the higher-order information volume of classic fuzzy membership function is that:

$$H_\infty(\mu, v) = -\sum_{i=1}^{\infty} \mu_i \log \mu_i - \mathop{lim}_{i \to \infty} (1-\mu_i) \log \frac{1-\mu_i}{2} \tag{12}$$

Algorithm 1 Higher-order information volume of fuzzy membership function $H_\infty(\mu, v)$

Input: : MF μ and non-MF v
Output: : Higher-order information volume of fuzzy membership function $H_\infty(\mu, v)$
1: $i = 1$ //Initial time of separation
2: $\mu_1 = \mu$, $v_1 = v$, $\pi_1 = 1 - \mu - v$ //Initial states
3: $k_1 = \mu$, $k_2 = v$, $k_3 = 1 - \mu - v$ //Separating proportion
4: **repeat**
5: $i{+}{+}$
6: $\mu_i = k_1 \pi_{i-1}$, $v_i = k_2 \pi_{i-1}$, $\pi_i = k_3 \pi_{i-1}$ //Separating π_{i-1}
7: $H_i(\mu, v) = -\sum_{j=1}^{i} \mu_j LOG \mu_j - \sum_{j=1}^{i} v_j LOG v_j - \pi_i LOG \frac{\pi_i}{C}$ //$xLOGy$ invokes **Algorithm 2**
8: **until** $\Delta_i = H_i(\mu, v) - H_{i-1}(\mu, v) < \varepsilon$ //Allowable error ε
9: **return** $H_\infty(\mu, v) = H_i(\mu, v)$

Algorithm 2 Modified logarithm $xLOGy$

Input: : Real numbers x and y
Output: : Modified logarithm $xLOGy$
if $x == 0$ && $y == 0$ **then**
$ans = 0$ //The expression $0 \log 0$ is defined as 0
else
$ans = x * \log(y)$
end if
return $xLOGy = ans$

In addition, given an allowable error ε, the higher-order information volume of fuzzy membership function $H_\infty(\mu, v)$ can also be calculated by **Algorithm 1**. It should be noted that, in **Algorithm 1**, step 7 invokes modified logarithm $xLOGy$ in **Algorithm 2** to avoid the computation $0 \log 0$.

It can be derived from **Definition 3.2** and **Algorithm 1** that, when $i = 1$, the higher-order information volume of fuzzy membership function $H_\infty(\mu, v)$ degenerates into the first-order information volume of fuzzy membership function $H_1(\mu, v)$, which can be written as follows:

$$H_\infty(\mu, v)|_{i=1} = H_1(\mu, v) \tag{13}$$

For better understanding, the calculating procedure of $H_\infty(\mu, v)$ is illustrated in Figure 1. This figure clearly shows that the hesitancy degree is continuously separated based on the proportion k_1, k_2 and k_3.

4 Numerical examples and discussions

In this section, some numerical examples are shown to illustrate above conceptions. In the following examples, let the universe discourse be X and the base of the logarithmic function be 2.

Example 4.1:

Consider that a classic fuzzy membership function is $\langle x_1, 0.65 \rangle$ where $x_1 \in X$. The first-order information volume of this classic fuzzy membership function can be calculated as follows:

Because this is a classic fuzzy membership function, the cardinality C is 2, and $\pi = 1 - \mu = 0.35$. Then, the first-order information volume of this classic fuzzy membership function is that:

$$\begin{aligned} H_1(\mu, v) &= -\mu \log \mu - (1 - \mu) \log \frac{1 - \mu}{C} \\ &= -0.65 \log_2(0.65) - 0.35 \log_2(\frac{0.35}{2}) = 1.2841 \end{aligned} \tag{14}$$

Example 4.2:

Let an intuitionistic fuzzy membership function be $\langle x_2, 0.65, 0.05 \rangle$ where $x_2 \in X$. The first-order information volume of this intuitionistic fuzzy membership function can be calculated as follows:

