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Approximate Membership Function Shapes of Solutions to Intuitionistic Fuzzy Transportation Problems

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Abstract

In this paper, proposing a mathematical model with disjunctive constraint system, and providing approximate membership function shapes to the optimal values of the decision variables, we improve the solution approach to transportation problems with trapezoidal fuzzy parameters. We further extend the approach to solving transportation problems with intuitionistic fuzzy parameters; and compare the membership function shapes of the fuzzy solutions obtained by our approach to the fuzzy solutions to full fuzzy transportation problems yielded by approaches found in the literature.
Keywords: full fuzzy transportation problem, extension principle, intuitionistic fuzzy numbers.

1 Introduction

Fuzzy mathematical programming is widely studied in the literature. Recently, Ghanbari et al. [6] surveyed fuzzy linear programming, and provided detailed descriptions of the relevant models and their solutions. Fuzzy linear transportation problems, as part of fuzzy linear programming problems were also analyzed in [6]. Fuzzy linear fractional programming problems including fractional transportation problems in fuzzy environment were recently reviewed by Stanojević et al. [14]. We briefly present below the papers relevant to our study.

Liu and Kao [10] proposed a solution approach to linear transportation problems with trapezoidal fuzzy parameters. Their approach was based on the extension principle, and provided the fuzzy set of the optimal solution values. Later on, Liu [9] proposed a similar solution approach to linear fractional transportation problems with fuzzy parameters. Both approaches solved crisp primal-dual-like-pair models for fixed α-cut intervals of the parameters. In this paper we improve Liu and Kao's approach [10] by optimizing their objective function over a wider feasible set using a disjunctive system of inequalities in the primal model. We also suggest improvements to Liu's method [9] by fixing some coefficients of the objective function to their upper limits instead of considering them as variables in the dual model.

Anukokila and Radhakrishnan [1] solved a full fuzzy fractional transportation problem using fuzzy goal programming. They used trapezoidal fuzzy numbers to describe the fuzzy goals.

Stanojević and Stanojević [16] proposed a Monte Carlo simulation algorithm to disclose the shapes of the fuzzy sets optimal solution and solution values to a full fuzzy linear programming problem. Further on, employing the same Monte Carlo simulation algorithm, Stanojević and Stanojević [15] empirically constructed the envelope for the optimal solution values to a full fuzzy transportation problem, thus identifying a shortcoming of the solution approach provided in [10]. In this paper we overcome the shortcoming proposing a disjunctive mathematical model as foundation to the solution approach.

Full intuitionistic fuzzy transportation problem was addressed in the recent literature. Singh and Yadav [13] proposed a method based on the analogy to the modified distribution method used to solve crisp transportation problems. Ebrahimnejad and Verdegay [5] proposed a solution approach based on the accuracy function used for ordering the trapezoidal intuitionistic fuzzy numbers. Mishra and Kumar [12] provided a procedure to balance the full intuitionistic fuzzy transportation problem, thus extended the applicability of Mahmoodirad et al.'s approach [11] that was able to solve only balanced forms of full intuitionistic fuzzy transportation problems. Approaching the full intuitionistic fuzzy transportation problem in the way we introduce in this paper, we avoid the balancing step and provide an extension-principle-based optimal solution fuzzy set to the problem.

A brief presentation of basic notation and terminology related to fuzzy numbers and the extension principle can be found in Section 2. Section 3 describes the main solution approaches to fuzzy transportation problems related to our study. Section 4 presents the improvements we propose to the existing methods, the shortcomings we overcome by our solution approach, and the relevance of our new theoretical results to solving full intuitionistic fuzzy transportation problems. Section 5 reports our numerical results. Our conclusions and directions for further research are included in Section 6.

2 Preliminaries

Fuzzy sets were introduced by Zadeh [17], and their applicability to mathematical programming was emphasized by Zimmermann [19] and [20]. The general aspects of fuzzy sets theory that contributed to the development of soft computing and artificial intelligence were recently described in [4].

A fuzzy subset $\widetilde{A}$ of a universe X is a collection of pairs $\left(x, \mu_{\widetilde{A}}(x)\right)$ such that $x \in X$, and $\mu_{\widetilde{A}}(x) \in [0,1]$. The value $\mu_{\widetilde{A}}(x)$ is called the membership degree of the element x in $\widetilde{A}$, and $\mu_{\widetilde{A}} : X \to [0,1]$ is called the membership function of the fuzzy subset $\widetilde{A}$ in the universe X. When there is no confusion about the universe to whom a fuzzy subset belongs, that fuzzy subset can be called fuzzy set.

To aggregate the fuzzy sets, Bellman and Zadeh [3] formulated the following extension principle: the fuzzy set $\widetilde{B}$ of the universe Y that is the result of evaluating the function f at the fuzzy sets $\widetilde{A}_1, \widetilde{A}_2, \dots, \widetilde{A}_r$ over their universes $X_1, X_2, \dots, X_r$ is defined through its membership function as

$$\mu_{\widetilde{B}}(y) = \begin{cases} \sup\limits_{(x_1,\dots,x_r)\in f^{-1}(y)} \left(\min\left\{\mu_{\widetilde{A}_1}(x_1), \dots, \mu_{\widetilde{A}_r}(x_r)\right\}\right), & f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}$$

The intuitionistic fuzzy sets were introduced in [2] as a generalization to the fuzzy set. An intuitionistic fuzzy subset $\widetilde{A}^I$ of a universe X is a collection of triples $\left(x, \mu_{\widetilde{A}^I}(x), \nu_{\widetilde{A}^I}(x)\right)$ such that $x \in X$, $\mu_{\widetilde{A}^I}(x), \nu_{\widetilde{A}^I}(x) \in [0,1]$, and $0 \le \mu_{\widetilde{A}^I}(x) + \nu_{\widetilde{A}^I}(x) \le 1$. The value $\mu_{\widetilde{A}^I}(x)$ is called the membership degree of the element x in $\widetilde{A}^I$, the value $\nu_{\widetilde{A}^I}(x)$ is called the non-membership degree of the element x in $\widetilde{A}^I$, $\mu_{\widetilde{A}^I} : X \to [0,1]$ is the membership function, and $\nu_{\widetilde{A}^I} : X \to [0,1]$ is the non-membership function of $\widetilde{A}^I$ in X. For each $x \in X$ the value $h(x) = 1 - \mu_{\widetilde{A}^I}(x) - \nu_{\widetilde{A}^I}(x)$ is called the degree of hesitancy of x in $\widetilde{A}^I$.

