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Game Theoretic Distributed Power Control Algorithms for Uplink Wireless Data in Flat Fading Channels

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Abstract: In this paper we present a game-theoretic power control algorithms for wireless data in CDMA cellular systems under two realistic channels: **(a1)** Fast flat fading channel and **(a2)** Slow flat fading channel. The fading coefficients under both **(a1)** and **(a2)** are studied for three appropriate small scale channel models that are used in the CDMA cellular systems: Rayleigh channel, Rician channel and Nakagami channel. This work is inspired by the results presented by [1] under non-fading channels. In other words, we study the impact of the realistic channel models on the findings in [1] through the followings: we evaluate the average utility function, the average number of bits received correctly at the receiver per one Joule expended, for each channel model. Then, using the average utility function we study the existence, uniqueness of Nash equilibrium (NE) if it exists, and the social desirability of NE in the Pareto sense. Results show that in a non-cooperative game (NPG) the best policy for all users in the cell is to target a fixed signal-to-interference and noise ratio (SINR) similar to what was shown in [1] for non-fading channel. The difference however is that the target SINR in fading channels is much higher than that in a non-fading channel. Also, for spreading gain less than or equal to 100, both NPG and non-cooperative power control game with pricing (NPGP) perform poorly, where all the terminals except the nearest one were not able to attain their corresponding minimum SINR even if sending at the maximum powers in their strategy spaces.
Keywords: Code-division-multiple-access (CDMA), utility function, power control, game theory, non-cooperative game (NPG), wireless data.

1 Introduction

The mathematical theory of games was introduced by Von Neumann and Morgenstern in 1944 [18], and by the late 1970's became an important tool whenever a player's decision depends on what the other players did or will do. A core idea of game theory is how strategic interactions between rational agents (players) generate outcomes according to the players' utilities [10], [19]. Game theory thus forms a suitable framework to obtain more insight into the interactions of self-interested rational agents with potentially conflicting interests. A player in a non-cooperative game responds to the actions of other players by choosing a strategy (from his strategy space) in an attempt to maximize a utility function that quantifies its level of satisfaction.

In a cellular system each user desires to have a high SINR at the base station (BS) coupled with the lowest possible transmit power. It is important in such systems to have high SINR, as this will reflect a low error rate, a more reliable system, and high channel capacity, so that more users can be served per cell [11]. It is also important to decrease the transmit power to lengthen battery life and to alleviate the near-far problem [17]. In power control algorithms exploiting game theory however, the tendency of each user to maximize his/her utility function in response to other users' actions, leads to a sequence of power vectors that converges to a point where no user has incentive to individually increase his power. This operating point is called a Nash equilibrium. Due to the lack of cooperation between the users this point may not be efficient, in the sense that it may not be the most desirable social point [3]. In Pareto sense, the most desirable social point is actually the power vector that Pareto dominates all other power vectors.

It should be noted that extensive work has been done on non-game theoretic power control algorithms for wireless data and multimedia CDMA cellular networks, e.g. [9, 10, 15, 16]. The power control problem for wireless data CDMA systems was first addressed in the game theoretic framework in [1]- [8]. In this paper the work in [1], which only dealt with deterministic (non-fading) channels, is extended to a realistic wireless CDMA channels by considering the following cases of fading models: A Rayleigh fast/slow flat fading channel model, a Rician[1] fast/slow flat fading channel model and a Nakagamifast/slow flat fading channel model. Where we use the same utility function and evaluate its average in the fading channels mentioned above, then we use these averaged utility functions to study the existence, uniqueness and social desirability of NE operating point under each channel model.

The remaining of this paper is organized as follows: In section 2 we present the utility function and the system model studied in this paper. In section 3 we evaluate the performance of the system for the channel models mentioned above. Non-cooperative power control game (NPG) and Non-cooperative power control game with pricing (NPGP) are discussed briefly in sections 4 and 5, respectively. We then point out the constraints on the new modified strategy spaces to guarantee the existence and uniqueness of Nash equilibrium points for NPG and NPGP under the assumed channel models in section 6. Simulation results are outlined in section 7, and our conclusions are given in section 8.

2 Utility Function and System Model

In general utility functions are used to quantify the satisfaction level a player achieves by choosing an action from its strategy profile, given the other players' actions. A utility function thus maps the player's preferences onto the real line. A formal definition of a utility function may be found in [10].

In a CDMA cellular system, a number of users sharing the spectrum and air interface. Henceforth, each user's transmission adds to the interference of all users at the BS. Each user desires to achieve a high quality of reception at the BS, i.e., a high SINR, while using the minimum possible amount of power in order to extend the battery's life. The conflicting goal of each user to have a high SINR at the BS makes the game theoretic framework suitable for studying and solving the problem.

In this paper we consider the same system model and the same utility function of [1]: Uplink single-cell direct sequence code division multiple access (DS-CDMA) system with N users, where each user transmits frames (packets) of M bits with L information bits. The rate of transmission is R bits/sec for all users. Let P_c represent the *average* probability of correct reception of a frame at the BS, and let p represent the average transmit power level. The utility function for a CDMA system is given by:

$$u = \frac{L\,R}{M\,p}\, f(\gamma) \tag{1}$$

[1]For space limitation, we omit the findings related to Rician channel model. The reader can find these findings in the supplementary document [22].

where $f(\gamma)$ is an efficiency function that approximates P_c, u thus represents the number of information bits successfully received at the BS per joule of consumed energy. With the assumption of no error correction, and *random* packet correct reception rate $\tilde{P}_c$, i.e., $P_c = E\{\tilde{P}_c\}$, is then given by $\prod_{l=1}^{M}(1-\tilde{P}_e(l))$, where $\tilde{P}_e(l)$ is the *random* bit error rate (BER) of the lth bit at a given SINR γ_i. P_e is the average BER, that is $P_e = E\{\tilde{P}_e\}$ (c. f. (9) and (23)). We are assuming that all users in a cell are using non-coherent binary frequency shift Keying (BFSK) modulation, and are transmitting at the same rate R. It should be noted that the efficiency function $f(\gamma)$ has the same expression of P_c in terms of $\tilde{P}_e$, except that $\tilde{P}_e$ is replaced by $2\tilde{P}_e$ [1].

3 Evaluation of The Performance

In this section we find closed-form formulas of the average utility functions under the six assumed channel models. We then use these formulas to study the existence and uniqueness of Nash equilibrium point in section 6. The SINR γ_i at the receiver for the ith user is assumed to be large ($\gamma_i \gg 1$) to combat the fading effect, it is given by [13]:

$$\gamma_i = \frac{W}{R}\frac{p_i\, h_i\, \alpha_i^2}{\sum_{k\neq i}^{N} p_k\, h_k\, \alpha_k^2 + \sigma^2} \tag{2}$$

Where α_i is the path fading coefficient between ith user and the BS and is constant for each bit in a fast flat fading channels **(a1)**, while it is constant for each packet in a slow flat fading channels **(a2)**. W is the spread spectrum bandwidth, p_k is the transmitted power of the kth user, h_k is the path gain between the BS and the kth user, and σ^2 is the variance of the AWGN (additive-white-gaussian-noise) representing the background thermal noise in the receiver. For simplicity we express the interference from all other users as x_i, i.e.

$$x_i = \sum_{k\neq i}^{N} p_k\, h_k\, \alpha_k^2 \tag{3}$$

therefore (2) can be written as:

$$\gamma_i = \gamma_i(\alpha_i, x_i) = \frac{W}{R}\frac{p_i\, h_i}{x_i + \sigma^2}\ \ \alpha_i^2 = \gamma_i'\alpha_i^2 \tag{4}$$

For a given α_i and x_i, the BER, $\tilde{P}(e|\alpha_i, x_i)$, of the ith user using BFSK is given by [13]:

$$\tilde{P}(e|\alpha_i, x_i) = \frac{1}{2}\, e^{-\frac{\gamma_i(\alpha_i, x_i)}{2}} \tag{5}$$

The average BER and average utility functions for this modulation scheme is evaluated next under the previously mentioned channel models.