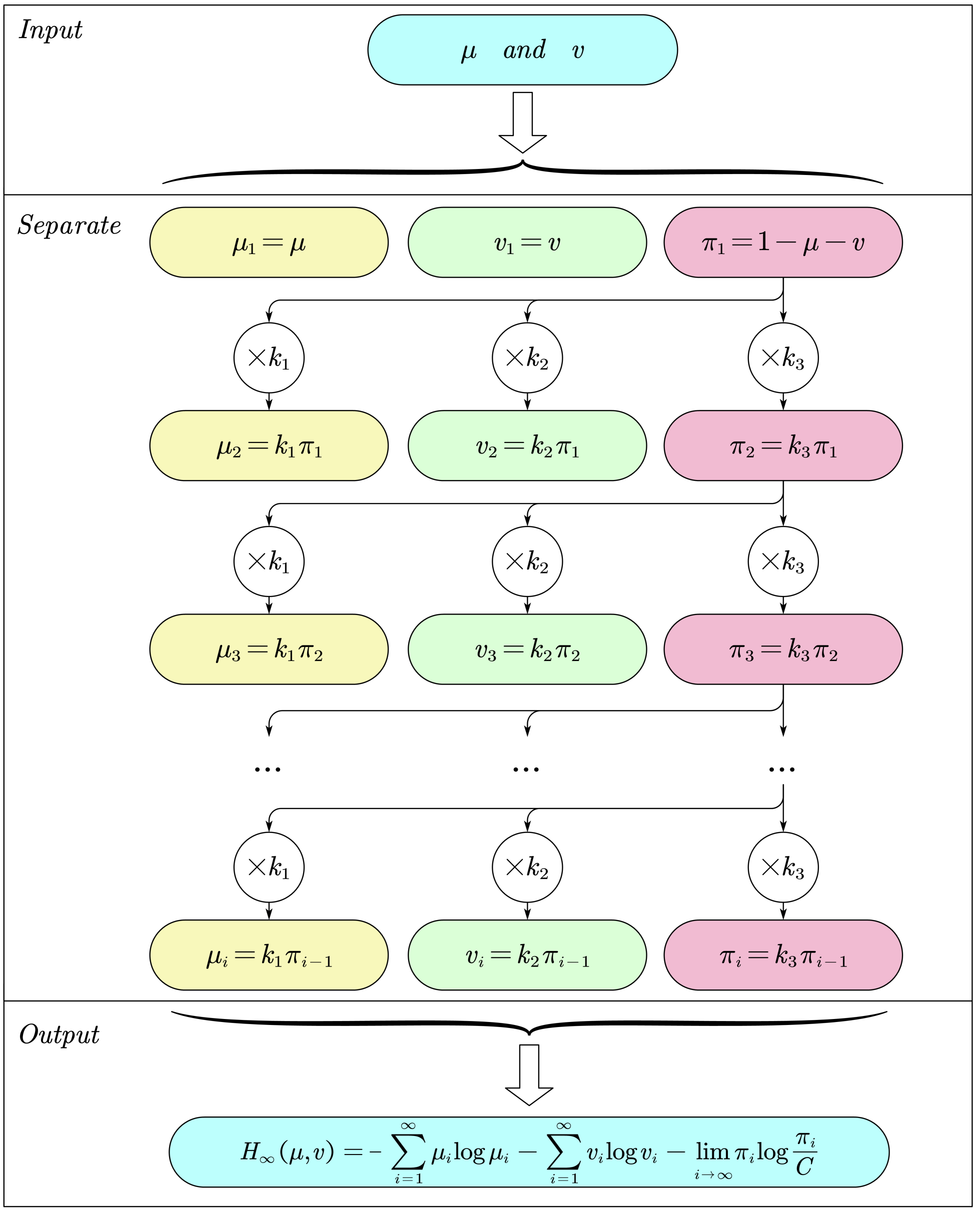

Figure 1: The calculating procedure of $H_\infty(\mu, v)$

Because this is an intuitionistic fuzzy membership function, the cardinality C is 3, and $\pi = 1 - \mu - v = 1 - 0.65 - 0.05 = 0.3$. Then, the first-order information volume of this intuitionistic fuzzy membership function is that:

$$\begin{aligned} H_1(\mu, v) &= -\mu \log \mu - v \log v - \pi \log \frac{\pi}{C} \\ &= -0.65 \log_2(0.65) - 0.05 \log_2(0.05) - 0.3 \log_2(\frac{0.3}{3}) = 1.6166 \end{aligned} \tag{15}$$