Fuzzy numbers, as particular fuzzy sets were introduced in [18]. They generalize the real numbers in a specific direction toward modeling uncertainty and incertitude. A fuzzy set $\widetilde{A}$ of the universe of real numbers R is called fuzzy number if and only if: (i) it is fuzzy normal (i.e. at least one $x_0 \in R$ exists such that $\mu_{\widetilde{A}}(x_0) = 1$) and fuzzy convex (i.e. $\forall x_1, x_2 \in R$, $\forall \lambda \in [0,1]$, $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\right\}$); (ii) $\mu_{\widetilde{A}}$ is upper semi-continuous; and (iii) its support $\left\{x \in R | \mu_{\widetilde{A}}(x) > 0\right\}$ is bounded. Similarly, an intuitionistic fuzzy set of R is an intuitionistic fuzzy number if and only if its membership function fulfills all conditions in the definition of a fuzzy number; $\nu_{\widetilde{A}}(x_0) = 0$; and the non-membership function is fuzzy concave (i.e. $\nu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq \max\left\{\nu_{\widetilde{A}}(x_1), \nu_{\widetilde{A}}(x_2)\right\}$), lower semi-continuous, and its support $\left\{x \in R | \nu_{\widetilde{A}}(x) < 1\right\}$ is bounded.

In what follows we describe the trapezoidal fuzzy numbers that are used in our numerical examples. The graph of the non-zero piece of the membership function of a trapezoidal fuzzy number forms a trapezoid with the abscissa. One way to denote a trapezoidal fuzzy number $\widetilde{A}$ is by using an quadruple of real numbers (a_1, a_2, a_3, a_4), where $[a_2, a_3]$ is the interval of the elements with the membership degree equal to 1, and the interval (a_1, a_4) represents the support of the fuzzy number. The algebraic definition of the membership function is $\mu_{\widetilde{A}} : R \to [0,1]$,

$$\mu_{\widetilde{A}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1), & a_1 \leq x < a_2, \\ 1, & a_2 \leq x < a_3, \\ (a_4 - x)/(a_4 - a_3), & a_3 \leq x < a_4, \\ 0, & \text{otherwise.} \end{cases}$$

For any trapezoidal fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4)$ its α-cut is the interval

$$\left[\widetilde{A}\right]_\alpha = [\alpha(a_2 - a_1) + a_1, \alpha(a_3 - a_4) + a_4].$$

For each $\alpha \in [0,1]$ we have $\min \mu_{\widetilde{A}}^{-1}(\alpha) = \alpha(a_2 - a_1) + a_1$, and $\max \mu_{\widetilde{A}}^{-1}(\alpha) = \alpha(a_3 - a_4) + a_4$.

For any two trapezoidal fuzzy numbers $\widetilde{A} = (a_1, a_2, a_3, a_4)$ and $\widetilde{B} = (b_1, b_2, b_3, b_4)$, the basic arithmetic operations between them (addition, subtraction and multiplication by a scalar) are derived using the extension principle, and are defined as follows:

$$\begin{aligned} &\widetilde{A} + \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4), \\ &\widetilde{A} - \widetilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1), \\ &k\widetilde{A} = (ka_1, ka_2, ka_3, ka_4),\ k \text{ real scalar},\ k \geq 0, \\ &k\widetilde{A} = (ka_4, ka_3, ka_2, ka_1),\ k < 0. \end{aligned} \tag{1}$$

A trapezoidal fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4)$ is reduced to a triangular fuzzy number if $a_2 = a_3$.

A trapezoidal intuitionistic fuzzy number $\widetilde{A}^I$ has the membership and non-membership functions defined as below:

$$\mu_{\widetilde{A}^I}(x) = \begin{cases} (x - a_1)/(a_2 - a_1), & a_1 \leq x < a_2, \\ 1, & a_2 \leq x < a_3, \\ (a_4 - x)/(a_4 - a_3), & a_3 \leq x < a_4, \\ 0, & \text{otherwise,} \end{cases}$$

$$\nu_{\widetilde{A}^I}(x) = \begin{cases} (a_2' - x)/(a_2' - a_1'), & a_1' \leq x < a_2', \\ 0, & a_2' \leq x < a_3', \\ (x - a_3')/(a_4' - a_3'), & a_3' \leq x < a_4', \\ 1, & \text{otherwise,} \end{cases}$$

respectively, where $a_1' \leq a_1 \leq a_2' \leq a_2 \leq a_3 \leq a_3' \leq a_4 \leq a_4'$. Such trapezoidal intuitionistic fuzzy number is denoted by

$$\widetilde{A}^I = (a_1, a_2, a_3, a_4; a_1', a_2', a_3', a_4').$$

The basic arithmetic operations for trapezoidal intuitionistic fuzzy numbers are direct extensions of (1). Given two trapezoidal intuitionistic fuzzy numbers $\widetilde{A}^I$ and $\widetilde{B}^I$ and a real scalar k,

$$\begin{aligned}
&\widetilde{A}^I + \widetilde{B}^I = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4; a'_1+b'_1, a'_2+b'_2, a'_3+b'_3, a'_4+b'_4),\\
&\widetilde{A}^I - \widetilde{B}^I = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1; a'_1-b'_4, a'_2-b'_3, a'_3-b'_2, a'_4-b'_1),\\
&k\widetilde{A}^I = (ka_1, ka_2, ka_3, ka_4; ka'_1, ka'_2, ka'_3, ka'_4),\ k \geq 0,\\
&k\widetilde{A}^I = (ka_4, ka_3, ka_2, ka_1; ka'_4, ka'_3, ka'_2, ka'_1),\ k < 0.
\end{aligned}$$

A trapezoidal intuitionistic fuzzy number $\widetilde{A}^I = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4)$ is reduced to a triangular intuitionistic fuzzy number if $a_2 = a'_2 = a_3 = a'_3$.

3 Existing approaches to fuzzy transportation problems

In a classic linear transportation problem the total cost of transferring commodities from m sources with certain supply capacities $(a_i)_{i=\overline{1,m}}$ to n destinations with certain demands $(b_j)_{j=\overline{1,n}}$ has to be minimized. When the total supply is equal to the total demand (i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$), the problem is balanced and its general model is given in (2).

$$\begin{aligned}
\min\ & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\\
\text{s.t.}\ & \sum_{j=1}^{n} x_{ij} = a_i, \quad i=\overline{1,m},\\
& \sum_{i=1}^{m} x_{ij} = b_j, \quad j=\overline{1,n},\\
& x_{ij} \geq 0, \quad i=\overline{1,m}, j=\overline{1,n},
\end{aligned} \tag{2}$$

where $(x_{ij})_{i=\overline{1,m}}^{j=\overline{1,n}}$ is the decision variable representing the amount of commodities transported from the source i to the destination j, and $(c_{ij})_{i=\overline{1,m}}^{j=\overline{1,n}}$ is the unit transportation cost from the source i to the destination j. The condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ assures a non-empty feasible set to Problem (2).