3.1 Rayleigh Flat Fading Channel

In this case α_i is modelled as a Rayleigh random variable with a probability distribution given by:

$$f^{\alpha_i}(\omega) = \frac{\omega}{\sigma_r^2}\ e^{-(1/2\sigma_r^2)\omega^2}, \quad i = 1, 2, \cdots, N \tag{6}$$

Where $\sigma_r^2 = E\{\alpha_i^2\}/2$ is the measure of the spread of the distribution. In all following calculations, and as a consequence of the multiplicative effect of small and large scale models, it is assumed that $\sigma_r^2 = 1/2$. Using (4) and (6) the distribution of γ_i for a given x_i becomes: $f^{\gamma_i|x_i}(\omega) = \frac{1}{\gamma_i'}\ e^{-(\frac{1}{\gamma_i'})\omega}$

Rayleigh Fast Flat Fading Channel

For the lth bit in the frame, we can rewrite the SINR (4) and the interference (3) for the ith user as follows: $\gamma_i(l) = \frac{W}{R}\frac{p_i\, h_i\, \alpha_i^2(l)}{x_i(l)+\sigma^2}$, and $x_i(l) = \sum_{k\neq i}^{N} p_k\, h_k\, \alpha_k^2(l)$. In this paper, we assume that the fading channel is fast enough to have the fading coefficients $\{\alpha_i(l)\}_{l=1}^{M}$ statistically independent, that is iid (identical independent distributed) random variables. Henceforth, the *averaged* correct reception P_c is given as $(1-P_e)^M$, while the efficiency function $f(\gamma_i)$ is given as $(1-2P_e)^M$, where P_e is *averaged* BER for each bit in the frame, that is $P_e = E\{\tilde{P}_e\}$. We will calculate the averaged P_e next. We can find the conditioned error probability $\tilde{P}(e|x_i)$ by taking the average of (5) with respect to $f^{\gamma_i|x_i}(\omega)$:

$$\begin{aligned}\tilde{P}(e|x_i) &= E\left\{\tilde{P}(e|\gamma_i, x_i)\right\} = \int_0^\infty \tilde{P}(e|\omega, x_i)\, f^{\gamma_i|x_i}(\omega) d\omega \\ &= \frac{1}{2\gamma_i'} \int_0^\infty e^{-(\frac{2+\gamma_i'}{2\gamma_i'})\omega} d\omega = \frac{1}{2+\gamma_i'}\end{aligned} \tag{7}$$

Notice that we dropped the bit index l because the average BER does not depend on l. For large SINR, (7) behaves like:

$$\tilde{P}(e|x_i) \approx \frac{1}{\gamma_i'} = \frac{x_i + \sigma^2}{\frac{W}{R}\, p_i\ h_i} \tag{8}$$

Now, we can find the *averaged* BER P_e by taking the expectation of (8):

$$P_e = E\left\{\tilde{P}(e|x_i)\right\} = \frac{E\{x_i\} + \sigma^2}{\frac{W}{R}\, p_i\, h_i} = \frac{1}{\overline{\gamma_i}} \tag{9}$$

where $\overline{\gamma_i}$ is the ratio of the mean of the received power from user i to the mean of the interference at the receiver and given by:

$$\overline{\gamma_i} = \frac{W}{R} \frac{p_i\, h_i}{\sum_{k\neq i}^{N} p_k\, h_k + \sigma^2} \tag{10}$$

Therefore, the average utility function of the ith user is given by:

$$\boxed{u_i = \frac{L\ R}{M\ p_i}\ (1 - \frac{2}{\overline{\gamma_i}})^M} \tag{11}$$

Rayleigh Slow Flat Fading Channel

In a slow flat fading channel model, α_i is assumed to be constant for each packet/frame. The averaged efficiency function $f(\gamma_i)$ is therefore given as the expectation of $(1-2\tilde{P}(e|\alpha_i, x_i))^M$ with respect to the random variables α_i and x_i. One can evaluate $u_i(\mathrm{p}|x_i)$ as follows:

$$\begin{aligned}u_i(\mathrm{p}|x_i) &= \int_0^\infty u_i(\mathrm{p}|\omega, x_i)\, f^{\gamma_i|x_i}(\omega)\, d\omega = \int_0^\infty \frac{L\ R}{M\ p_i}\ (1-e^{-\omega/2})^M\ \frac{1}{\gamma_i'}\ e^{-(\frac{1}{\gamma_i'})\omega}\ d\omega \\ &= \frac{L\ R}{M\ p_i\, \gamma_i'} \sum_{k=0}^{M} (-1)^k \binom{M}{k} \times \int_0^\infty e^{-(\frac{k}{2}+\frac{1}{\gamma_i'})\,\omega}\ d\omega = \frac{L\ R}{M\ p_i} \sum_{k=0}^{M} \binom{M}{k} \frac{2\,(-1)^k}{k\,\gamma_i' + 2}\end{aligned} \tag{12}$$

For large SINR ($\gamma_i' \gg 1$), (12) can be approximated by:

$$u(\mathrm{p}|x_i) \approx \frac{L\ R}{M\ p_i} \left(1 + \frac{1}{\gamma_i'}\ \sum_{k=1}^{M} \binom{M}{k}\ \frac{2\,(-1)^k}{k}\right) \tag{13}$$

Averaging (13) with resect to x_i we obtain the average utility function for high SINR:

$$\begin{aligned} u_i &= E\{u_i(\mathrm{p}|x_i)\} \approx \frac{L\ R}{M\ p_i}\left(1+\frac{E\{x_i\}+\sigma^2}{\frac{W}{R}\,p_i\ h_i}\ \sum_{k=1}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k}\right) \\ &= \frac{L\ R}{M\ p_i}\left(1+\frac{1}{\gamma_i}\ \sum_{k=1}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k}\right) \\ &\boxed{u_i\ \approx\ \frac{L\ R}{M\ p_i}\left(1-\frac{\beta}{\gamma_i}\right)} \end{aligned} \quad (14)$$

where $\beta = -\sum_{k=1}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k} > 0$.

3.2 Nakagami Flat Fading Channel

Here, the fading coefficient α_i is modelled as a Nakagami random variable with a probability distribution given by [13]: $f^{\alpha_i}(\omega) = \frac{2m^m}{\Gamma(m)\Omega^m}\,\omega^{2m-1}\,e^{(-\frac{m}{\Omega})\omega^2}$; $\ i = 1, 2, \cdots, N$ where $\Omega = E\{\alpha_i^2\}$ controls the spread of the distribution. The fading figure $m = \frac{\Omega^2}{E\{(\alpha_i^2-\Omega)^2\}}$ is a measure of the severity of the fading channel, where $m = \infty$ corresponds to a nonfading channel. In the following it is assumed that $\Omega = 1$. Then the distribution of γ_i for fixed x_i is given as: $f^{\gamma_i|x_i}(\omega) = \frac{1}{\Gamma(m)}\left(\frac{m}{\gamma_i'}\right)^m\ \omega^{m-1}\ e^{-(\frac{m}{\gamma_i'})\,\omega}$

Nakagami Fast Flat Fading Channel

We find the conditioned error probability $\tilde{P}(e|x_i)$ as:

$$\begin{aligned} \tilde{P}(e|x_i) &= \int_0^\infty \tilde{P}(e|\omega, x_i)\,f^{\gamma_i|x_i}(\omega)d\omega = \frac{1}{2\Gamma(m)}\ \left(\frac{m}{\gamma_i'}\right)^m \int_0^\infty \omega^{m-1}\ e^{-(\frac{\gamma_i'+2m}{2\gamma_i'})\,\omega}\,d\omega \\ &= \frac{1}{2}\left(\frac{2m}{2m+\gamma_i'}\right)^m \end{aligned} \quad (15)$$

For fixed m and $\gamma_i' \gg 1$, (15) can be rewritten as:

$$\tilde{P}(e|x_i) \approx \frac{1}{2}\left(\frac{2m}{\gamma_i'}\right)^m \quad (16)$$

To find the *average* P_e, we need to find the mean of $(x_i + \sigma^2)^m$. Here, x_i is a summation of independent random variables each distributed according to a Gamma density function. This makes the evaluation of $(x_i + \sigma^2)^m$ tedious and it may be easier to find an approximate density function of x_i. To do this, let us recall Esseen's inequality which estimates the deviation of the exact distribution of a sum of independent variables from the normal distribution [21].

