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Generalized Ordered Propositions Fusion Based on Belief Entropy

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Abstract: A set of ordered propositions describe the different intensities of a characteristic of an object, the intensities increase or decrease gradually. A basic support function is a set of truth-values of ordered propositions, it includes the determinate part and indeterminate part. The indeterminate part of a basic support function indicates uncertainty about all ordered propositions. In this paper, we propose generalized ordered propositions by extending the basic support function for power set of ordered propositions. We also present the entropy which is a measure of uncertainty of a basic support function based on belief entropy. The fusion method of generalized ordered proposition also be presented. The generalized ordered propositions will be degenerated as the classical ordered propositions in that when the truth-values of non-single subsets of ordered propositions are zero. Some numerical examples are used to illustrate the efficiency of generalized ordered propositions and their fusion.
Keywords: Dempster-Shafer evidence theory, ordered proposition, uncertainty measure, belief entropy, information fusion.

1 Introduction

In resent year, with the intensification of competition in the modern information war, information technology has developed rapidly, and the amount of information has increased explosively. Thus, as the critical technologies for information collection, storage and processing, the essentiality of information modeling and fusion has gradually increased.

There are many methods to model information, such as probability theory [10], Dempster-Shafer evidence theory [7,23], rough sets [40], fuzzy sets [6,7,9,19,19,22–24,24], Z-numbers [17,37,37], D numbers [1,5,76] and as so on. A specialized fusion algorithm is used for each method. Ordered proposition is a new approach to model information which is proposed by Liu *et al.* [35]. A set of ordered propositions describe the different intensities of a characteristic of a objects, the intensities increase or decrease gradually. For example, consumers evaluate the quality of a product on a rank of "Wonderful, Good, Indifferent, Weak". A set of ordered propositions can be expressed as a basic support function (similar to belief function in Dempster-Shafer evidence theory), whose elements represent the truth-value (belief value) of each proposition. The truth-values of a basic support function must be convex, because a subject cannot be two degrees in same characteristic. Such as, we cannot say the quality of a product is both wonderful and indifference simultaneously.

A basic support function is divided into determinate part and indeterminate part [35]. The determinate part is the sum of truth-values of each ordered proposition. The sum of indeterminate part and determinate part is one. In the ordered propositions fusion, the indeterminate part is prorated to each proposition and itself. Therefore, the indeterminate part can express the uncertainty for all ordered propositions. In this paper, we define the generalized ordered propositions, they extend the indeterminate part to all non-single subsets of ordered propositions. The truth-value of a non-single subset expresses the uncertainty of the propositions in it. For example, the "Wonderful, Good" express "the quality of this product is wonderful" or "the quality of this product is good". In order to ensure the convex property of a basic support function, the indeterminate part is listed separately. The generalized ordered propositions will be degenerated as the classical ordered propositions in that when the truth-values of non-single subsets of ordered propositions are zero.

The ordered propositions fusion is an important and extensive problem [35]. Previously, a fusion algorithm based on centroid is proposed [42], which fuse the basic support functions of two sets of ordered propositions and ensure the convexity. However, this approach has a few shortages [35]. In order to address these shortages, a new fusion method based on consistency and uncertainty measurements was presented by Liu *et al.* for the fusion of ordered proposition [35]. They also introduced entropy to measure the uncertainty of the basic support function based on Shannon entropy [35]. But this entropy only considered the determinate part of a basic support function, the indeterminate part is ignored. In Dempster-Shafer evidence theory, an entropy is presented to measure the uncertainty of a belief function, named Deng entropy [6]. When we add the groups of propositions in ordered propositions, the basic support function is more similar with the belief function. In this paper, we introduce a new entropy to measure the uncertainty of a basic support function based on belief entropy. It will be degenerated as the entropy which is proposed by Liu *et al.* in that when the indeterminate part of a basic support function is zero. Additionally, the fusion method of generalized ordered propositions based on consistency and uncertainty measurements is introduced. When the truth-values of non-single subsets of ordered propositions are zero, the fusion result is same as the fusion result of Liu *et al.*'s method.

The rest part of this paper is organized as follows. Section 2 briefly discusses the definitions and properties of ordered propositions, Dempster-Shafer evidence theory and belief entropy. Section 3 introduces the definition and properties of generalized ordered propositions. Section 4 discusses the proposed method for measuring uncertainty of a basic support function. The fusion method of generalized ordered proposition is described in Section 5. Section 6 presents some numerical examples. Finally, this paper is concluded in Section 7.

2 Preliminaries

2.1 Ordered propositions

In this section, some background knowledge about ordered propositions is briefly introduced [35].

Definition 1 (Ordered propositions)**.** For a set of propositions $p_1, p_2, \cdots, p_n$, the truth-value of p_i is denoted as $\lambda(p_i)$. $\lambda(p_k) = max\{\lambda(p_1), \cdots, \lambda(p_n)\}$. $p_1, p_2, \cdots, p_n$ are ordered propositions if [35]

(1) $\forall i = 1, 2, \cdots, n$, all subjects described in p_i are S;

(2) $\forall i = 1, 2, \cdots, n$, s_i describes the same characteristics or features of S;

(3) $\forall i = 1, 2, \cdots, k-1$, $\lambda(p_i) \leq \lambda(p_{i+1})$; and $\forall i = k, k+1, \cdots, n-1$, $\lambda(p_i) \geq \lambda(p_{i+1})$.