Generally, if the transportation problem is unbalanced, depending on the inequality between the total supply and total demand, only one of Models (2-a) and (2-b) has a non-empty feasible set and should be considered in the optimization process.

$$\begin{aligned}
\min\ & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\\
\text{s.t.}\ & \sum_{j=1}^{n} x_{ij} \leq a_i, \quad i=\overline{1,m},\\
& \sum_{i=1}^{m} x_{ij} = b_j, \quad j=\overline{1,n},\\
& x_{ij} \geq 0, \quad i=\overline{1,m},\\
& \qquad\qquad j=\overline{1,n}.
\end{aligned} \tag{2-a}$$

$$\begin{aligned}
\min\ & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\\
\text{s.t.}\ & \sum_{j=1}^{n} x_{ij} = a_i, \quad i=\overline{1,m},\\
& \sum_{i=1}^{m} x_{ij} \leq b_j, \quad j=\overline{1,n},\\
& x_{ij} \geq 0, \quad i=\overline{1,m},\\
& \qquad\qquad j=\overline{1,n}.
\end{aligned} \tag{2-b}$$

In fuzzy transportation problems the parameters $(a_i)_{i=1,\ldots,m}$, $(b_j)_{j=1,\ldots,n}$ and $(c_{ij})_{i=\overline{1,m}}^{j=\overline{1,n}}$ become fuzzy numbers denoted by $(\widetilde{a}_i)_{i=\overline{1,m}}$, $\left(\widetilde{b}_j\right)_{j=\overline{1,n}}$ and $(\widetilde{c}_{ij})_{i=\overline{1,m}}^{j=\overline{1,n}}$ respectively. When fuzzy decision variables are considered, they are denoted by $(\widetilde{x}_{ij})_{i=\overline{1,m}}^{j=\overline{1,n}}$, and used in fuzzy models analog to Models (2), (2-a) and (2-b).

Among crisp linear TPs the balanced problems are efficiently solved using MODI (MOdified DIstribution) method; while the other variants are solved either directly using a simplex method or via an equivalent balancing transformation. The theoretical foundation of our novel solution approach will show that balancing a transportation problem with fuzzy parameters is an unnecessary step.

Liu and Kao [10] proposed a solution approach to Problem (3) and imposed $\sum_{i=1}^{m}\widetilde{a}_i \geq \sum_{j=1}^{n}\widetilde{b}_j$ to assure the feasibility of the model.

$$\begin{array}{lll} \min & \sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{c}_{ij}x_{ij} & \\ \text{s.t.} & \sum_{j=1}^{n}x_{ij} \leq \widetilde{a}_i, & i=\overline{1,m}, \\ & \sum_{i=1}^{m}x_{ij} \geq \widetilde{b}_j, & j=\overline{1,n}, \\ & x_{ij} \geq 0, & i=\overline{1,m}, j=\overline{1,n}, \end{array} \tag{3}$$

Their extension-principle-based solution concept is defined by the membership function of the optimal solution value fuzzy set

$$\mu_{\widetilde{f}}(z) = \begin{cases} \max\limits_{(a,b,c)|z=\min\limits_{x\in X_{a,b}} c^T x} \left(\mu_{\left(\widetilde{a},\widetilde{b},\widetilde{c}\right)}(a,b,c)\right), & \exists a,b,c|z=\min\limits_{x\in X_{a,b}} c^T x, \\ 0, & \text{otherwise}, \end{cases} \tag{4}$$

where

$$\mu_{\left(\widetilde{a},\widetilde{b},\widetilde{c}\right)}(a,b,c) = \min\left\{\mu\left(c_{ij}\right)|i=\overline{1,m}, j=\overline{1,n}\right\}\bigcup\left\{\mu_{\widetilde{a}_i}\left(a_i\right)|i=\overline{1,m}\right\}\bigcup\left\{\mu_{\widetilde{b}_j}\left(b_j\right)|j=\overline{1,n}\right\}$$

and

$$X_{a,b} = \left\{\left(x_{ij}\right)_{i=\overline{1,m}}^{j=\overline{1,n}} \in M_{m\times n}(R)\,|\sum_{j=1}^{n}x_{ij} \leq a_i, \sum_{i=1}^{m}x_{ij} \geq b_j, x_{ij} \geq 0, i=\overline{1,m}, j=\overline{1,n}\right\}.$$

They derived an approximate membership function shape of the optimal solution value fuzzy set by solving the following pair of optimization problems for fixed values of the parameter α.

$$\begin{array}{lll} \min & \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij}\right)_{\alpha}^{L}x_{ij} & \\ \text{s.t.} & \sum_{j=1}^{n}x_{ij} \leq a_i, & i=\overline{1,m}, \\ & \sum_{i=1}^{m}x_{ij} \geq b_j, & j=\overline{1,n}, \\ & \left(a_i\right)_{\alpha}^{L} \leq a_i \leq \left(a_i\right)_{\alpha}^{U}, & i=\overline{1,m}, \\ & \left(b_j\right)_{\alpha}^{L} \leq b_j \leq \left(b_j\right)_{\alpha}^{U}, & j=\overline{1,n}, \\ & \sum_{i=1}^{m}a_i \geq \sum_{j=1}^{n}b_j, & \\ & x_{ij} \geq 0, \quad i=\overline{1,m}, & j=\overline{1,n}, \end{array} \tag{4-a}$$

$$\begin{array}{lll} \max & -\sum_{i=1}^{m}a_iu_i + \sum_{j=1}^{n}b_jv_j, & \\ \text{s.t.} & -u_i + v_j \leq \left(c_{ij}\right)_{\alpha}^{U}, & i=\overline{1,m}, \\ & & j=\overline{1,n}, \\ & \left(a_i\right)_{\alpha}^{L} \leq a_i \leq \left(a_i\right)_{\alpha}^{U}, & i=\overline{1,m}, \\ & \left(b_j\right)_{\alpha}^{L} \leq b_j \leq \left(b_j\right)_{\alpha}^{U}, & j=\overline{1,n}, \\ & \sum_{i=1}^{m}a_i \geq \sum_{j=1}^{n}b_j, & \\ & u_i \geq 0, & i=\overline{1,m}, \\ & v_j \geq 0, & j=\overline{1,n}. \end{array} \tag{4-b}$$

It is clear that Models (4-a) and (4-b) are not a primal-dual pair, but the former was obtained from the crisp form of Model (3) with lower and upper bounds for $\left(a_i\right)_{i=\overline{1,m}}$ and $\left(b_j\right)_{j=\overline{1,n}}$ that became variables, and $\left(c_{ij}\right)_{i=\overline{1,m}}^{j=\overline{1,n}}$ equal to $\left(\left(\widetilde{c}_{ij}\right)_{\alpha}^{L}\right)_{i=\overline{1,m}}^{j=\overline{1,n}}$; and the later was constructed as a dual to the crisp form of Model (3) with lower and upper bounds for $\left(a_i\right)_{i=\overline{1,m}}$ and $\left(b_j\right)_{j=\overline{1,n}}$ that became variables, and $\left(c_{ij}\right)_{i=\overline{1,m}}^{j=\overline{1,n}}$ equal to $\left(\left(\widetilde{c}_{ij}\right)_{\alpha}^{U}\right)_{i=\overline{1,m}}^{j=\overline{1,n}}$.