Theorem 1. *let $Y_1, \cdots, Y_N$ be independent random variables with $EY_j = 0$, $E|Y_j|^3 < \infty$ $(j = 1, \cdots, N)$. Let $\sigma_j^2 = EY_j^2$, $B_N = \sum_{j=1}^N \sigma_j^2$, $L_N = B_N^{-3/2}\sum_{j=1}^N E|Y_j|^3$. Let $\psi_N(z)$ be the c.f. (cumulative distribution) of the random variable $B_N^{-1/2}\sum_{j=1}^N Y_j$. Then*

$$|\psi_N(z) - e^{-z^2/2}| \leq 16\,L_N\ |z|^3\ e^{-z^2/3} \quad (17)$$

Define $\tilde{Y}_k = p_k h_k \alpha_k^2$ and $Y_k = \tilde{Y}_k - p_k h_k$. By simple calculations we can find that $\tilde{Y}_k, (k = 1, \cdots, N)$ are Gamma distributed random variables, such that $f^{\tilde{Y}_k}(\omega) = \frac{(m/p_k h_k)^m}{\Gamma(m)} \; \omega^{m-1} \, e^{-(m/p_k h_k)\omega}$ and $E\tilde{Y}_k = p_k h_k$, which means that $Y_k, (k = 1, \cdots, N)$ are zero mean random variables. Note that $\sigma_k^2 = EY_k^2 = (p_k h_k)^2/m, \forall k = 1, \cdots, N$, and therefore, $B_N = \frac{1}{m}\sum_{k=1}^{N}(p_k h_k)^2$. It is fairly simple to find out that the third moment $E|Y_k|^3 = EY_k^3 = \frac{2(p_k h_k)^3}{m^2}$ $(Y_k \geq 0)$, and $L_N = \frac{2\sum_{k=1}^{N}(p_k h_k)^3}{\sqrt{m}\,(\sum_{k=1}^{N}(p_k h_k)^2)^{3/2}}$. For large N, L_N has a very small value, i.e., $L_N << 1$. Examining (17) for small values of z, L_N takes care of the bound and making it very small, while for large values of z, the exponential will decrease the bound and make it approach zero. In conclusion, we can approximate x_i as a Gaussian random variable with mean ζ_{x_i} and variance $\overline{\sigma}_{x_i}^2$ given by:

$$\zeta_{x_i} = E\{x_i\} = E\left\{\sum_{k\neq i}^{N} \alpha_k^2 p_k\, h_k\right\} = \sum_{k\neq i}^{N} p_k\, h_k\, E\{\alpha_k^2\} = \sum_{k\neq i}^{N} p_k\, h_k \tag{18}$$

and

$$\overline{\sigma}_{x_i}^2 = E\{x_i^2\} - \zeta_{x_i}^2 = E\left\{\sum_{l\neq i}^{N}\sum_{k\neq i}^{N} p_l h_l p_k\, h_k\, \alpha_l^2\, \alpha_k^2\right\} - \zeta_{x_i}^2 = \frac{1}{m}\sum_{k\neq i}^{N}(p_k\, h_k)^2 \tag{19}$$

where (19) was obtained using the fact that α_k and α_l are statistically independent for all $k \neq l$. So, we can write f^{x_i}, the PDF of x_i, as follows: $f^{x_i}(w) = \frac{\delta_i}{\sqrt{2\pi}\overline{\sigma}_{x_i}}\, e^{-\frac{(w-\zeta_{x_i})^2}{2\overline{\sigma}_{x_i}^2}}$, where $w \geq 0$ and $\delta_i = 2/(1 + Erf[\zeta_{x_i}/\sqrt{2}\,\sigma_{x_i}])$ is a scaling factor such that $f^{x_i}(w)$ is a valid PDF. $Erf[.]$ is the error function. By examining equations (18) and (19), one can see that $\zeta_{x_i} \gg \sigma_{x_i}$, therefore $\delta_i \approx 1$. Averaging (16) over $f^{x_i}(\omega)$ we obtain the average error probability P_e for high SINR below:

$$\begin{aligned} P_e &\approx \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_0^\infty \left(x_i + \sigma^2\right)^m \times \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_i}}\; e^{-\frac{(x_i-\zeta_{x_i})^2}{2\overline{\sigma}_{x_i}^2}}\; dx_i \\ &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_{\sigma^2}^\infty y^m \times \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_i}}\; e^{-\frac{(y-(\zeta_{x_i}+\sigma^2))^2}{2\overline{\sigma}_{x_i}^2}}\; dy \\ &\approx \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_0^\infty y^m \times \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_i}}\; e^{-\frac{(y-(\zeta_{x_i}+\sigma^2))^2}{2\overline{\sigma}_{x_i}^2}}\; dy \end{aligned} \tag{20}$$

where we used the change of variable $y = x_i + \sigma^2$ and the last approximation in (20) used the fact that $\sigma^2 \ll 1$. By examining (20) one can see that it is the mth moment of a random variable y normally distributed with mean $\zeta_y = \zeta_{x_i} + \sigma^2$ and variance $\sigma_y^2 = \overline{\sigma}_{x_i}^2$. Therefore, the average P_e is given by:

$$\begin{aligned} P_e &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m E\{y^m\} = \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m E\left\{((y-\zeta_y)+\zeta_y)^m\right\} \\ &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \sum_{k=0}^{m}\binom{m}{k}\zeta_y^{m-k}\, C_k = \frac{1}{2}\left(\frac{2\, m\, \zeta_y}{\frac{W}{R}\, p_i\, h_i}\right)^m \sum_{k=0}^{m}\binom{m}{k}\frac{C_k}{\mu_y^k} \\ &= 2^{m-1}\left(\frac{m}{\overline{\gamma}_i}\right)^m \sum_{k=0}^{m}\binom{m}{k}\frac{C_k}{\zeta_y^k} \end{aligned} \tag{21}$$

where $\overline{\gamma_i}$ is given in (10), and C_k is the kth central moment and it is given by [13]:

$$C_k = \begin{cases} 1.3\cdots(k-1)\,\overline{\sigma}_{x_i}^k & k \text{ even} \\ 0 & k \text{ odd} \end{cases}$$

By splitting up the summation in (21), we obtain:

$$\sum_{l=0}^{m} \binom{m}{l} \frac{C_l}{\zeta_y^l} = 1 + \binom{m}{2} \frac{\overline{\sigma}_{x_i}^2}{(\sigma^2 + \sum_{k\neq i}^N p_k\, h_k)^2} + \cdots + \binom{m}{m'} \frac{1.3\cdots(m'-1)\overline{\sigma}_{x_i}^{m'-1}}{(\sigma^2 + \sum_{k\neq i}^N p_k\, h_k)^{m'}} \tag{22}$$

where $m' = m$ if m is even and $m' = m-1$ if m is odd. Since $\overline{\sigma}_x^2$ (see (19)) is very small compared to ζ_{x_i} (see (18)), we can approximate the summation by its leading term which is 1. Therefore the average P_e at high SINR behaves like:

$$P_e \approx 2^{m-1} \left(\frac{m}{\overline{\gamma_i}}\right)^m \tag{23}$$

And the average utility function of the ith user is given by:

$$\boxed{u_i = \frac{L\ R}{M\ p_i} \left(1 - 2^m \left(\frac{m}{\overline{\gamma_i}}\right)^m\right)^M} \tag{24}$$

Notice that if we set $m = 1$, we obtain the same performance as in the Rayleigh fast flat fading case.