Definition 2 (Basic support function)**.** For a set of ordered propositions $P = \{p_1, p_2, \cdots, p_n\}$, a function λ is called the basic support function of the ordered propositions if [35]

(1) λ is defined on $\{\overline{P}\} \cup \{\{p_i\} | 1 \leq i \leq n\}$, where $\overline{P}$ indicates indeterminacy;

(2) $\lambda(p_i) \geq 0, 1 \leq i \leq n$;

(3) $\sum_{1\le i\le n}\lambda(p_i)\le 1$;

(4) $\lambda(\overline{P})=1-\sum_{1\le i\le n}\lambda(p_i)$.

Definition 3 (Determinate part and indeterminate part)**.** For a basic support function λ, the determinate part $\lambda(P)$ and indeterminate part $\lambda(\overline{P})$ are defined as [35]

$$\lambda(S)=\sum_{i=1,\cdots,n}\lambda(p_i),\quad \lambda(\overline{P})=1-\lambda(P). \tag{1}$$

Definition 4 (Mean value)**.** The mean value of a basic support function λ is defined as [35]

$$\overline{\lambda}=\frac{\sum_{i=1}^{n}\lambda(p_i)}{n}. \tag{2}$$

Definition 5 (Measure of convexity)**.** The measure of convexity of a basic support function λ is defined as [35]

$$convex(\lambda)=max\{\lambda(p_1),\lambda(p_2),\cdots,\lambda(p_n)\}-\overline{\lambda}. \tag{3}$$

It was clear that the maximum of the measure of convexity is $1-\overline{\lambda}$. Thus, the normalized $convex(\lambda)$ as follows: [35]

$$NC(\lambda)=(max\{\lambda(p_1),\lambda(p_2),\cdots,\lambda(p_n)\}-\overline{\lambda})/(1-\overline{\lambda}). \tag{4}$$

Definition 6 (Center of a basic support function)**.** For a basic support function $\lambda=(\lambda(p_1),\lambda(p_2),\cdots,\lambda(p_n))$, the center of λ is defined as [35]

$$CI(\lambda)=\begin{cases} argmax_{i=1,\cdots,n}\lambda(p_i), & NC(\lambda)\ge\theta\\ \dfrac{\sum_{i=1,\cdots,n\wedge\lambda(p_i)\ge\tau\cdot\overline{\lambda}}\lambda(p_i)\times i}{\sum_{i=1,\cdots,n\wedge\lambda(p_i)\ge\tau\cdot\overline{\lambda}}\lambda(p_i)}, & otherwise,\end{cases} \tag{5}$$

θ is set to 0.55 in [35], $1<\tau\le 1.5$.

In order to model the complex information of interaction, complex networks are proposed [4, 20,21,40,65,69]. The measure of consistency is essential to information, affected by the reliability of the information source [9, 11, 17, 32, 46, 64, 66, 74]. The reliability of obtaining data is very important for information fusion [41].

Definition 7 (Measure of consistency)**.** If $CI(\lambda_1)$ and $CI(\lambda_2)$ are the centers of the basic support functions λ_1 and λ_2.The consistency between λ_1 and λ_2 is defined as [35]

$$\Delta G(\lambda_1,\lambda_2)=|CI(\lambda_1)-CI(\lambda_2)|/(n-1). \tag{6}$$

If $\Delta G=1$, then λ_1 and λ_2 are totally conflicting. If $\Delta G=0$, then λ_1 and λ_2 are consistent. Otherwise, if

$0<\Delta G<1$, then λ_1 and λ_2 are partially conflicting. The consistency between λ_1 and λ_2 can be divided into 3 degrees [35].

$0\le\Delta G\le\delta_1$ indicates the consistency between λ_1 and λ_2 is high.

$\delta_1\le\Delta G\le\delta_2$ indicates the consistency between λ_1 and λ_2 is medium.

$\delta_2\le\Delta G\le 1$ indicates the consistency between λ_1 and λ_2 is poor.

2.2 Dempster-Shafer evidence theory

Evidence theory is widely used in many applications such as target recognition [29,30], decision making [1,11], uncertain processing [3,13,16,16,20,21,26–28,31,35], risk management [18,36], fault diagnosis [4,15,25,56,60] and as so on. The frame of discernment Θ is the exhaustive hypotheses of variable, X.
$\Theta = \{x_1, x_2, \cdots, x_i, \cdots, x_n\}$. The power set of Θ is $2^\Theta = \{\emptyset, \{x_1\}, \cdots, \{x_n\}, \{x_1, x_2\}, \cdots, \{x_1, x_2, \cdots, x_i\}, \cdots, \Theta\}$, where $\emptyset$ is an empty set [7,23].