Stanojević and Stanojević [15] provided empirical solutions to the special class of full fuzzy transportation problems. Quadratic optimization models were derived from the original transportation model considering the cost coefficients, demands and supplies as decision variables. The numerical results obtained in [15] by employing a Monte-Carlo-simulation-based algorithm (introduced in [16]) in solving relevant instances represent a good validation test for our results that will be reported in Section 5.

Ebrahimnejad and Verdegay [5] proposed a solution approach to balanced full trapezoidal intuitionistic fuzzy transportation problems

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{c}_{ij}^{I}\widetilde{x}_{ij}^{I} \\ \text{s.t.} \quad & \sum_{j=1}^{n}\widetilde{x}_{ij}^{I} = \widetilde{a}_{i}^{I}, \quad i=\overline{1,m}, \\ & \sum_{i=1}^{m}\widetilde{x}_{ij}^{I} = \widetilde{b}_{j}^{I}, \quad j=\overline{1,n}, \\ & \widetilde{x}_{ij}^{I} \geq 0, \qquad i=\overline{1,m}, j=\overline{1,n}. \end{aligned} \tag{5}$$

They interpreted the trapezoidal inutitionistic fuzzy valued objective function as a multiple objective function due to its eight components; and converted it into a single objective function

$$\frac{1}{8}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{4}\left(c_{ij,k}x_{ij,k}+c'_{ij,k}x'_{ij,k}\right)$$

using the weighted sum method, and assigning an equal weight $\frac{1}{8}$ to all components. The single objective function is then optimized over the feasible set described in (5). Each equality of (5) is component by component transformed to eight crisp constraints

$$\begin{aligned} & \sum_{j=1}^{n}x_{ij,k} = a_{i,k}, \quad \sum_{j=1}^{n}x'_{ij,k} = a'_{i,k}, \quad i=\overline{1,m}, k=\overline{1,4}, \\ & \sum_{i=1}^{m}x_{ij,k} = b_{j,k}, \quad \sum_{i=1}^{m}x'_{ij,k} = b'_{j,k}, \quad j=\overline{1,n}, k=\overline{1,4}. \end{aligned}$$

The non-negativity of the decision variables were transformed to non-negativity of all components and the needed inequalities between the components of decision variables were included.

Mahmoodirad et al. [11] addressed the same problem (5) but used triangular intuitionistic fuzzy numbers as parameters, and solved it in the same way. Mishra and Kumar [12] proposed a method to transforming an unbalanced fully intuitionistic fuzzy transportation problem into a balanced one. Their method is an analog extension of the method introduced by Kumar and Kaur [7] to transportation problems with fuzzy parameters.

4 Our novel solution approach

Taking into consideration one neglected fact of the extension principle, we first improve Liu and Kao's solution approach [10] by introducing a disjunctive system of constraints in Model (4-a), and propagating it through duality to Model (4-b).

Further on, we provide approximate membership function shapes for the optimal solution fuzzy sets of the decision variables, thus showing that the original transportation problem with fuzzy parameters implicitly assumes that the decision variables are also fuzzy sets.

Finally, incorporating the novel ideas we extend the solution approach to the class of intuitionistic fuzzy transportation problem.

4.1 Mathematical model with disjunctive constraint system

Any crisp transportation problem can be solved choosing one of Models (2), (2-a) and (2-b): if the problem is balanced the first model is chosen, if sources together supply more commodities than the total demand then the second model is used, otherwise the third one must be employed.

A similar flexibility should be assured when proposing fuzzy models to real systems. Unfortunately, assuming $\sum_{i=1}^{m}\widetilde{a}_i \geq \sum_{j=1}^{n}\widetilde{b}_j$ when modeling a transportation problem with imprecise information about supplies and demands brings a weakness to the solution approach since the inequality between fuzzy numbers can be interpreted in a wide variety of ways. Moreover, using the crisp models (4-a) and (4-b) in solving Model (3), as proposed in [10], accentuates the weakness since the crisp models contain the crisp constraint $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ that restricts the feasible sets more than demanded by the extension principle.

As a more flexible alternative to Models (3-a) and (3-b), we propose two new models (9) and (10) whose solutions improve the solutions derived in [10]. The new models are based on the crisp model

$$\begin{array}{lll} \min & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} & \\ \text{s.t.} & (x_{i,j})_{i=\overline{1,m}}^{j=\overline{1,n}} \in X_{a,b}^{disj.}, & \\ & x_{ij} \geq 0, & i=\overline{1,m}, j=\overline{1,n}, \end{array} \tag{6}$$

where

$$X_{a,b}^{disj.} = \left\{ (x_{i,j})_{i=\overline{1,m}}^{j=\overline{1,n}} \in M_{m\times n}(R) \,|\, \sum_{j=1}^{n} x_{ij} \leq a_i, \sum_{i=1}^{m} x_{ij} \geq b_j, i=\overline{1,m}, j=\overline{1,n} \right\} \cup \left\{ (x_{i,j})_{i=\overline{1,m}}^{j=\overline{1,n}} \in M_{m\times n}(R) \,|\, \sum_{j=1}^{n} x_{ij} \geq a_i, \sum_{i=1}^{m} x_{ij} \leq b_j, i=\overline{1,m}, j=\overline{1,n} \right\}. \tag{7}$$

We make use of a binary variable y, and a great scalar constant M, to describe $X_{a,b}^{disj.}$ by the following conjunctive system of constraints

$$\begin{array}{ll} a_i - (1-y)M \leq \sum_{j=1}^{n} x_{ij} \leq a_i + yM, & i=\overline{1,m}, \\ b_j - yM \leq \sum_{i=1}^{m} x_{ij} \leq b_j + (1-y)M, & j=\overline{1,n}, \\ y \in \{0,1\}. & \end{array} \tag{8}$$

Further on, releasing $(a_i)_{i=\overline{1,m}}$ and $(b_j)_{j=\overline{1,n}}$ to vary between their lower and upper bounds, and fixing the coefficients of the objective function to their lower and upper bounds respectively, we derive the final new models (9) and (10).