Though the parameter C can be determined by the cardinality of fuzzy sets, for the sake of simple and easy comparison between different types of fuzzy sets, we suggest to use $C = 3$ in both cases. Then, under the condition of $C = 3$, the result in **Example 4.1** can be calculated below:

$$\begin{aligned} H_1(\mu, v) &= -\mu \log \mu - (1 - \mu) \log \frac{1 - \mu}{C} \\ &= -0.65 \log_2(0.65) - 0.35 \log_2(\frac{0.35}{3}) = 1.4888 \end{aligned} \tag{16}$$

This result is smaller than the result 1.6166 in **Example 4.2**, which means that the first-order information volume of $\langle x_1, 0.65\rangle$ is smaller than that of $\langle x_2, 0.65, 0.05\rangle$.

Example 4.3:

Consider that a classic fuzzy membership function is $\langle x_3, 0.3\rangle$ where $x_3 \in X$, and the allowable error $\varepsilon = 0.00001$. Because this is a classic fuzzy membership function, the cardinality C is 2, and $\pi = 1-\mu = 1-0.3 = 0.7$. Then, based on **Algorithm 1**, the calculating procedure for the higher-order information volume of this classic fuzzy membership function is shown in Table 1, and the convergence tendency of $H_\infty(\mu, v)$ is illustrated in Figure 2.

Table 1: The calculating procedure of $H_\infty(\mu, v)$ in **Example 4.3**

i	$H_i(\mu, v)$	i	$H_i(\mu, v)$	i	$H_i(\mu, v)$
1	1.58129	12	2.91082	23	2.93711
2	1.98819	13	2.91886	24	2.93727
3	2.27303	14	2.92449	25	2.93738
4	2.47241	15	2.92844	26	2.93745
5	2.61198	16	2.93120	27	2.93751
6	2.70968	17	2.93313	28	2.93755
7	2.77806	18	2.93448	29	2.93757
8	2.82594	19	2.93543	30	2.93759
9	2.85945	20	2.93609	31	2.93761
10	2.88290	21	2.93655	32	2.93761
11	2.89932	22	2.93688		