Definition 8 (Basic probability assignment (BPA))**.** A basic probability assignment function $m : 2^\Theta \to [0, 1]$, which satisfies [7,23]:

$$m(\Theta) = 0 \quad \sum_{A \in 2^\Theta} m(A) = 1 \quad 0 \leq m(A) \leq 1, \tag{7}$$

the mass $m(A)$ indicates how strongly the evidence supports A.

2.3 Belief entropy

Shannon entropy is widely used to measure the uncertainty of a probability. In addition, a belief entropy named Deng entropy is proposed to measure the uncertainty of a BPA [6].

Definition 9 (Belief entropy)**.** For a BPA, m, defined on the frame of discernment Θ, it's belief entropy is defined as [6]

$$E_d(m) = -\sum_{A \subseteq \Theta} m(A) \ln \frac{m(A)}{2^{|A|} - 1}, \tag{8}$$

where A is the focal element of m, $|A|$ is the cardinality of A.

3 Generalized ordered propositions

3.1 Definitions

Definition 10 (Generalized ordered propositions)**.** For a set of propositions $p_1, p_2, \cdots, p_n$, it's power set, $\{\emptyset, \{p_1\}, \{p_2\}, \cdots, \{p_n\}, \{p_1, p_2\}, \cdots, \{p_1, \cdots, p_n\}\}$, let $\lambda(p_i, p_j \cdots)$ represent the truth-value of $\{p_i, p_j, \cdots\}$ and $\lambda(p_k) = max\{\lambda(p_1), \cdots, \lambda(p_n)\}$. $p_1, p_2, \cdots, p_n$ are generalized ordered propositions, if

(1)$\forall i = 1, 2, \cdots, n$, all subjects described in p_i are S;
(2)$\forall i = 1, 2, \cdots, n$, p_i describes the same characteristics or features of S;
(3)$\forall i = 1, 2, \cdots, m-1$, $\lambda(p_i) \leq \lambda(p_{i+1})$; and $\forall i = m, m+1, \cdots, n-1$, $\lambda(p_i) \geq \lambda(p_{i+1})$.

Definition 11 (Basic support function of the generalized ordered propositions)**.** For a set of generalized ordered propositions $P = \{p_1, p_2, \cdots, p_n\}$, it's power set $2^P = \{\emptyset, \{p_1\}, \{p_2\}, \cdots, \{p_n\}, \{p_1, p_2\}, \{p_1, p_3\}, \cdots, \{p_1, p_2, \cdots, p_n\}\}$ a function λ is called a basic support function of the generalized ordered propositions if

(1) λ is defined on 2^P;
(2) $\lambda(A) \geq 0, A \subseteq P$;
(3) $\lambda(\emptyset) = 0$;
(4) $\sum_{1 \leq i \leq n} \lambda(A) = 1$, where $A \subseteq P$;

Take the example of "the quality of a product", the basic support function is $\{(0.1, 0.3, 0.2, 0.0), (\lambda(p_1, p_2) = 0.2, \lambda(p_2, p_3) = 0.2)\}$.

$\lambda(p_1) = 0.1$ means the truth-value of 1st proposition "the quality of a product is wonderful" is 0.1;

$\lambda(p_2) = 0.3$ means the truth-value of 2nd proposition "the quality of a product is good" is 0.3;

$\lambda(p_3) = 0.2$ means the truth-value of 3rd proposition "the quality of a product is indifference" is 0.2;

$\lambda(p_4) = 0.0$ means the truth-value of 4rd proposition "the quality of a product is weak" is 0.0;

$\lambda(p_1, p_2) = 0.2$ means the uncertain truth-value of 1st proposition and 2nd proposition is 0.2;

$\lambda(p_2, p_3) = 0.2$ means the uncertain truth-value of 2st proposition and 3rd proposition is 0.2.

3.2 Properties

Definition 12 (Determinate part and indeterminate part). For a basic support function of generalized ordered proposition, the determinate part and indeterminate part is

$$\lambda(P) = \sum_{i=1}^{n} \lambda(p_i), \quad \lambda(\overline{P}) = \sum_{A \subseteq P \wedge A \neq \{q_1\}, \cdots, \{q_n\}} \lambda(A) = 1 - \lambda(P). \tag{9}$$

Definition 13 (Mean value). The mean value of a basic support function λ of generalized ordered propositions is

$$\overline{\lambda} = \frac{\sum_{i=1}^{n} \lambda(p_i)(1 + \sum_{p_i \subset A} \lambda(A))}{n}, \tag{10}$$

where $A \subsetneq \{p_1, p_2, \cdots, p_n\}$.

Definition 14 (Degree of convexity). The degree of convexity of a basic support function λ of generalized ordered propositions is:

$$convex(\lambda) = max_{i=1,\cdots,n}\{\lambda(p_i)(1 + \sum_{p_i \subset A} \lambda(A))\} - \overline{\lambda}, i = 1, 2, \cdots, n, \tag{11}$$

where $A \subsetneq \{p_1, p_2, \cdots, p_n\}$.