Model (9) is used for minimizing the minimal cost with respect to variables x, y, a, and b; while Model (10) is used for maximizing the minimal cost with respect to variables w, u, v, p, q, a, and b.

$$\begin{array}{lll} \min & \sum_{i=1}^{m}\sum_{j=1}^{n} (c_{ij})_{\alpha}^{L} x_{ij} & \\ \text{s.t.} & a_i - (1-y)M \leq \sum_{j=1}^{n} x_{ij} \leq a_i + yM, & i=\overline{1,m}, \\ & b_j - yM \leq \sum_{i=1}^{m} x_{ij} \leq b_j + (1-y)M, & j=\overline{1,n}, \\ & (a_i)_{\alpha}^{L} \leq a_i \leq (a_i)_{\alpha}^{U}, & i=\overline{1,m}, \\ & (b_j)_{\alpha}^{L} \leq b_j \leq (b_j)_{\alpha}^{U}, & j=\overline{1,n}, \\ & x_{ij} \geq 0, & i=\overline{1,m}, j=\overline{1,n}, \\ & y \in \{0,1\}, & \end{array} \tag{9}$$

Solving Models (9) and (10) for fixed values of $\alpha \in [0,1]$ we obtain an approximate shape of the membership function $\mu_{\widetilde{f}}$.

$$
\begin{array}{lll}
\max & w & \\
\text{s.t.} & -u_i + v_j \le (c_{ij})^U_\alpha , & i=\overline{1,m}, j=\overline{1,n}, \\
& p_i - q_j \le (c_{ij})^U_\alpha , & i=\overline{1,m}, j=\overline{1,n}, \\
& -\sum_{i=1}^{m} a_i u_i + \sum_{j=1}^{n} b_j v_j \ge w, & i=\overline{1,m}, j=\overline{1,n}, \\
& \sum_{i=1}^{m} a_i p_i - \sum_{j=1}^{n} b_j q_j \ge w, & i=\overline{1,m}, j=\overline{1,n}, \\
& (a_i)^L_\alpha \le a_i \le (a_i)^U_\alpha , & i=\overline{1,m}, \\
& (b_j)^L_\alpha \le b_j \le (b_j)^U_\alpha , & j=\overline{1,n}, \\
& (1-y)\,M \le \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j \le yM, & \\
& u_i, q_j \ge 0, & i=\overline{1,m}, j=\overline{1,n}, \\
& y \in \{0,1\}. &
\end{array}
\tag{10}
$$

In the same way, a disjunctive system of constraints can be incorporate in the methodology proposed in [9] to improve solutions to linear fractional transportation problems.

4.2 Membership function shapes of the optimal solution values of the decision variables

Our next goal is to define the membership functions of the optimal solutions fuzzy sets, and propose optimization models to approximate the membership function shapes of the optimal solution values of the decision variables.

Preliminary ideas about the membership function of the vector of the decision variables in a fuzzy linear programming problem were given in [16]. Using a Monte Carlo simulation algorithm the envelopes of the membership functions of the variables were drafted. We next define the exact membership function for each decision variable separately as

$$
\mu_{\widetilde{x}_i}(t) = \begin{cases} \max\limits_{(a,b,c) \mid t = \arg_i \min\limits_{y \in X^{disj.}_{a,b}} c^T y} \left(\mu_{\left(\widetilde{a},\widetilde{b},\widetilde{c}\right)}(a,b,c) \right), & \exists a,b,c \mid t = \arg_i \min\limits_{y \in X_{a,b}} c^T y \\ 0, & \text{otherwise} \end{cases}, i=\overline{1,m}, \tag{11}
$$

where $\arg_i \min\limits_{y \in X_{a,b}} c^T y$ represents the optimal value of the i-th decision variable obtained when minimizing $c^T y$ over the feasible set $X^{disj.}_{a,b}$, and $\mu_{\left(\widetilde{a},\widetilde{b},\widetilde{c}\right)}(a,b,c)$ has the same meaning as in the definition of the membership function of optimal solution values (4).

Defining the membership functions of the decision variables with the help of (11) we round out a solution concept to both classes of problems: with fuzzy parameters and full fuzzy. We proceed to formalize mathematical models able to derive the shapes of decision variables membership functions.

For any fixed value of $\alpha \in [0,1]$ and fixed indexes $k=\overline{1,m}$ and $l=\overline{1,n}$ we construct the pair of the optimization models

$$
\begin{array}{ll}
\min & x_{kl} \\
\text{s.t.} & (a,b,c,x,y) \in P,
\end{array}
\tag{12}
$$

and

$$
\begin{array}{ll}
\max & x_{kl} \\
\text{s.t.} & (a,b,c,x,y) \in P,
\end{array}
\tag{13}
$$

where P is the constraints set described by System (14). To describe System (14) we use the minimal and maximal values ($z^{\min}_\alpha$ and $z^{\max}_\alpha$) of the objective function yielded by solving Models (9) and (10) respectively.

$$
\begin{array}{ll}
a_i - (1-y)\,M \le \sum\limits_{j=1}^{n} x_{ij} \le a_i + yM, & i=\overline{1,m},\\
b_j - yM \le \sum\limits_{i=1}^{m} x_{ij} \le b_j + (1-y)\,M, & j=\overline{1,n},\\
(b_j)_\alpha^L \le b_j \le (b_j)_\alpha^U, & j=\overline{1,n},\\
(c_{ij})_\alpha^L \le c_{ij} \le (c_{ij})_\alpha^U, & i=\overline{1,m}, j=\overline{1,n}\\
z_\alpha^{\min} \le \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} c_{ij}x_{ij} \le z_\alpha^{\max}, & \\
x_{ij} \ge 0, & i=\overline{1,m}, j=\overline{1,n},\\
y \in \{0,1\}. &
\end{array}
\tag{14}
$$

4.3 Extension to intuitionistic fuzzy transportation problems

Both pairs of models (9)-(10) and (12)-(13) can be extended to provide the approximate membership function shapes of the optimal solution values of the objective function and decision variables to a full intuitionistic fuzzy transportation problem. The extension is made by adding two more pairs of optimization models that derive the non-membership function shapes of the optimal solution values.

Since in all Models (9), (10), (12) and (13) the lower and upper bounds of $(a_i)_{i=\overline{1,m}}$, $(b_j)_{j=\overline{1,n}}$ and $(c_{ij})_{i=\overline{1,m}}^{j=\overline{1,n}}$ are defined with the help of the membership functions of problem's parameters $\left(\mu_{\widetilde{a}_i}\right)_{i=\overline{1,m}}$, $\left(\mu_{\widetilde{b}_j}\right)_{j=\overline{1,n}}$ and $\left(\mu_{\widetilde{c}_{ij}}\right)_{i=\overline{1,m}}^{j=\overline{1,n}}$ and their corresponding α-cuts, in the case of intuitionistic fuzzy parameters, the non-membership functions $\left(\nu_{\widetilde{a}_i}\right)_{i=\overline{1,m}}$, $\left(\nu_{\widetilde{b}_j}\right)_{j=\overline{1,n}}$ and $\left(\nu_{\widetilde{c}_{ij}}\right)_{i=\overline{1,m}}^{j=\overline{1,n}}$ together with their α-cuts dictate the lower and upper bounds of the variables $(a_i)_{i=\overline{1,m}}$, $(b_j)_{j=\overline{1,n}}$ and $(c_{ij})_{i=\overline{1,m}}^{j=\overline{1,n}}$ in the additional models.