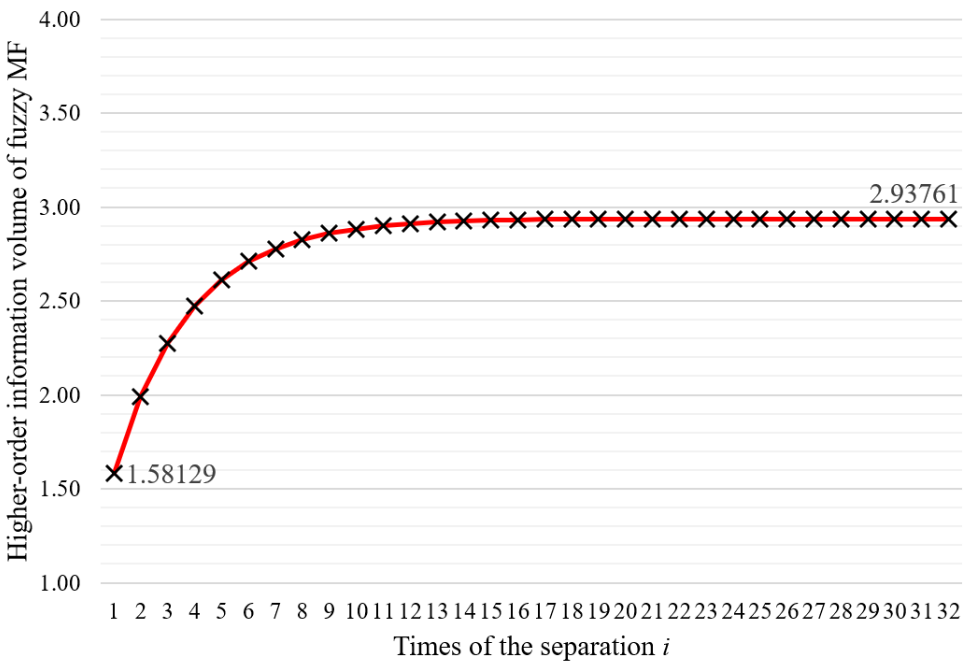

Figure 2: The convergence tendency of $H_\infty(\mu, v)$ in **Example 4.3**

According to Table 1 and Figure 2, by continuously separating the hesitancy degree, the higher-order information volume of this classic fuzzy membership function becomes larger and larger, and finally converges to 2.93761.

Example 4.4:

Let an intuitionistic fuzzy membership function be $\langle x_4, 0.2, 0.15\rangle$ where $x_4 \in X$, and the allowable error $\varepsilon = 0.00001$. Because this is an intuitionistic fuzzy membership function, the cardinality C is 3, and $\pi = 1 - \mu - v = 1 - 0.2 - 0.15 = 0.65$. Then, based on **Algorithm 1**, the calculating procedure for the higher-order information volume of this intuitionistic fuzzy membership function is shown in Table 2, and the convergence tendency of $H_\infty(\mu, v)$ is illustrated in Figure 3.

Table 2: The calculating procedure of $H_\infty(\mu, v)$ in **Example 4.4**

i	$H_i(\mu, v)$	i	$H_i(\mu, v)$	i	$H_i(\mu, v)$
1	2.30912	11	3.63589	21	3.65375
2	2.77983	12	3.64223	22	3.65384
3	3.08579	13	3.64634	23	3.65389
4	3.28466	14	3.64902	24	3.65393
5	3.41393	15	3.65076	25	3.65395
6	3.49795	16	3.65189	26	3.65397
7	3.55257	17	3.65263	27	3.65398
8	3.58807	18	3.65311	28	3.65398
9	3.61114	19	3.65342		
10	3.62614	20	3.65362		