The normalized $convex(\lambda)$ is

$$NC(\lambda) = (max_{i=1,\cdots,n}\{\lambda(p_i)(1 + \sum_{p_i \subset A} \lambda(A))\} - \overline{\lambda})/(1 - \overline{\lambda}), i = 1, 2, \cdots, n. \tag{12}$$

Definition 15 (Center of a basic support function). A basic support function of generalized ordered propositions $\lambda = \{(\lambda(p_1), \lambda(p_2), \cdots, \lambda(p_n)), (\lambda(p_1, p_2), \cdots, \lambda(p_1, p_2, \cdots, p_n))\}$, the center of λ is

$$CI(\lambda) = \begin{cases} argmax_{i=1,\cdots,n}\lambda(p_i)(1 + \sum_{p_i \subset A} \lambda(A)), & NC(\lambda) \geq \theta \\ \dfrac{\sum_{i=1,\cdots,n \wedge \lambda(p_i) \geq \tau \cdot \overline{\lambda}} \lambda(p_i)(1 + \sum_{p_i \subset A} \lambda(A)) \times i}{\sum_{i=1,\cdots,n \wedge \lambda(p_i)(1+\sum_{p_i \subset A} \lambda(A)) \geq \tau \cdot \overline{\lambda}} \lambda(p_i)(1 + \sum_{p_i \subset A} \lambda(A))}, & otherwise, \end{cases} \tag{13}$$

where $A \subsetneq \{p_1, p_2, \cdots, p_n\}$.

4 Uncertainty measure

Uncertainty can evaluate the quality of information [2,3,13,15,16,31,32,39,39,47,48,50,61,71]. The more uncertainty, the less information [7,8]. A method to measure the uncertainty of a basic support function of ordered propositions based on Shannon entropy is proposed by Liu *et al.* [35].

Definition 16 (Liu *et al.*'s entropy)**.** For a basic support function $\lambda = (\lambda(p_1), \lambda(p_2), \cdots, \lambda(p_n))$, $\lambda \neq (\lambda(p_1) = 0, \lambda(p_2) = 0, \cdots, \lambda(p_n) = 0)$ and $n \geq 2$. Let $\lambda(p_k) = max\{\lambda(p_1), \lambda(p_2), \cdots, \lambda(p_n)\}$, $1 \leq k \leq n$. If $\beta\lambda(p_k) \leq \lambda(p_j) \leq \lambda(p_k), \beta \geq 0.9$ and $1 \leq j \leq n$, then $\lambda(p_j)$ is quasi-maxima. Let n' is the total number of maxima and quasi-maxima. The Liu *et al.*'s entropy of λ is defined as: [35]

$$E(\lambda) = \begin{cases} -\sum_{i=1}^{n} \lambda(p_i) \ln \lambda(p_i), & n' = 1, \\ -\sum_{i=1}^{n} \lambda(p_i) \ln \lambda(p_i) + (\ln n + \sum_{i=1}^{n} \lambda(p_i) \ln \lambda(p_i)))(n'/n)^{\alpha}, & 2 \leq n' \leq n, \end{cases} \tag{14}$$

where $\alpha = 0.1$.

When indeterminate part of a basic support function is equal to zero, this entropy can accurately measure the uncertainty of a basic support function. For example, given two basic support functions $\mu_1 = (0.005, 0.99, 0.005, 0.0, 0.0)$, $\mu_2 = (0.0049995, 0.990001, 0.0049995, 0.0, 0.0)$, we can given $E(\mu_1) = 0.062933$ and $E(\mu_2) = 0.062928$ using Eq. (14). $E(\mu_1)$ is greater than $E(\mu_2)$, this means that the uncertainty of μ_1 is higher than the uncertainty of μ_2. The result is reasonable. When there are multiple maxima of a basic support function, Liu *et al.*'s method can also measure uncertainty accurately. Take two basic support functions $\mu_3 = (0.5, 0.5, 0.0, 0.0)$, $\mu_4 = (0.15, 0.7, 0.1, 0.05)$, then $E(\mu_3) = 1.34$ and $E(\mu_4) = 0.914$. It is reasonable that $E(\mu_3) > E(\mu_4)$.

However, when indetermination part of a basic support function is not equal to zero, this entropy doesn't apply to measure uncertainty of a basic support function. For example, for two basic support functions $\mu_5 = (0.2, 0.3, 0.0, 0.0)$ and $\mu_6 = (0.7, 0.1, 0.1, 0)$, then $E(\mu_5) = 0.6831$, $E(\mu_6) = 0.7103$. $E(\mu_5) < E(\mu_6)$, this means that the degree of uncertainty of μ_6 is higher. It is obviously counterintuitive. In order to take into considered not only the determinate part but also indeterminate part, we present the a new method to measure uncertainty of a basic support function of generalized ordered proposition based on belief entropy [1, 6].