The solution approach to full intuitionistic fuzzy transportation problem defined through its intuitionistic fuzzy parameters $\left(\widetilde{a}_i^I\right)_{i=\overline{1,m}}$, $\left(\widetilde{b}_j^I\right)_{j=\overline{1,n}}$ and $\left(\widetilde{c}_{ij}^I\right)_{i=\overline{1,m}}^{j=\overline{1,n}}$ representing the supplies of all sources, demands of all destinations, and unit transportation costs from each source to each destination respectively is described by Algorithm 1.

Using notation from Algorithm 1 the approximate membership function of optimal objective values is described by

$$\left\{\left(z_{\mu,\alpha_s}^{\min}, \alpha_s\right) | s=\overline{1,p}\right\} \bigcup \left\{\left(z_{\mu,\alpha_s}^{\max}, \alpha_s\right) | s=\overline{1,p}\right\};$$

the approximate non-membership function of optimal objective values is described by

$$\left\{\left(z_{\nu,\alpha_s}^{\min}, 1-\alpha_s\right) | s=\overline{1,p}\right\} \bigcup \left\{\left(z_{\nu,\alpha_s}^{\max}, 1-\alpha_s\right) | s=\overline{1,p}\right\};$$

the approximate membership functions of the optimal values of the decision variables are described by

$$\left\{\left(x_{(k,l),\mu,\alpha_s}^{\min}, \alpha_s\right) | s=\overline{1,p}\right\} \bigcup \left\{\left(x_{(k,l),\mu,\alpha_s}^{\max}, \alpha_s\right) | s=\overline{1,p}\right\}, k=\overline{1,m}, l=\overline{1,n};$$

and the approximate non-membership functions of the optimal values of the decision variables are described by

$$\left\{\left(x_{(k,l),\nu,\alpha_s}^{\min}, 1-\alpha_s\right) | s=\overline{1,p}\right\} \bigcup \left\{\left(x_{(k,l),\nu,\alpha_s}^{\max}, 1-\alpha_s\right) | s=\overline{1,p}\right\}, k=\overline{1,m}, l=\overline{1,n}.$$

5 Numerical results

To report our numerical results we follow the same structure as in presenting our theoretical results. We first solve an example of a transportation problem with trapezoidal fuzzy parameters from [10], showing our improvements. Then we solve two examples of intuitionistic fuzzy transportation problems one form [5], and second from [5], [8] and [11] in order to illustrate the advantages of the new approach.

Algorithm 1 Solution approach to full fuzzy intuitionistic transportation problem

Require: the membership and non-membership functions of the intuitionistic fuzzy parameters, and p α-levels $\alpha_1 < \alpha_2 < \ldots < \alpha_p$ chosen for the desired approximation.

for $s = \overline{1,p}$ **do**
  Set $(a_i)^L_{\alpha_s} = \min \mu^{-1}_{\widetilde{a}_i}(\alpha_s)$, $(b_j)^L_{\alpha_s} = \min \mu^{-1}_{\widetilde{b}_j}(\alpha_s)$, $(c_{ij})^L_{\alpha_s} = \min \mu^{-1}_{\widetilde{c}_{ij}}(\alpha_s)$,
  $(a_i)^U_{\alpha_s} = \max \mu^{-1}_{\widetilde{a}_i}(\alpha_s)$, $(b_j)^U_{\alpha_s} = \max \mu^{-1}_{\widetilde{b}_j}(\alpha_s)$, $(c_{ij})^U_{\alpha_s} = \max \mu^{-1}_{\widetilde{c}_{ij}}(\alpha_s)$, $i = \overline{1,m}$, $j = \overline{1,n}$.
  Solve Models (9) and (10) and derive the optimal values $z^{\min}_{\mu,\alpha_s}$ and $z^{\max}_{\mu,\alpha_s}$ respectively.
  for $k = \overline{1,m}$ and $l = \overline{1,n}$ **do**
    Set $z^{\min}_{\alpha} = z^{\min}_{\mu,\alpha_s}$ and $z^{\max}_{\alpha} = z^{\max}_{\mu,\alpha_s}$.
    Solve Models (12) and (13) and derive the optimal values $x^{\min}_{(k,l),\mu,\alpha_s}$, $x^{\max}_{(k,l),\mu,\alpha_s}$ respectively.
  end for
  Set $(a_i)^L_{\alpha_s} = \min\left(1 - \nu^{-1}_{\widetilde{a}_i}(\alpha_s)\right)$, $(b_j)^L_{\alpha_s} = \min\left(1 - \nu^{-1}_{\widetilde{b}_j}(\alpha_s)\right)$, $(c_{ij})^L_{\alpha_s} = \min\left(1 - \nu^{-1}_{\widetilde{c}_{ij}}(\alpha_s)\right)$,
  $(a_i)^U_{\alpha_s} = \max\left(1 - \nu^{-1}_{\widetilde{a}_i}(\alpha_s)\right)$, $(b_j)^U_{\alpha_s} = \max\left(1 - \nu^{-1}_{\widetilde{b}_j}(\alpha_s)\right)$, $(c_{ij})^U_{\alpha_s} = \max\left(1 - \nu^{-1}_{\widetilde{c}_{ij}}(\alpha_s)\right)$,
  $i = \overline{1,m}$, $j = \overline{1,n}$.
  Solve Model (9) and (10) and derive the optimal values $z^{\min}_{\nu,\alpha_s}$ and $z^{\max}_{\nu,\alpha_s}$ respectively.
  for $k = \overline{1,m}$ and $l = \overline{1,n}$ **do**
    Set $z^{\min}_{\alpha} = z^{\min}_{\nu,\alpha_s}$ and $z^{\max}_{\alpha} = z^{\max}_{\nu,\alpha_s}$.
    Solve Models (12) and (13) and derive the optimal values $x^{\min}_{(k,l),\nu,\alpha_s}$, $x^{\max}_{(k,l),\nu,\alpha_s}$ respectively.
  end for
end for

Ensure: the approximate membership and non-membership functions of optimal objective values and values of the decision variables.

Table 1: Trapezoidal fuzzy parameters of the transportation problem used for Example 1

$\widetilde{c}$	$j=1$	$j=2$	$j=3$	$\widetilde{a}$
$i=1$	$(10,10,10,10)$	$(50,50,50,50)$	$(80,80,80,80)$	$(70,90,90,100)$
$i=2$	$(60,70,80,90)$	$(60,60,60,60)$	$(20,20,20,20)$	$(40,60,70,80)$
$\widetilde{b}$	$(30,40,50,70)$	$(20,30,40,50)$	$(40,50,50,80)$	

Example 1 was first addressed in [10]. The problem was formulated using Model (3) and the fuzzy parameters reported in Table 1. Models (4-a) and (4-b) were used in [10] to derive the shape of the membership function of optimal values. Tackling the essence of the problem, and allowing more flexibility in modeling, we make use of Models (9) and (10) to derive a similar shape.