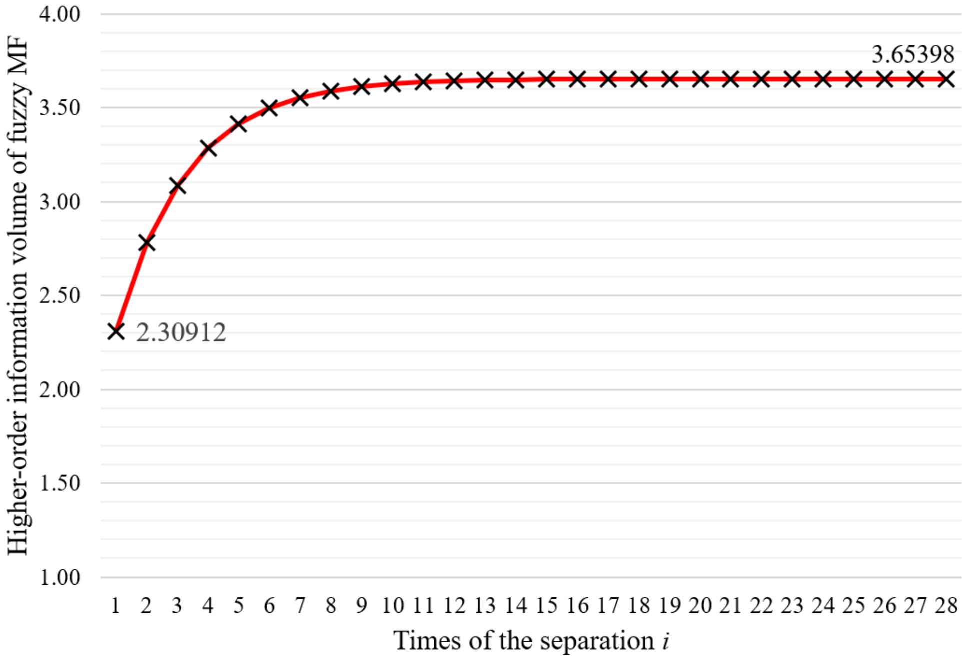

Figure 3: The convergence tendency of $H_\infty(\mu, v)$ in **Example 4.4**

According to Table 2 and Figure 3, by continuously separating the hesitancy degree, the higher-order information volume of this intuitionistic fuzzy membership function becomes larger and larger, and finally converges to 3.65398.

Although the parameter C can be determined by the cardinality of fuzzy sets, for simply and easily comparing the information volume between different types of fuzzy sets, we suggest to use $C = 3$ in both cases. Then, under the condition of $C = 3$, the result of **Example 4.3** can be obtained, whoso calculating procedure is shown in Table 3 and the convergence tendency is illustrated in Figure 4.

According to Table 3 and Figure 4, when $C = 3$, the result in **Example 4.3** finally converges to 2.93761. This result is smaller than the result 3.65398 in **Example 4.4**, which means that the higher-order information volume of $\langle x_3, 0.3\rangle$ is smaller than that of $\langle x_4, 0.2, 0.15\rangle$.

Example 4.5:

Consider that an intuitionistic fuzzy membership function is $\langle x_5, 0.8, 0.2\rangle$ where $x_5 \in X$, and the allowable error $\varepsilon = 0.00001$. Because this is an intuitionistic fuzzy membership function, the cardinality C is 3, and $\pi = 1 - \mu - v = 1 - 0.8 - 0.2 = 0$. Then, based on **Algorithm 1**, the higher-order information volume of this intuitionistic fuzzy membership function can be calculated as follows.

When $i = 1$, the associated higher-order information volume is that:

Table 3: The calculating procedure of $H_\infty(\mu, v)$ in **Example 4.3** when $C = 3$

i	$H_i(\mu, v)$	i	$H_i(\mu, v)$	i	$H_i(\mu, v)$
1	1.99076	12	2.91891	23	2.93727
2	2.27483	13	2.92453	24	2.93738
3	2.47367	14	2.92846	25	2.93745
4	2.61286	15	2.93121	26	2.93751
5	2.71029	16	2.93314	27	2.93755
6	2.77850	17	2.93449	28	2.93757
7	2.82624	18	2.93543	29	2.93759
8	2.85966	19	2.93609	30	2.93761
9	2.88305	20	2.93656	31	2.93761
10	2.89943	21	2.93688		
11	2.91089	22	2.93711		