Definition 17 (The entropy based on belief entropy)**.** For a basic support function of generalized ordered propositions $\lambda = \{(\lambda(p_1), \lambda(p_2), \cdots, \lambda(p_n)), (\lambda(p_1, p_2), \lambda(p_1, p_3), \cdots, \lambda(p_1, p_2, \cdots, p_n))\}$, $\lambda \neq (\lambda(p_1) = 0, \lambda(p_2) = 0, \cdots, \lambda(p_n) = 0)$ and $n \geq 2$. Let $\lambda(p_k) = max\{\lambda(p_1), \lambda(p_2), \cdots, \lambda(p_n)\}$, $1 \leq k \leq n$. If $\beta\lambda(p_k) \leq \lambda(p_j) \leq \lambda(p_k), \beta \geq 0.9$ and $1 \leq j \leq n$, then $\lambda(q_j)$ is quasi-maxima. Let n' is the total number of maxima and quasi-maxima. The entropy of λ is defined as:

$$E_d(\lambda) = \begin{cases} -\sum_{i=1}^{n} \lambda(A) \ln(\frac{\lambda(A)}{2^{|A|-1}}), & n' = 1, \\ -\sum_{i=1}^{n} \lambda(A) \ln(\frac{\lambda(A)}{2^{|A|-1}}) + (\ln n + \lambda(A) \ln(\frac{\lambda(A)}{2^{|A|-1}}))(n'/n)^{\alpha}, & 2 \leq n' \leq n, \end{cases} \tag{15}$$

where $A \subseteq \{q_1, 1_2, \cdots, q_n\}$, $|A|$ is the number of elements of A, $\alpha = 0.1$.

Using Eq.(15) to calculate the uncertainty of μ_5 and μ_6, the results are $E_d(\mu_5) = 2.3837$, $E_d(\mu_6) = 1.2114$. $E_d(\mu_5) > E_d(\mu_6)$, it is reasonable. For two basic support functions of generalized ordered propositions $\mu_7 = \{(0.2, 0.5, 0.1, 0.0), (\mu_7(p_1, p_2) = 0, 1, \mu_7(p_2, p_3) = 0.1)\}$ and $\mu_8 =$

$\{(0.2, 0.6, 0.1, 0.0), (\mu_8(p_1, p_2) = 0.1)\}$. The results are $E_d(\mu_7 = 1.5790$ and $E_d(\mu_8) = 1.1988$ using Eq. (12). $E_d(\mu_7) > E_d(\mu_8)$, this means that the degree of uncertainty of μ_7 is higher than μ_8.

5 Fusion of generalized ordered propositions

For a set of generalized ordered propositions $P = \{p_1, p_2, \cdots, p_n\}$, let λ_1 and λ_2 are two basic support functions of P. Denote the fusion result of λ_1 and λ_2 is ω. The processes of method for fusion of basic support functions of generalized ordered propositions is shown in Fig. 5. The steps of this method can be explained as follows:

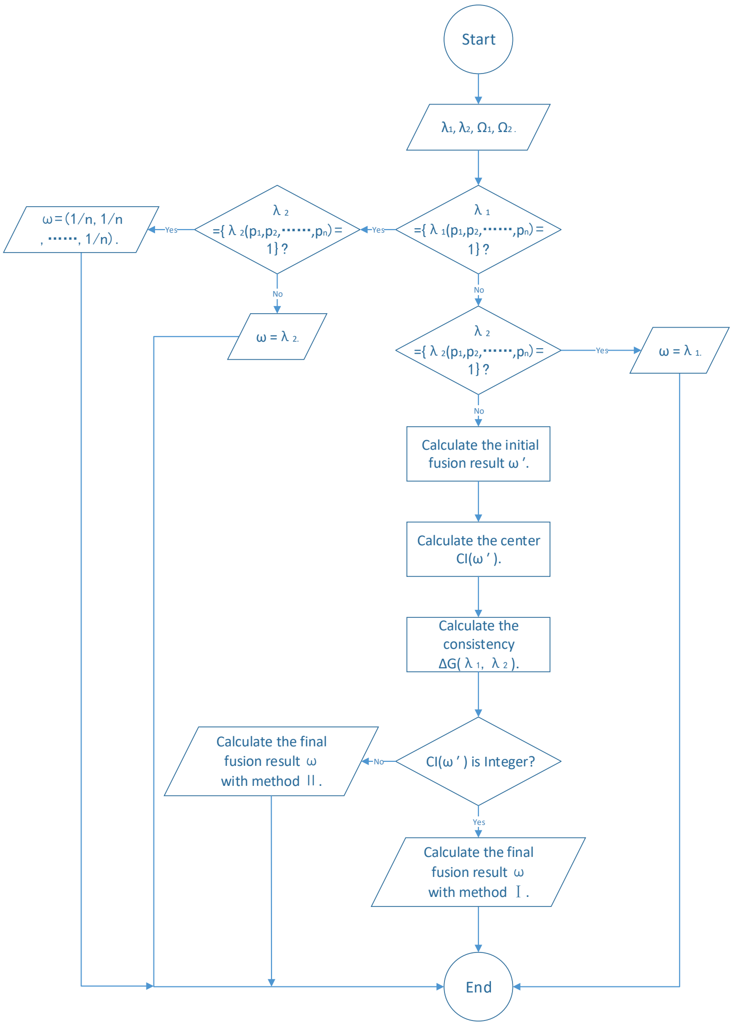

Figure 1: The processes of proposed method

Step 1: Give two basic support functions λ_1, λ_2 of a set of generalized ordered propositions $P = \{p_1, p_2, \cdots, p_n\}$, and the weights Ω_1, Ω_2 of two basic support functions respectively.