Nakagami Slow Flat Fading Channel

As done earlier, $u_i(\mathrm{p}|x_i)$ can be determined as follows:

$$\begin{aligned} u_i(\mathrm{p}|x_i) &= \int_0^\infty u_i(\mathrm{p}|\omega, x_i) f^{\gamma_i|x_i}(\omega) d\omega \\ &= \int_0^\infty \frac{L\ R}{M\ p_i}\ (1 - e^{-\omega/2})^M\ \frac{1}{\Gamma(m)} \left(\frac{m}{\gamma_i'}\right)^m\ \omega^{m-1}\, e^{-(\frac{m}{\gamma_i'})\,\omega}\ d\omega \end{aligned} \tag{25}$$

By factorizing $(1 - e^{-\gamma_i/2})^M$ and using the identity $\int_0^\infty y^n e^{-a\,y}\, dy = \frac{\Gamma(n+1)}{a^{n+1}}$ we obtain:

$$u_i(\mathrm{p}|x_i) = \frac{L\,R}{M\,p_i} \sum_{k=0}^{M} (-1)^k \binom{M}{k} \left(\frac{2\,m}{k\,\gamma_i' + 2\,m}\right)^m \tag{26}$$

For fixed m and high SINR, $\gamma_i' \gg 1$ (26) can be approximated as:

$$u_i(\mathrm{p}|x_i) \approx \frac{L\,R}{M\,p_i} \left[1 + (\frac{1}{\gamma_i'})^m \sum_{k=1}^{M} (-1)^k \binom{M}{k} \left(\frac{2\,m}{k}\right)^m\right] \tag{27}$$

Averaging (27) with respect to the distribution of x_i and using the same argument as in (20), (21) and (22) we end up with the final approximate averaged utility function given by:

$$\begin{aligned} u_i &\approx \frac{L\,R}{M\,p_i} \left[1 + (\frac{1}{\overline{\gamma_i}})^m \sum_{k=1}^{M} (-1)^k \binom{M}{k} \left(\frac{2\,m}{k}\right)^m\right] \\ \Aboxed{u_i &\approx \frac{L\,R}{M\,p_i} \left[1 - \xi\ (\frac{1}{\overline{\gamma_i}})^m\right]} \end{aligned} \tag{28}$$

where $\xi = -\sum_{k=1}^{M} (-1)^k \binom{M}{k} \left(\frac{2\,m}{k}\right)^m > 0$.

In the following two sections, we introduce briefly both NPG and NPGP games.

4 Non-Cooperative Power Control Game (NPG)

Let $\mathcal{N} = \{1, 2, \cdots, N\}$ be the index set of the users currently served in the cell and $\{P_j\}_{j\in\mathcal{N}}$ represent the set of strategy spaces of all users in the cell. Let $G = [\mathcal{N}, \{P_j\}, \{u_j(.)\}]$ denote a noncooperative game, where each user, based on local information, chooses a power level from a convex set $P_j = [p_{j-min}, p_{j-max}]$ and where p_{j-min} and p_{j-max} are the minimum and the maximum power levels in the jth user strategy space, respectively. Assuming that the power vector $\mathrm{p} = [p_1, p_2, \cdots, p_N]$ is the result of NPG, the utility of user j is given by [1]:

$$u_j(\mathrm{p}) = u_j(p_j, \mathrm{p}_{-j}) \tag{29}$$

where p_j is the power of user j, and p_{-j} is the vector of powers transmitted by all other users. The right side of (29) emphasizes the fact that user j can only control his own power. We rewrite (1) for user j as:

$$u_j(p_j, \mathrm{p}_{-j}) = \frac{L\,R}{M\,p_j}\,f(\gamma_j) \tag{30}$$

The formal expression for the NPG is given in [1] as:

$$G: \quad \max_{p_j \in \mathrm{P}_j} u_j(p_j, \mathrm{p}_{-j}), \;\; \text{for all } \; j \in \mathcal{N} \tag{31}$$

This game will produce a sequence of power vectors until it converges to a point where all users are satisfied with their utility level. This operating point is called a Nash equilibrium operating point of NPG. In the next subsection, we define the Nash equilibrium point and describe its physical interpretation.

4.1 Nash Equilibrium in NPG

Definition 2. *[1] A power vector* $\mathrm{p} = [p_1, p_2, \cdots, p_N]$ *is a Nash equilibrium of the* NPG defined above *if for every* $j \in \mathcal{N}, u_j(p_j, \mathrm{p}_{-j}) \geq u_j(p'_j, \mathrm{p}_{-j})$ *for all* $p'_j \in \mathrm{P}_j$.

One interpretation of Nash equilibrium is that no user can increase its utility by changing its power level unilaterally. If we multiply the power vector p by a constant $0 < \lambda < 1$ we may get higher utilities for all users, as was the case in nonfading channels. This means that the Nash equilibrium is not efficient, that is, the resulting p is not the most desired social operating point. This results from the lack of cooperation between the users currently in the system. To impose a level of cooperation between users in order to reach a Pareto dominant Nash point, a pricing technique was introduced in [1]. We discus this modified NPG game next.

5 Non-Cooperative Power Control Game with Pricing (NPGP)

In NPGP each user maximizes the difference between his/her utility function and a pricing function. This aims to allow more efficient use of the system resources within the cell, as each user is made aware of the cost of aggressive resources use, and of the harm done to other users in the cell. We use here a linear pricing function, i.e., a pricing factor multiplied by the transmit power. The base station broadcasts the pricing factor to help the users currently in the cell reach a Nash equilibrium that improves the aggregate utilities of all users at power levels lower than those of the pure NPG. In other words, the resulting power vector of NPGP is Pareto dominant compared to the resulting power vector of NPG, but is still not Pareto optimal in the sense that we may multiply the resulting power vector of NPGP by a constant $0 < \lambda < 1$ to get higher utilities for all users. Let $G_c = [\mathcal{N}, \{\mathrm{P}_j\}, \{u_j^c(.)\}]$ represent an N-player noncooperative power control game with pricing (NPGP), where the utilities are [1]:

$$u_j^c(\mathrm{p}) = u_j(\mathrm{p}) - c\, p_j \text{ for all } j \in \mathcal{N} \tag{32}$$

where c is a positive number chosen to get the best possible improvement in the performance. Therefore, NPGP with a linear pricing function can be expressed as:

$$G_c : \quad \max_{p_j \in \mathrm{P}_j} \{u_j(\mathrm{p}) - c\, p_j\} \text{ for all } j \in \mathcal{N} \tag{33}$$

6 Existence and Uniqueness of Nash Equilibrium Point

In this section we show that NPG and NPGP introduced by [1] admit a unique Nash equilibrium points under the assumed channel models. However, to guarantee the existence and uniqueness of NE point in both games, the terminals' strategy spaces defined in [1] should be constrained more. That is, some transmit power values which were allowed in a non-fading channel, may not be allowed under a fading channel. In the following, we refer to the unconstrained maximizing transmit power level of user i by p_i^{max}. P_i refers to the convex strategy space of user i.

Lemma 3. *In NPG under Rayleigh fast flat fading channel with the average utility function u_i given in (11), the existence of a Nash equilibrium point is guaranteed if and only if the strategy space is modified to $P_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})\}$, where $\overline{\gamma}_{i-min} = 2(M+1) - \sqrt{2M(M+1)}$ and $\overline{\gamma}_{i-max} = 2(M+1) + \sqrt{2M(M+1)}$. The best response vector of all users $r^1(p) = (r_1^1(p), r_2^1(p), \cdots, r_N^1(p))$, where $r_i^1(p) = min(p_i^{max}, p_{i-max})$, and*

$$p_i^{max} \quad = \quad 2(M+1)\, I_i, \;\; I_i = \frac{R\,(\sum_{k \neq i}^{N} h_k p_k + \sigma^2)}{W\, h_i} \tag{34}$$

is a standard interference function, therefore by [12], the Nash equilibrium point is unique. I_i is the effective interference for user i.