The numerical results are reported in Table 2 and graphed in Figure 1. Analyzing the results obtained for the optimal values one can notice a difference only on the first component of the trapezoidal fuzzy numbers reported in Table 2: 1500 for our approach, and 2100 for the Liu and Kao's approach [10]. It means that our approach provided the smallest value 1500 of the transportation cost that can be reached for a certain combination of the parameters, and was not revealed by the approach from the literature.

Figure 1 shows graphically the differences between the membership functions of the optimal values of both objective function and decision variables. In the case of the objective values the compared shapes differ only on their left side for $\alpha \in [0, 0.4]$. No complete information about the optimal values of the decision variables were provided in [10]. The values reported in [10] for $\alpha = 0$ and $\alpha = 1$ can be seen within the approximated enveloped area provided by our approach. Therefore the optimal values

Table 2: Numerical results for Example 1

$\widetilde{z}_{\min}$	μ
Liu and Kao [10]	$(2100, 2900, 3500, 5800)$
Our approach	$(1500, 2900, 3500, 5800)$

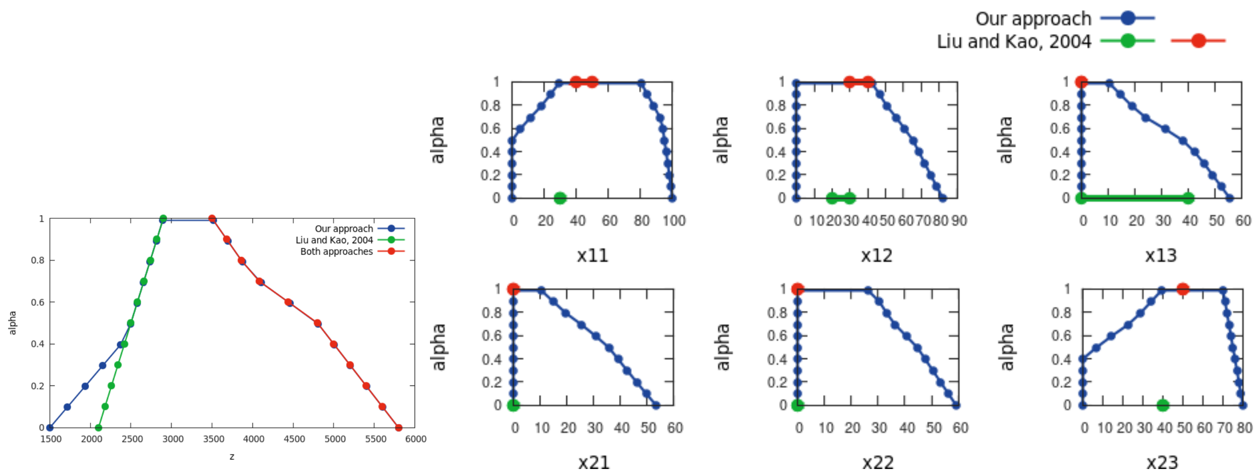

Figure 1: Approximated membership functions of the optimal solution values of the transportation problem with fuzzy parameters given in Table 1

Table 3: Intuitionistic fuzzy parameters of the transportation problem used for Example 2

$\widetilde{c}^I$	$i=1$	$i=2$	$\widetilde{b}^I$
$j=1$	$(10,20,30,40;5,15,35,45)$	$(60,70,80,90;55,65,85,95)$	$(30,50,70,90;20,40,80,100)$
$j=2$	$(50,60,70,80;45,55,75,85)$	$(70,80,90,100;65,75,95,105)$	$(20,30,40,50;15,25,45,55)$
$j=3$	$(80,90,100,110;75,85,105,115)$	$(20,30,40,50;15,25,45,55)$	$(50,60,70,80;45,55,75,85)$
$\widetilde{a}^I$	$(60,80,100,120;50,70,110,130)$	$(40,60,80,100;30,50,90,110)$	

of the decision variables cannot be derived by solving Models (4-a) and (4-b). Therefore, Models (9) and (10) bring essential improvements to the existing approach [10].

The similarity to Liu and Kao's results for the objective values shows that we solved the same problem like them, a transportation problem with fuzzy parameters. On the other side, the envelopes of the values of the decision variables show that in fact, the solving method assumes that the variables are also fuzzy quantities, thus the methodology can be applied to solve full fuzzy problems. This fact will be illustrated by the next example.

Example 2 is a full fuzzy problem recalled form [5] that uses Model (5) with the trapezoidal intuitionistic fuzzy parameters provided in Table 3.

The numerical results reported in [5] together with our results are shown in Table 4 and graphed in Figure 2. Our method finds the membership functions of the optimal values of the decision variables. Their shapes are non-linear but similar to the shapes of trapezoidal intuitionistic fuzzy numbers. Our approach identified smaller values with non-zero membership values that are optimal values for certain combinations of the given parameters, thus providing more relevant information about the minimal transportation cost of the modeled problem.

Analyzing Figure 2 we can notice an essential difference between the fuzzy sets provided as final fuzzy-valued decision variables by the existing approach and our approach. Our results fulfill the extension principle concept applied to the optimization problem as a whole, and not only to the arithmetic on the coefficients and variables as in [5].

Example 3 is a real life problem discussed in the recent papers [5], [8] and [11]. The triangular intuitionistic fuzzy parameters are given in Table 5, and a comparison of the results is shown in Table 6 and Figure 3.

Comparing the numerical results obtained for the objective function Kumar and Hussain's method

Table 4: Numerical results for Example 2

$\widetilde{z}^I_{\min}$	μ	ν
Ebrahimnejad and Verdegay [5]	$(3300,5800,9100,13200)$	$(2350,4450,11050,15550)$
Our approach	$(1400,3760,13500,8750)$	$(700,2280,9550,16000)$