Table 1: Process of calculating ω' by Eq. (16).

	λ_1 to ω'	λ_2 to ω'	ω'
Truth-value obtained by $\omega'(p_1)$	0.025	0.08625	0.11125
Truth-value obtained by $\omega'(p_2)$	0.345	0.345	0.69
Truth-value obtained by $\omega'(p_3)$	0.08625	0.025	0.11125
Truth-value obtained by $\omega'(p_4)$	0.025	0.025	0.05
Truth-value obtained by $\omega'(p_1, p_2)$	0	0.01875	0.01875
Truth-value obtained by $\omega'(p_2, p_3)$	0.01875	0	0.01875

Step 2: Determine whether λ_1 is equal to $\{\lambda_1(p_1, p_2, \cdots, p_n) = 1\}$ and if λ_2 is equal to $\{\lambda_2(p_1, p_2, \cdots, p_n) = 1\}$.

If $\lambda_1 = \{\lambda_1(p_1, p_2, \cdots, p_n) = 1\}$ and $\lambda_2 = \{\lambda_2(p_1, p_2, \cdots, p_n) = 1\}$, the fusion result $\omega = (1/n, 1/n, \cdots, 1/n)$. If $\lambda_1 = \{\lambda_1(p_1, p_2, \cdots, p_n) = 1\}$ but $\lambda_2 \neq \{\lambda_2(p_1, p_2, \cdots, p_n) = 1\}$, the fusion result $\omega = \lambda_2$. If $\lambda_2 = \{\lambda_2(p_1, p_2, \cdots, p_n) = 1\}$ but $\lambda_1 \neq \{\lambda_1(p_1, p_2, \cdots, p_n) = 1\}$, the fusion result $\omega = \lambda_1$. If $\lambda_1 \neq \{\lambda_1(p_1, p_2, \cdots, p_n) = 1\}$ and $\lambda_2 \neq \{\lambda_2(p_1, p_2, \cdots, p_n) = 1\}$, take the next step.

Step 3: Calculate the initial fusion result.

$$\omega'(A) = \begin{cases} \Omega_1 \cdot \lambda_1(A)(1 + \sum\limits_{A \subset B} \lambda_1(B)) + \Omega_2 \cdot \lambda_1(A)(1 + \sum\limits_{A \subset C} \lambda_1(C)), & |A| = 1, \\ \Omega_1 \cdot \lambda_1(A)(1 - \sum\limits_{p_i \subset A} \lambda_1(p_i)) + \Omega_2 \cdot \lambda_2(A)(1 - \sum\limits_{p_i \subset A} \lambda_1(p_i)), & 1 < |A| \leq n, \end{cases} \tag{16}$$

where $A, B, C \subseteq \{p_1, p_2, \cdots, p_n\}$, $i = 1, 2, \cdots, n$, $\Omega_1 + \Omega_2 = 1$.

For example, there are two basic support functions $\lambda_1 = \{(0.05, 0.6, 0.15, 0.05), (\lambda_1(p_2, p_3) = 0.15\}$ and $\lambda_2 = \{(0.15, 0.6, 0.05, 0.05), (\lambda_2(p_1, p_2) = 0.15)\}$. The weights are $\Omega_1 = \Omega_2 = 0.5$. The process of calculating initial fusion result ω' by using Eq. (16) is illustrated in Table 1.

Step 4: Calculate the center of initial fusion result ω' with Eq. (13), $CI(\omega')$.

Step 5: Calculate the consistency between λ_1 and λ_2 with Eq. (6), $\Delta G(\lambda_1, \lambda_2)$.

Step 6: Determine whether the center of initial fusion result $CI(\omega')$ is Integer. If $CI(\omega')$ is Integer, take the step 7, otherwise take the step 8.

Step 7: Calculate the final fusion result ω with method I.

Step 7.1: Positive regulation.

$$\omega(p_i) = \begin{cases} \sum\limits_{k=1}^{i} \frac{\omega'(p_k)[1 + \varphi(i-k)]}{\sum_{j=0}^{CI(\omega')-k}(1 + j\varphi)}, & if\ i < CI(\omega'), \\ \sum\limits_{k=1}^{CI(\omega)} \frac{\omega'(p_k)[1 + \varphi(i-k)]}{\sum_{j=0}^{CI(\omega')-k}(1 + j\varphi)} + \sum\limits_{k=CI(\omega')+1}^{n} \frac{\omega'(p_k)[1 + \varphi(k - CI(\omega'))]}{\sum_{j=0}^{k-CI(\omega')}(1 + j\varphi)} & if\ i = CI(\omega'), \\ \sum\limits_{k=i}^{n} \frac{\omega'(p_k)[1 + \varphi(k-i)]}{\sum_{j=0}^{k-CI(\omega')}(1 + j\varphi)} & if\ i > CI(\omega'), \end{cases} \tag{17}$$

$$\omega(A) = \omega'(A),$$

where $\varphi = 0.2, 0.1, 0$ when the consistency between two basic support function is high, medium, poor respectively, A is the non-simple subset of P.