Proof: In all following proofs we make use of the classical results of game theory, where the existence of a Nash equilibrium point is guaranteed if the utility function is quasiconcave and optimized on a convex strategy space. Thus, to prove the existence of Nash equilibrium point, it is enough to prove that the utility function u_i is concave (a concave function on some set is also a quasiconcave function on the same set) in p_i given p_{-i} on the convex set $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})\}$. Let us find the first and second order derivatives with respect to p_i as follows:

$$\frac{\partial u_i}{\partial p_i} = \frac{L\, R}{M\, p_i^2}\, (\frac{2(M+1)}{\overline{\gamma_i}} - 1)\, (1 - \frac{2}{\overline{\gamma_i}})^{M-1}, \tag{35}$$

then $\frac{\partial^2 u_i}{\partial p_i^2} = \frac{2L\, R\, (1-\frac{2}{\overline{\gamma_i}})^M\, [2(M+1)(2+M) - 4(1+M)\, \overline{\gamma_i} + \overline{\gamma_i}^2]}{M\, p_i^3 (-2+\overline{\gamma_i})^2}$. Therefore, $\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\, \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})$, where $\overline{\gamma}_{i-min} = 2(M+1) - \sqrt{2M(M+1)}$ and $\overline{\gamma}_{i-max} = 2(M+1) + \sqrt{2M(M+1)}$. This implies that the strategy space should be modified to the convex set $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})\}$ to guarantee the concavity of the utility function, and then to guarantee the existence of Nash equilibrium point.

To prove the uniqueness of Nash equilibrium point we need to prove that $r^1(\mathrm{p})$ is a standard function. By setting (35) to zero we find the maximizing transmit power level that lies in the convex strategy space P_i is given as in (34). Before proving that $r^1(\mathrm{p})$ is a standard interference function we introduce the definition of an arbitrary standard interference function $\phi(\mathrm{p})$ as follows:

Definition 4. *[12] An interference function $\phi(p)$ is called a standard interference function if it satisfies the following: 1) Positivity: $\phi(p) > 0$, i.e., each element is positive, 2) Monotonicity: if $p > \hat{p}$ then $\phi(p) \geq \phi(\hat{p})$ (component wise), and 3) Scalability: $\forall \delta > 1,\ \delta\phi(p) > \phi(\delta\, p)$ (component wise).*

To prove that $r^1(\mathrm{p})$ is a standard interference function we proceed as follows: The proof of positivity is trivial, since $\mathrm{P}_i \subset \mathbb{R}^+$ and $r_i^1(\mathrm{p}_{-i}) \in P_i, \forall\, i \in \mathcal{N}$, where $r_i^1(\mathrm{p}_{-i}) = r_i^1(\mathrm{p})$. Also, it is obvious that $p_i^{max}(\mathrm{p}) > p_i^{max}(\hat{\mathrm{p}})$ for all i if $\mathrm{p} > \hat{\mathrm{p}}$ by looking at (34), henceforth the monotonicity of $r^1(\mathrm{p})$ is satisfied. To prove the scalability, it is enough to prove that $p_i^{max}(\mathrm{p}_{-i})$ is a scalable function and then the scalability of $r^1(\mathrm{p})$ comes through. Let us rewrite equation (34) as follows: $p_i^{max}(\mathrm{p}_{-i}) = \frac{2R\,(M+1)\,(\sum_{k\neq i}^{N} h_k p_k + \sigma^2)}{W\, h_i}$ then

$$p_i^{max}(\delta \mathrm{p}_{-i}) = \frac{2R\,(M+1)\,(\,\delta \sum_{k\neq i}^{N} h_k p_k + \sigma^2)}{W\, h_i}, \tag{36}$$

while

$$\delta p_i^{max}(\mathrm{p}_{-i}) = \frac{2\,\delta\, R\,(M+1)\,(\,\sum_{k\neq i}^{N} h_k p_k + \sigma^2)}{W\, h_i} \tag{37}$$

It is clear that $\delta p_i^{max}(\mathrm{p}_{-i}) > p_i^{max}(\delta\, \mathrm{p}_{-i})$, therefore $r^1(\mathrm{p})$ is a standard interference function, and the Nash equilibrium point is unique. □

In the following lemmas we omit the proof of existence and/or uniqueness as they are similar to those of lemma 3.

Lemma 5. *In NPG under Rayleigh slow flat fading channel with the average utility function (14), the existence of a Nash equilibrium point is guaranteed if and only if the strategy space is modified to the convex set $P_i = \{p_i : \overline{\gamma_i} \in (1, 3\,\beta)\}$. The best response vector of all users $r^2(p) = (r_1^2(p), r_2^2(p), \cdots, r_N^2(p))$, where $r_i^2(p) = min(p_i^{max}, p_{i-max})$, and $p_i^{max} = 2\,\beta\, I_i$, is a standard interference function, therefore by [12] Nash equilibrium point is unique.*

Proof: Similarly, we need to find the first and second order derivatives of u_i with respect to p_i: $\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\left(\frac{2\beta}{\overline{\gamma_i}} - 1\right)$, then $\frac{\partial^2 u_i}{\partial p_i^2} = \frac{2\,L\,R}{M\,p_i^3}\left(1 - \frac{3\beta}{\overline{\gamma_i}}\right)$, which means that $\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\, \overline{\gamma_i} \in (1, 3\,\beta)$ so the convex strategy space should have the following form: $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in (1, 3\,\beta)\}$ to guarantee the concavity of u_i and then to guarantee the existence of Nash point. □

Lemma 6. *In NPG under Nakagami fast flat fading channel with the average utility function u_i given in (24) with $m = 2$, the existence of a Nash equilibrium point is guaranteed if the strategy space is modified to the following convex set $P_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})\}$, where $\overline{\gamma}_{i-min} = \sqrt{8}\,\sqrt{2 + 5M - \sqrt{M\,(8 + 17M)}}$ and $\overline{\gamma}_{i-max} = \sqrt{8}\,\sqrt{2 + 5M + \sqrt{M\,(8 + 17M)}}$. The best response vector of all users $r^5(p) = (r_1^5(p), r_2^5(p), \cdots, r_N^5(p))$, where $r_i^5(p) = min(p_i^{max}, p_{i-max})$, and $p_i^{max} = 4\sqrt{1 + 2M}\, I_i$, is a standard interference function, therefore by [12] Nash equilibrium point is unique.*

Proof: We find the first and second order derivatives of u_i in (24) after setting $m = 2$ with respect to p_i as follows:

$$\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\,(\frac{16(2M+1)}{\overline{\gamma}_i^2} - 1)\,(1 - \frac{16}{\overline{\gamma}_i^2})^{M-1}, \tag{38}$$

then

$$\frac{\partial^2 u_i}{\partial p_i^2} = \frac{1}{M\,p_i^3(-16 + \overline{\gamma}_i^2)^2}\,\left(2\,L\,R\,(1 - \frac{16}{\overline{\gamma}_i^2})^M \times \left[256(1+M)(2M+1) - 16(2+5M)\,\overline{\gamma}_i^2 + \overline{\gamma}_i^4\right]\right) \tag{39}$$

and this implies that $\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\, \overline{\gamma_i} \in \left(\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max}\right)$,
where $\overline{\gamma}_{i-min} = \sqrt{8(2+5M) - 8\sqrt{M\,(8+17M)}}$ and $\overline{\gamma}_{i-max} = \sqrt{8(2+5M) + 8\sqrt{M\,(8+17M)}}$.
Henceforth, the strategy space should have the following convex set: $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max}\right)\}$ to guarantee that u_i is strict concave on P_i, then a Nash equilibrium exists. □

Lemma 7. *In NPG under Nakagami slow flat fading channel with the average utility function u_i given in (28), a Nash equilibrium point is guaranteed if and only if the strategy space is the following convex set $P_i = \{p_i : \overline{\gamma_i} \in \left(1, \sqrt{6\,\xi}\right)\}$. The best response vector of all users $r^6(p) = (r_1^6(p),\, r_2^6(p),\, \cdots,\, r_N^6(p))$, where $r_i^6(p) = min(p_i^{max}, p_{i-max})$, and $p_i^{max} = \sqrt{3\xi}\, I_i$, is a standard interference function, therefore by [12] Nash equilibrium point is unique.*

Proof: The first derivative and second order derivatives of u_i after setting $m = 2$ with respect to p_i are given by: $\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\left(\frac{3\,\xi}{\overline{\gamma}_i^2} - 1\right)$, and $\frac{\partial^2 u_i}{\partial p_i^2} = \frac{2\,L\,R}{M\,p_i^3}\left(1 - \frac{6\,\xi}{\overline{\gamma}_i^2}\right)$, therefore $\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\, \overline{\gamma_i} \in \left(1, \sqrt{6\,\xi}\right)$. As a result, the convex strategy space should be $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \left(1, \sqrt{6\,\xi}\right)\}$ to guarantee the strict concavity of u_i and then the existence of a Nash equilibrium point is guaranteed. □

Now, we turn to the existence and uniqueness of Nash equilibrium point of NPGP under the assumed channel models discussed above.