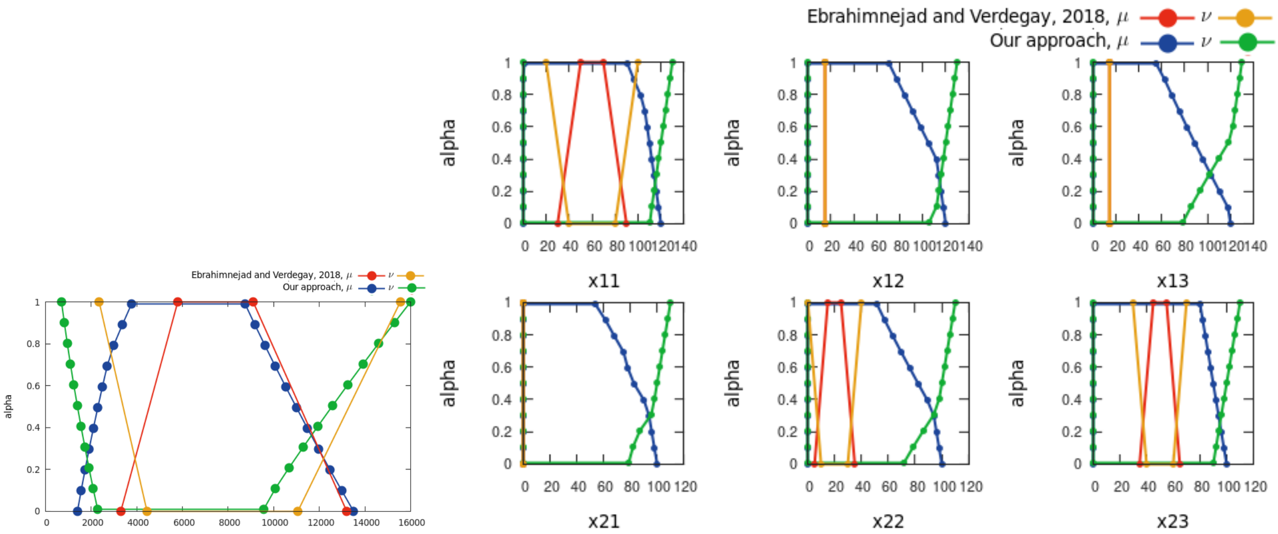

Figure 2: Approximated membership functions of the optimal solution values of the transportation problem with intuitionistic fuzzy parameters given in Table 3

Table 5: Triangular intuitionistic fuzzy parameters of the transportation problem used for Example 3

$\widetilde{c}^I$	$j=1$	$j=2$	$j=3$	$\widetilde{a}^I$
$i=1$	$(1,4,9;0,4,12)$	$(3,13,14;2,13,15)$	$(4,6,16;1,6,33)$	$(6,7,10;2,7,11)$
$i=2$	$(4,5,7;1,5,9)$	$(5,10,15;0,10,39)$	$(7,16,24;0,16,41)$	$(6,15,23;1,15,29)$
$i=3$	$(1,3,6;0,3,10)$	$(5,13,21;5,13,35)$	$(8,18,27;6,18,48)$	$(2,10,16;0,10,21)$
$\widetilde{b}^I$	$(3,8,16;0,8,19)$	$(1,6,7;0,6,14)$	$(10,18,26;3,18,28)$	

[8] is the best considering the minimal value with maximal amplitude. However, some of the values of the decision variables provided by Kumar and Hussain's method are negative, fact that cannot by acceptable in any model of transportation problem. Our approach provides non-negative fuzzy intuitionistic numbers to all decision variables, and smaller minimal value with maximal amplitude than the methods in [5] and [11].

In the same way as in the previous example, the results obtained by our approach are better then those found in the literature since our methodology follows the extension principle applied to the optimization problem as a whole, while the others were focused just on following the arithmetic rules on the coefficients and variables.

6 Final remarks

The main contribution of this paper is three-fold: (i) we involved a mathematical model with disjunctive system of constraints in modeling transportation problems in fuzzy environment, thus improving an existing solution approach to transportation problems with fuzzy parameters; (ii) we proposed mathematical models able to provide approximate membership functions shapes to the optimal values of the decision variables, thus extending the solution approach to full fuzzy transportation problems; (iii) we expanded our approach to solving full intuitionistic fuzzy transportation problems consistently respecting the extension principle.

We included in the paper several numerical examples from the literature in order to illustrate our

Table 6: Numerical results for Example 3

$\widetilde{z}^I_{\min}$	μ	ν
Kumar and Hussain [8]	$(137,292,502)$	$(12,292,961)$
Ebrahimnejad and Verdegay [5]	$(63,313,773)$	$(2,313,1726)$
Mahmoodirad et al. [11]	$(63,310,757)$	$(2,310,1806)$
Our approach	$(32,305.4,765)$	$(0,305.4,1697)$

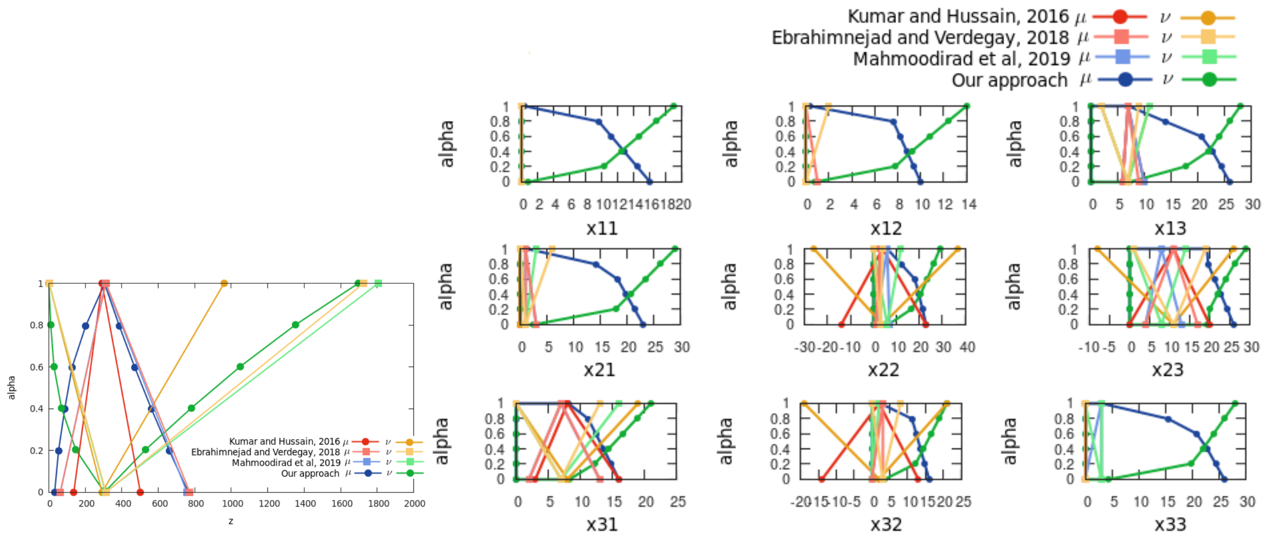

Figure 3: Intuitionistic fuzzy numbers of the minimal objective values obtained by approximating the results of our approach compared to the results reported in [5], [8] and [11]

theoretical results, and provide a visualization of the improvements brought in by our novel solution approach.

Our future research will focus on extending the ideas on which our solution approach is based to a wider class of optimization problems under uncertainty.

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Author contributions

The authors contributed equally to this work.

Conflict of interest

The authors declare no conflict of interest.

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