Step 7.2: Negative regulation.

When the consistency between two basic support functions is poor, the measure of uncertainty is used to compress the curve of truth-value of ω vertically until the entropy of ω approximately equals the entropy of ω', that is $|E(\omega) - E(\omega')| \leq \epsilon$. This process is called negative regulation and outlined in Algorithm 1.

Algorithm 1 The procedure of negative regulation.

Input: The initial fusion result ω' and basic support function ω after positive regulation
Output: The final fusion result ω

$\delta \leftarrow 1$
while $|E(\omega) - E(\omega')| \leq \epsilon$ **do**
  $I \leftarrow$ index of maximum truth-value of ω
  $k \leftarrow 1$
  for $k = I$ to $n-1$ **do**
    if $\omega(p_k) > \omega(p_{k+1})$ **then**
      $\omega(p_k) = \omega(p_k) - \frac{\delta\omega(p_{k+1})(\omega(p_k)-\omega(p_{k+1}))}{\omega(p_k)+\omega(p_{k+1})}$
      $\omega(p_{k+1}) = \omega(p_{k+1}) + \frac{\delta\omega(p_{k+1})(\omega(p_k)-\omega(p_{k+1}))}{\omega(p_k)+\omega(p_{k+1})}$
    end if
  end for
  for $k = I; k > 1; k--$ **do**
    if $\omega(p_k) > \omega(p_{k-1})$ **then**
      $\omega(p_k) = \omega(p_k) - \frac{\delta\omega(p_{k-1})(\omega(p_k)-\omega(p_{k-1}))}{\omega(p_k)+\omega(p_{k-1})}$
      $\omega(p_{k-1}) = \omega(p_{k-1}) + \frac{\delta\omega(p_{k-1})(\omega(p_k)-\omega(p_{k-1}))}{\omega(p_k)+\omega(p_{k-1})}$
    end if
  end for
  if $E(\omega) < E(\omega') - \epsilon$ **then**
    $\delta \leftarrow 1$
  end if
  if $E(\omega) > E(\omega') + \epsilon$ **then**
    $\delta \leftarrow \delta/2$
  end if
end while

Step 8: Calculate the final fusion result ω with method II.

Step 8.1: Positive regulation.

Denote $a = \lceil CI(\omega) \rceil$, $b = \lfloor CI(\omega') \rfloor$ for convenience, thus

$$\omega(p_i) = \begin{cases} \sum_{k=1}^{i} \frac{\omega'(p_k)[1+\varphi(i-k)]}{((\sum_{j=0}^{a-k}(1+j\varphi))-\varphi)}, & if\ i < b, \\ \omega'(p_b) + \Gamma(a - CI(\omega')), & If\ i = b \wedge \omega'(p_b) \neq \omega'(p_a), \\ \omega'(p_a) + \Gamma(CI(\omega') - b), & If\ i = a \wedge \omega'(p_b) \neq \omega'(p_a), \\ \omega'(p_b) + \Gamma/2, & If\ i = b \wedge \omega'(p_b) = \omega'(p_a), \\ \omega'(p_a) + \Gamma/2, & If\ i = a \wedge \omega'(p_b) = \omega'(p_a), \\ \sum_{k=i}^{n} \frac{\omega'(p_k)[1+\varphi(k-i)]}{((\sum_{j=0}^{k-b}(1+j\varphi))-\varphi)}, & if\ i > a, \end{cases} \tag{18}$$

$$\omega(A) = \omega'(A),$$

Table 2: The fusion process and result of example (1)

variables	values
λ_1	$\{(0.1, 0.4, 0.2, 0.1), (\lambda_1(p_1, p_2) = 0.1, \lambda_1(p_2, p_3) = 0.1)\}$
λ_2	$\{(0.1, 0.4, 0.2, 0.1), (\lambda_2(p_1, p_2) = 0.1, \lambda_2(p_2, p_3) = 0.1)\}$
$\overline{\lambda_1} = \overline{\lambda_2}$	0.2275
$NC(\lambda_1) = NC(\lambda_2)$	0.3269
$CI(\lambda_1) = CI(\lambda_2)$	2.4615
$\Delta G(\lambda_1, \lambda_2)$	0
ω'	$\{(0.11, 0.48, 0.22, 0.1), (\omega'(p_1, p_2) = 0.05, \omega'(p_2, p_3) = 0.04)\}$
$\overline{\omega'}$	0.2419
$NC(\omega')$	0.3711
$CI(\omega')$	2.3407
ω	$\{(0.0324, 0.5777, 0.2705, 0.1482), (\omega(p_1, p_2) = 0.05, \omega(p_2, p_3) = 0.04)\}$