Lemma 8. *In NPGP under Rayleigh fast flat fading channel model with utility function $u_i^c = u_i - c\,p_i$, where u_i is given in (11), a Nash equilibrium point existence is guaranteed if and only if the strategy space is the following convex set: $P_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max}\right)\}$, where $\overline{\gamma}_{i-min} = 2\,(M+1) - \sqrt{2\,M(M+1)}$ and $\overline{\gamma}_{i-max} = 2\,(M+1)$. The best response vector of all users $r^7(p) = (r_1^7(p), r_2^7(p), \cdots, r_N^7(p))$, where $r_i^7(p) = min(p_i^{max}, p_{i-max})$, and*

$$p_i^{max} \approx \frac{-6\,2^{1/3}\,a + 2^{2/3}(27\,b_i + \sqrt{108\,a^3 + 729\,b_i^2}\;)^{2/3}}{6\,(27\,b_i + \sqrt{108\,a^3 + 729\,b_i^2}\;)^{1/3}};\;\; a = \frac{L\,R}{M\,c},\;\; b_i = \frac{2\,(M+1)\,L\,R\,I_i}{M\,c} \tag{40}$$

is a standard interference function, therefore by [12] Nash equilibrium point is unique.

Proof: Let us find the maximizing power p_i^{max} in terms of the SINR $\overline{\gamma_i}$ as follows:

$$\frac{\partial u_i^c}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\,(\frac{2\,(M+1)}{\overline{\gamma_i}} - 1)\,(1 - \frac{2}{\overline{\gamma_i}})^{M-1} - c = 0, \tag{41}$$

then

$$p_i^{max} = \sqrt{\frac{L\,R}{M\,c}\,(\frac{2\,(M+1)}{\overline{\gamma_i}} - 1)\,(1 - \frac{2}{\overline{\gamma_i}})^{M-1}} \tag{42}$$

For p_i^{max} to be feasible, i.e., real and positive we need to satisfy the following condition on the strategy space: $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in (1, M+1)\}$. But, to guarantee the concavity of the utility function u_i^c, we have to have $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max}\right)\}$, where $\overline{\gamma}_{i-min} = 2\,(M+1) - \sqrt{2\,M(M+1)}$ and $\overline{\gamma}_{i-max} = 2\,(M+1) + \sqrt{2\,M(M+1)}$. Therefore, to fulfill the two conditions the convex strategy space should be the intersection of the two sets, that is $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max}\right)\}$, where $\overline{\gamma}_{i-min} = 2\,(M+1) - \sqrt{2\,M(M+1)}$ and $\overline{\gamma}_{i-max} = 2\,(M+1)$. Since $\overline{\gamma_i} >> 1$ on the convex strategy space P_i given above, one can approximate p_i^{max}, the solution of (41), as the feasible solution of the following equation:

$$p_i^3 + \frac{L\,R}{M\,c}\,p_i - \frac{2\,(M+1)\,L\,R\,I_i}{M\,c} = 0 \tag{43}$$

This equation has only one real and positive solution which is given in (40). It is fairly easy to prove that r^7(p) with p_i^{max} as given in (40) is a standard interference function. Therefore, Nash equilibrium point is unique. □

In the following lemmas we may omit the proof of existence and/or uniqueness if it can be argued the same way as in lemma 8.

Lemma 9. *In NPGP under Rayleigh slow flat fading channel model with utility function* $u_i^c = u_i - c\, p_i$*, where* u_i *is given in (14), a Nash equilibrium point existence is guaranteed if and only if the strategy space is the following convex set:* $P_i = \{p_i : \overline{\gamma_i} \in (1, 2\,\beta)\}$*. The best response vector of all users* $r^8(p) = (r_1^8(p), r_2^8(p), \cdots, r_N^8(p))$*, where* $r_i^8(p) = min(p_i^{max}, p_{i-max})$*, and*

$$p_i^{max} = \frac{-6\, 2^{1/3}\, a + 2^{2/3}(27\, b_i + \sqrt{108\, a^3 + 729\, b_i^2}\ )^{2/3}}{6\,(27\, b_i + \sqrt{108\, a^3 + 729\, b_i^2}\ )^{1/3}};\ a = \frac{L\,R}{M\,c},\ b_i = \frac{2\,L\,R\,\beta\,I_i}{M\,c} \tag{44}$$

is a standard interference function, therefore by [12] Nash equilibrium point is unique. We have equality in (44) because there was no approximation in getting p_i^{max}*.*

Lemma 10. *In NPGP under Nakagami fast flat fading channel model with utility function* $u_i^c = u_i - c\, p_i$*, where* u_i *is given in (24), a Nash equilibrium point existence is guaranteed if and only if the strategy space is the convex set:* $P_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-min}, \overline{\gamma}_{i-max})\}$*, where* $\overline{\gamma}_{i-min} = \sqrt{8}\sqrt{2 + 5M - \sqrt{M\,(8 + 17M)}}$ *and* $\overline{\gamma}_{i-max} = 4\sqrt{1 + 2M}$*. The best response vector of all users* $r^{11}(p) = (r_1^{11}(p),\, r_2^{11}(p),\, \cdots,\, r_N^{11}(p))$*, where* $r_i^{11}(p) = min(p_i^{max}, p_{i-max})$*, and*

$$p_i^{max} \approx \sqrt{\frac{L\,R}{2\,M\,c}}\sqrt{-1 + \sqrt{1 + \frac{64\,(1 + 2M)\,I_i^2\,M\,c}{L\,R}}} \tag{45}$$

is a standard interference function, therefore by [12] Nash equilibrium point is unique.

Proof: The maximizer transmit power p_i^{max} is the feasible solution of $\frac{\partial u_i}{\partial p_i} - c = 0$, where $\frac{\partial u_i}{\partial p_i}$ is given in (38), and results in a polynomial of degree $2M + 4$. It is a tedious and may be impossible to find a closed-form for the feasible solution of this polynomial. Recall that $\overline{\gamma_i} > 4$ to guarantee $u_i(\mathrm{p}) > 0$, so the maximizer transmit power level p_i^{max} can be approximated by the feasible solution of the following equation.

$$p_i^4 + \frac{L\,R}{M\,c}\,p_i^2 - \frac{16\,(1 + 2M)\,L\,R\,I_i^2}{M\,c} = 0 \tag{46}$$

The only feasible solution of the equation above is given by (45). □

Lemma 11. *In NPGP under Nakagami slow flat fading channel model with utility function* $u_i^c = u_i - c\, p_i$*, where* u_i *is given in (28), a Nash equilibrium point existence is guaranteed if and only if the strategy space is the following convex set:* $P_i = \{p_i : \overline{\gamma_i} \in (1, \sqrt{3\,\xi})\}$*. The best response vector of all users* $r^{12}(p) = (r_1^{12}(p),\, r_2^{12}(p),\, \cdots,\, r_N^{12}(p))$*, where* $r_i^{12}(p) = min(p_i^{max}, p_{i-max})$*, and*

$$p_i^{max} = \sqrt{\frac{L\,R}{2\,M\,c}}\sqrt{-1 + \sqrt{1 + \frac{12\,\xi\,I_i^2\,M\,c}{L\,R}}} \tag{47}$$

is a standard interference function, therefore by [12] Nash equilibrium point is unique.