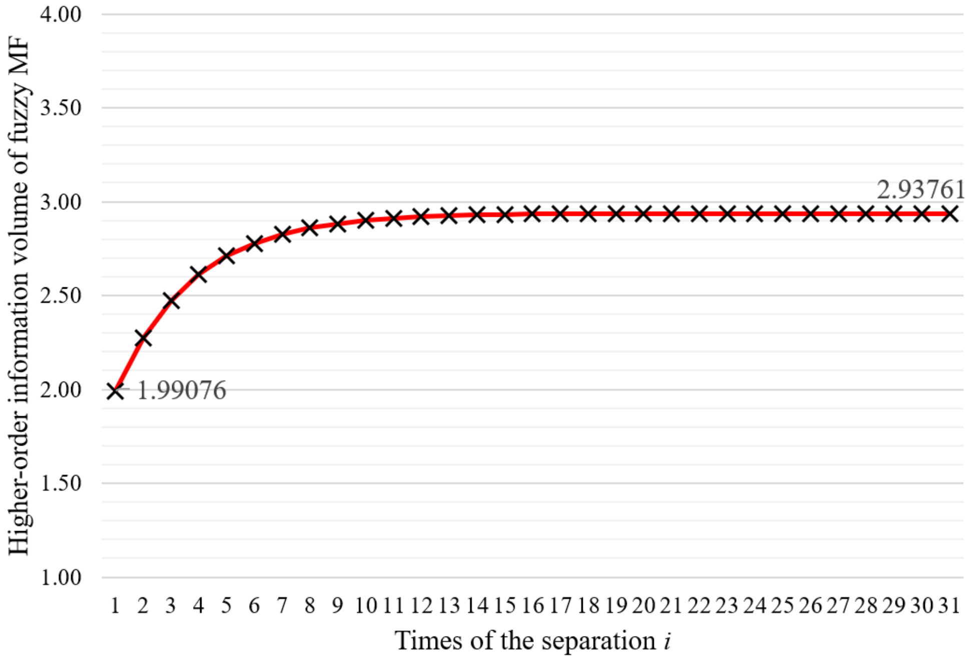

Figure 4: The convergence tendency of $H_\infty(\mu, v)$ in **Example 4.3** when $C = 3$

$$\begin{aligned} H_\infty(\mu, v)|_{i=1} &= -\sum_{j=1}^{1} \mu_j \log \mu_j - \sum_{j=1}^{1} v_j \log v_j - \pi_1 \log \frac{\pi_1}{C} \\ &= -0.8 \log_2(0.8) - 0.2 \log_2(0.2) - 0 = 0.7219 \end{aligned} \tag{17}$$

When $i = 2$, the associated higher-order information volume is that:

$$\begin{aligned} H_\infty(\mu, v)|_{i=2} &= -\sum_{j=1}^{2} \mu_j \log \mu_j - \sum_{j=1}^{2} v_j \log v_j - \pi_2 \log \frac{\pi_2}{C} \\ &= -(0.8 \log_2(0.8) + 0) - (0.2 \log_2(0.2) + 0) - 0 = 0.7219 \end{aligned} \tag{18}$$

Then, calculate the increment of higher-order information volume:

$$\begin{aligned} \Delta_2 &= H_2(\mu, v) - H_1(\mu, v) \\ &= 0.7219 - 0.7219 = 0 < \varepsilon \end{aligned} \tag{19}$$

Hence, the algorithm is convergent, and the higher-order information volume of this intuitionistic fuzzy membership function is 0.7219.

Because the hesitancy degree $\pi = 0$, the three propositions of intuitionistic fuzzy membership function, namely 'support', 'oppose' and 'hesitant', degenerate into two propositions, which are 'support' and 'oppose'. Since these two propositions are exclusive, the intuitionistic fuzzy membership function $\langle x_5, 0.8, 0.2\rangle$ can be seen as the probability distributions $P_1 = 0.8, P_2 = 0.2$ which are also exclusive. Then, the associated Shannon entropy [29] is that

$$\begin{aligned} H_s(P) &= -P_1 \log P_1 - P_2 \log P_2 \\ &= -0.8 \log_2(0.8) - 0.2 \log_2(0.2) = 0.7219 \end{aligned} \tag{20}$$

which is the same as the higher-order information volume $H_\infty(\mu, v)$.

This example shows that, when the hesitancy degree $\pi = 1 - \mu - v = 0$, the value of $H_\infty(\mu, v)$ is identical to Shannon entropy.

5 Conclusion

In this paper, a novel information volume of fuzzy membership function is proposed. The proposed measure not only takes the first-order information volume, but also the higher-order information volume into consideration. The major contributions and some desirable properties of the proposed method are listed below:

(1) The first-order information volume of the fuzzy membership function is presented.

(2) The higher-order information volume of the fuzzy membership function is proposed.

(3) When the hesitancy degree equals to zero, the information volume of fuzzy membership function is identical to Shannon entropy.

(4) When the times of separation equals to one, the higher-order information volume of fuzzy membership function degenerates into the first-order information volume of fuzzy membership function.

Some examples, especial on classic fuzzy sets and intuitiontistic fuzzy sets, are used to show the efficiency of the proposed measure.

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Author contributions

The authors contributed equally to this work.

Conflict of interest

The authors declare no conflict of interest.

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