where

$$\Gamma = \Gamma_1 + \Gamma_2, \quad \Gamma_1 = \sum_{k=1}^{b-1} \frac{\omega'(p_k)[1 + \varphi(b-k)]}{(\sum_{j=0}^{a-k}(1 + j\varphi)) - \varphi} + \sum_{a+1}^{n} \frac{\omega'(p_k)[1 + \varphi(k-a)]}{(\sum_{j=0}^{k-b}(1 + \varphi j)) - \varphi},$$

$$\Gamma_2 = \sum_{k=1}^{b-1} \frac{\omega'(p_k)[1 + \varphi(a-k) - \varphi]}{(\sum_{j=0}^{a-k}(1 + j\varphi)) - \varphi} + \sum_{a+1}^{n} \frac{\omega'(p_k)[1 + \varphi(k-a)]}{(\sum_{j=0}^{k-b}(1 + \varphi j)) - \varphi},$$

$\varphi = 0.2, 0.1, 0$ when the consistency between two basic support function is high, medium, poor respectively, A is the non-simple subset of P.

Step 8.2: Negative regulation.

It is same as Step 7.2.

6 Numerical examples

(1) Two basic support functions are $\lambda_1 = \{(0.1, 0.4, 0.2, 0.1), (\lambda_1(p_1, p_2) = 0.1, \lambda_1(p_2, p_3) = 0.1)\}$ and $\lambda_2 = \{(0.1, 0.4, 0.2, 0.1), (\lambda_2(p_1, p_2) = 0.1, \lambda_2(p_2, p_3) = 0.1)\}$. The weights of λ_1 and λ_2 are $\Omega_1 = \Omega_2 = 0.5$. The fusion processes and results are shown in Table 2. λ_1 and λ_2 are consistent, and they all mean that the 2nd proposition is most likely to be correct. So the fusing basic support function should reach the maximum truth-value at the index 2. The results are reasonable.

(2) Two basic support functions are $\lambda_1 = (0, 0.1, 0.2, 0.7)$ and $\lambda_2 = \{(0.1, 0.1, 0.1, 0.6), (\lambda_2(p_3, p_4) = 0.1)\}$. The weights of λ_1 and λ_2 are $\Omega_1 = \Omega_2 = 0.5$. The fusion result is $\omega = \{(0.0096, 0.0394, 0.1172, 0.8188), (\omega(p_3, p_4) = 0.015)\}$. λ_1 and λ_2 are not exactly the same, but $NC(\lambda_1) = 0.6 > 0.55$ and $NC(\lambda_2) = 0.5512 > 0.55$, so $CI(\lambda_1) = CI(\lambda_2) = 4$ and $\Delta G(\lambda_1, \lambda_2) = 0$. Similar to the previous example, the fusing basic support function should reach the maximum truth-value at the index 4. So the results are reasonable.

(3) Two basic support functions are $\lambda_1 = (0.7, 0.2, 0.1, 0)$ and $\lambda_2 = \{(0.1, 0.1, 0.1, 0.6), (\lambda_2(p_3, p_4) = 0.1)\}$. The weights of λ_1 and λ_2 are $\Omega_1 = \Omega_2 = 0.5$. The fusion results are shown in Table 4. λ_1 and λ_2 are totally conflicting and the fusion result is $\omega = \{(0.1333, 0.4737, 0.2680, 0.11), (\omega(p_3, p_4) = 0.015)\}$. The result shows that the 2nd proposition is most likely to be true, which is logical. It is reasonable that the uncertainty of the result is high.

Table 3: The fusion process and result of example (3)

variables	values
λ_1	$(0.7, 0.2, 0.1, 0)$
λ_2	$\{(0.1, 0.1, 0.1, 0.6), (\lambda_2(p_3, p_4) = 0.1)\}$
ω'	$\{(0.4, 0.15, 0.105, 0.33), (\omega'(p_3, p_4) = 0.015\}$
$E_d(\omega')$	1.5185
ω	$\{(0.1333, 0.4737, 0.2680, 0.11), (\omega(p_3, p_4) = 0.015)\}$
$E_d(\omega)$	1.4832

7 Conclusion

In order to better model the uncertain information of the characteristics of a subject, we proposed the generalized ordered propositions based on classical ordered propositions. The generalized ordered propositions extended the indeterminate part of a basic support function to all groups of propositions, not just the universal set of propositions. Then we considered the determinate part, indeterminate part, mean, degree of convexity and center of a basic support function in the situation of generalized ordered propositions. These properties can also be applied in classical ordered propositions. Additionally, we found the existing entropy of a basic support function does not apply when the indeterminate part is not zero. To address this shortage, we presented a new entropy based on belief entropy. This entropy measures not only the uncertainty of the determinate part but also indeterminate part of a basic support function. When the indeterminate part equals to zero, this entropy is degenerated into the existing entropy. Finally, we instructed the fusion method of basic support functions in generalized ordered propositions based on consistency and uncertainty. The experimental results show that the method is effective.

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Conflict of interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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