Proof: The maximizer transmit power level p_i^{max} is the feasible solution of the following equation.

$$p_i^4 + \frac{L\,R}{M\,c}\,p_i^2 - \frac{3\xi\,L\,R\,I_i^2}{M\,c} = 0 \tag{48}$$

The only feasible solution of the equation above is as given by (47). It is simple to check that $r^{12}(\mathrm{p})$ with the maximizer power in (47) satisfies all the conditions of a standard interference function. Henceforth, the Nash equilibrium point is unique. □

Observing lemmas 3-7, we see that the maximizing SINR γ_i^{max} for all users are the same: $\gamma_i^{max} = 2(M+1)$, $\forall\, i \in \mathcal{N}$ under fast Rayleigh and fast Rician fading channels. On the other hand $\gamma_i^{max} = 2\,\beta$, $\forall\, i \in \mathcal{N}$ under slow Rayleigh and slow Rician fading channels. $\gamma_i^{max} = 4\sqrt{1+2M}\,\forall\, i \in \mathcal{N}$ under fast Nakagami fading channels, and $\gamma_i^{max} = \sqrt{3\,\xi}\,\forall\, i \in \mathcal{N}$ under slow Nakagami fading channels. For nonfading channels it was shown in [1] that $\gamma_i^{max} = 12.4$, $\forall\, i \in \mathcal{N}$. This implies, as expected, that in order to overcome the fading effect, users in fading channels have to target higher SINR values.

Next, we introduce an algorithm that converges to Nash equilibrium point of NPG and NPGP. We need to keep in mind that the strategy space denoted by P_i in the algorithm differs according to the channel model. The algorithm is the same as in [1] except that the strategy spaces are modified to the forms given in lemmas 3-7 to guarantee the existence of Nash equilibrium point under the studied channel models.

Assume user j updates its power level at time instances that belong to a set T_j, where $T_j = \{t_{j_1}, t_{j_2}, \cdots\}$, with $t_{j_k} < t_{j_{k+1}}$ and $t_{j_0} = 0$ for all $j \in \mathcal{N}$. Let $T = \{t_1, t_2, \cdots\}$ where $T = T_1 \bigcup T_2 \bigcup \cdots \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{\mathrm{p}}$ to be the smallest power vector in the modified strategy space $\mathrm{P} = \mathrm{P}_1 \bigcup \mathrm{P}_2 \bigcup \cdots \bigcup \mathrm{P}_N$.

Algorithm 12. *Consider non-cooperative game G as given in (31) and generate a sequence of power vectors as follows:*
Initialize
$k = 0$;
$N =$ *total numberof active users;*
$p[0] = \underline{p}$;
for $j = 1$ *to* N ***do***
set $T_j =$ *set of times user* j *updates its power;*
end for
set $T = \bigcup_{j \in N} T_j = \{t_1, t_2, t_3, ...\}$;
$k \leftarrow k + 1$;
STEP1 : ***for*** $j = 1$ *to* N ***do***
if $t_k \in T_j$;
set $p_j^{max}(t_k) = arg \max\limits_{p_j \in P_j} u_j(p_j, p_{-j}(t_{k-1}))$;
else $p_j^{max}(t_k) = p_j^{max}(t_{k-1})$;
end if;
end for
if $p(t_k) = p(t_{k-1})$
stop and declare $p(t_k)$ *as the NE Point;*
else
$k \leftarrow k + 1$
Goto STEP1

The next algorithm finds the best pricing factor c_{Best} for NPGP, keeping in mind that the strategy space should be according to lemmas 8-11.

Algorithm 13. *Initialize*
$\Delta c =$ *real positive number;*
$c = 0$;
use algorithm 12 to obtain $u_j^{c=0}$ *for all users* $j = 1$ *to* $j = N$;
STEP2:***for*** $j = 1$ *to* N ***do***
set $c_j \leftarrow c + \Delta c$; *{Broadcast the pricing factor to all users}* ;
end for
use algorithm 12 to obtain $u_j^{c+\Delta c}$ *for all users* $j = 1$ *to* $j = N$;
if $u_j^{c+\Delta c}$ *is Pareto dominates* u_j^c *for all users;*
goto STEP2
else
stop *and declare* $c_{Best} = c - \Delta c$

7 Simulation Results

We show the effects of time-varying, fast and slow fading wireless channels on the equilibrium utilities and powers, that are the outcomes of the extended NPG and NPGP algorithms (algorithms12 and 13) which were originally studied for non-fading wireless channels in [1].

The system studied is a single-cell DS-CDMA cellular mobile system with 9 stationary users, all are using the same data rate R and the same modulation scheme, non-coherent BFSK. The system parameters used in this study are given in Table 1. The distances between the 9 users and the BS are d = $[310, 460, 570, 660, 740, 810, 880, 940, 1000]$ in meters. The path attenuation between user j and the BS using the simple path loss model [17] is $h_j = 0.097/d_j^4$, where 0.097 approximates the shadowing effect. Results of simulations show that under Rayleigh, Rician, and Nakagami fast flat fading channels with spreading gain $W/R = 10^2$, users do not reach a Nash equilibrium point where all users except the nearest user to the BS are using the highest power level in the strategy space without achieving the maximizing SINRs ($\gamma_i^{max} = 2(M+1)$, $\forall i \in \mathcal{N}$).

In Fig.1 we demonstrate the equilibrium utilities and the equilibrium powers of NPG under a fast fading channels **(a1)** with the three small scale fading models with spreading gain $W/R = 10^3$. All users were able to achieve their maximizing SINR under two small scale fading models, namely, Nakagami and Rician channels. Under the Rayleigh channel model however some users failed to achieve the maximizing SINR. One can see in Fig.1 that the farthest 4 users in the Rayleigh channel were forced to send at their maximum allowable power to achieve their minimum SINRs. The equilibrium utilities and equilibrium powers of the NPGP under **(a1)** are shown in the left and right graphs of Fig.2, respectively. Results show that a Pareto improvement over NPG for Rayleigh, Rician, and Nakagami channels was obtained such that all users succeeded to attain SINRs more than their corresponding minimum SINRs ($\gamma_i > \gamma_{i-min}$, $\forall i \in \mathcal{N}$).

Then we present the effect of a slow flat fading channels **(a2)** on the equilibrium utilities and powers which are the outcomes of NPG algorithm 12 as shown in Fig. 3. This figure shows that, unlike fast fading channels, all users succeeded to achieve the maximizing SINR ($\gamma_i^{max} = 2\beta = 19.8619$ under Rician and Rayleigh channels and $\gamma_i^{max} = \sqrt{3\xi} = 25.1182$ under Nakagami channels). Left graph of Fig. 3 shows that under both Rayleigh and Nakagami channel models users were equally satisfied, i.e., the equilibrium utilities are the same for both models. However, under Rayleigh channels, the equilibrium powers are less than those under Nakagami channels. This could be due to the fact that users under Nakagami channels target a higher maximizing SINR as we just mentioned above.

As for the effect of slow fading channels on the outcomes of NPGP, equilibrium utilities and equilibrium powers, our simulations showed that Pareto improvement (dominance) over NPG was not possible under the three small scale models. At $c = c_{Best}$, simulations showed that the best policy is that all users to target a fixed SINR, that is $\gamma_i^{max} = 19.8619$ under Rician and Rayleigh channels and $\gamma_i^{max} = 25.1182$ under Nakagami channels, which is exactly the same situation as in NPG. To demonstrate this result for the three small scale models more clearly, we present Fig. 4 for Rician and Rayleigh channel models and Fig. 5 for Nakagami channel model. Fig. 4 shows that with $c = c_{Best}$ the maximizing transmit power p_i^{max} given in (44) behaves with respect to the effective interference I_i (feasible values of I_i) the same as $p_i^{max} = 2\,\beta\, I_i$ given in Lemmas 2 and 4. While Fig. 5 shows that p_i^{max} given in (47) behaves with feasible values of I_i the same as $p_i^{max} = \sqrt{3\,\xi}\, I_i$ given in Lemma 6. Surprisingly, both figures suggest that NPGP with linear pricing does not admit a Pareto dominance over the NE operating point of NPG in a slow flat fading channels under the three small scale fading models.

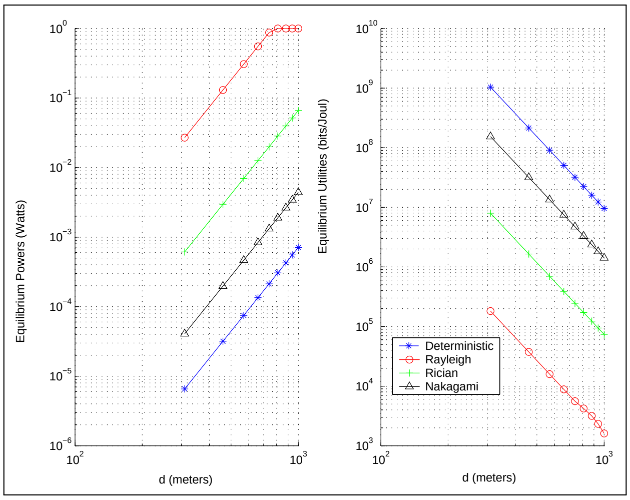

Figure 1: Equilibrium powers and equilibrium utilities of NPG for Rician fast flat fading channel gain (+), Rayleigh fast flat fading channel gain (o), Nakagami fast flat fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 10^3$.

Table 1: The values of parameters used in the simulations.

L, number of information bits	64
M length of the codeword	80
W, spread spectrum bandwidth	10^6, 10^7 Hz
R, data rate	10^4 bits/sec
σ^2, AWGN power at the BS	5×10^{-15}
N, number of users in the cell	9
s^2, specular component	1
W/R, spreading gain	10^2, 10^3
m, fading figure	2
p_{i-max}, maximum power in ith user's strategy space	1 Watts

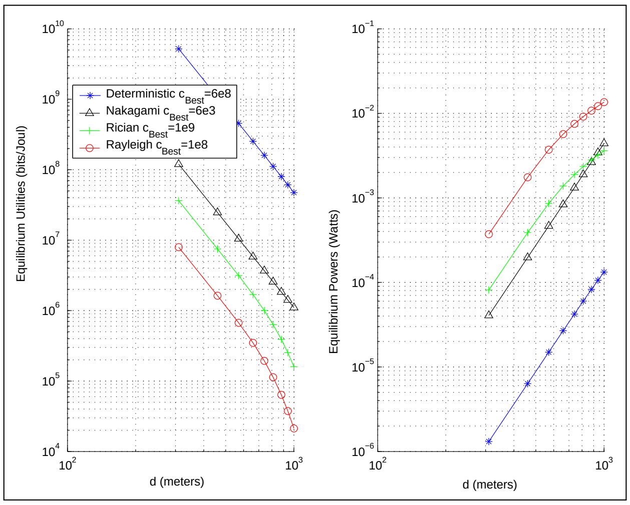

Figure 2: Equilibrium utilities and equilibrium powers of NPGP for Rician fast flat fading channel gain (+), Rayleigh fast flat fading channel gain (o), Nakagami fast flat fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 10^3$.

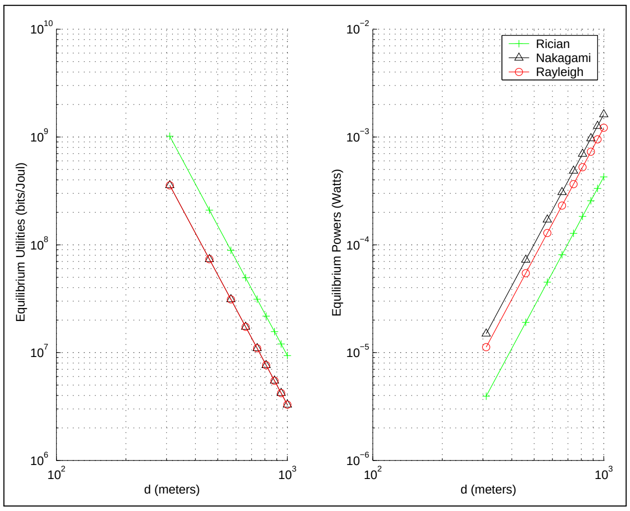

Figure 3: Equilibrium utilities and equilibrium powers of NPG for Rician slow flat fading channel gain (+), and Rayleigh slow flat fading channel gain (o), Nakagami slow flat fading (△) versus the distance of a user from the BS in meters with $W/R = 10^3$.

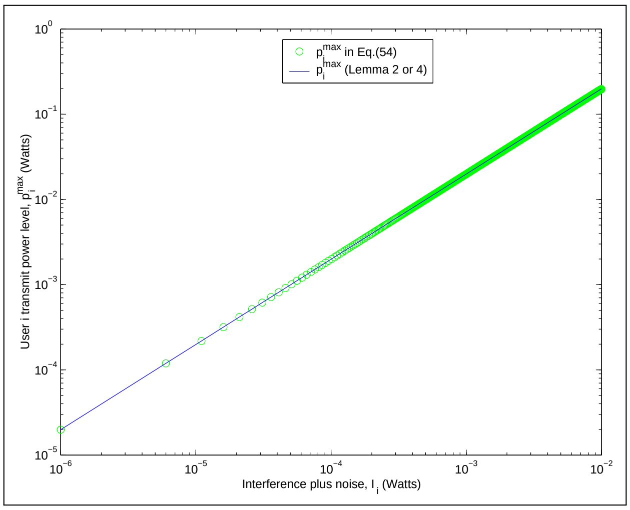

Figure 4: p_i^{max} as a function of I_i as in Eq. (44) (o), and the linear expression $p_i^{max} = 2\,\beta\,I_i$ given in Lemmas 2 and 4 (solid line) with $c = c_{Best}$.

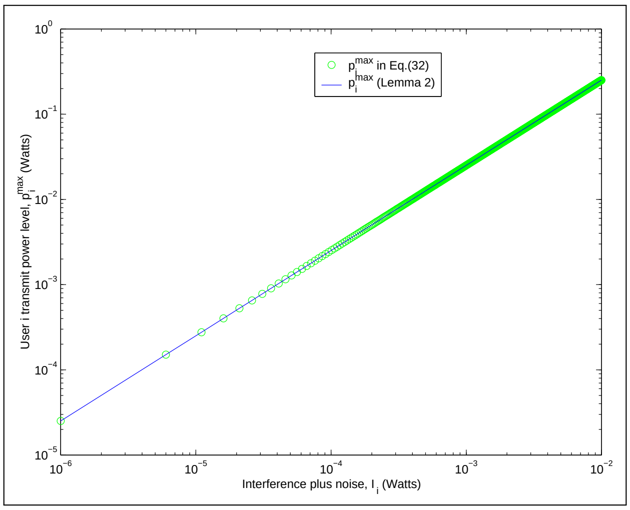

Figure 5: p_i^{max} as a function of I_i as in Eq. (47) (o), and the expression of $p_i^{max} = \sqrt{3\,\xi}\,I_i$ given in Lemma 6 (solid line) with $c = c_{Best}$.

8 Conclusions

We studied a noncooperative power control game (NPG) and noncooperative power control game with pricing (NPGP) introduced in [1] for realistic channel models, where we studied the impact of power statistical variation in Rayleigh, Rician and Nakagami fast/slow flat fading channels on the powers and utilities vectors at equilibrium. The results showed that an equilibrium with an equal maximizing SINR is not attainable in both games with spreading gain ($W/R = 10^2$). In fast fading with spreading gain $W/R = 10^3$, fixed target SINR NPG admitted NE point only under both Rician and Nakagami small scale models. And unlike fast fading, NPG admitted NE point under all small scale fading models in slow fading channels. Results demonstrated that in slow flat fading channels, NPGP with linear pricing does not exhibit a Pareto dominance over NPG outcomes at equilibrium.

In order to overcome the fading effects the SINRs targeted at equilibrium are higher for all users at equilibrium in the Rician, Rayleigh and Nakagami flat fading cases than SINRs under deterministic (nonfading) channels.

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