International Journal of Computers, Communications & Control
Vol. II (2007), No. 4, pp. 340-354

Bounded Controllers for Decentralized Formation Control of Mobile Robots with Limited Sensing

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Abstract: This paper presents a constructive method to design bounded cooperative controllers that force a group of N mobile robots with limited sensing ranges to stabilize at a desired location, and guarantee no collisions between the robots. The control development is based on new general potential functions, which attain the minimum value when the desired formation is achieved, and are equal to infinity when a collision between any robots occurs. Smooth and p times differential jump functions are introduced and embedded into the potential functions to deal with the robot limited sensing ranges. Formation tracking is also considered.
Keywords: Formation control, mobile robot, local potential function, nonholonomic mobile robot.

1 Introduction

Formation is an extremely useful tool mimicking from biological systems to man-made teams of vehicles, mobile sensors and embedded robotic systems to perform tasks such as jointly moving in a synchronized manner or deploying over a given region with applications to search, rescue, coverage, surveillance, reconnaissance and cooperative transportation. Formation control can be roughly understood as controlling positions of a group of the robots such that they stabilize/track desired locations relative to reference point(s), which can be another robot(s) within the team, and can either be stationary or moving. Three popular approaches to formation control are leader-following (e.g. [2], [3]), behavioral (e.g. [4], [5]), and use of virtual structures (e.g. [6], [7]. Most research works investigating formation control utilize one or more of these approaches in either a centralized or decentralized manner. Centralized control schemes, see e.g. [3], [11], and [10] where actual dynamics of nonholonomic robots were considered in the formation control design, use a single controller that generates collision free trajectories in the workspace. Although these guarantee a complete solution, centralized schemes require high computational power and are not robust due to the heavy dependence on a single controller. A nice application of formation control based on potential field method [3] and Lyapunov's direct method [17] to gradient climbing is recently addressed in [18]. However, the final configuration of formation cannot be foretold. On the other hand, decentralized schemes, see e.g. [8], [12], require less computational effort, and are relatively more scalable to the team size. The decentralized approach usually involves a combination of robot based local potential fields (e.g. [3], [15], [16]). The main problem with the decentralized approach, when collision avoidance is taken into account, is that it is extremely difficult to predict and control the critical points of the controlled systems. Recently, a method based on a different navigation function from [13] provided a centralized formation stabilization control design strategy is proposed in [11]. This work is extended to a decentralized version in [12]. However, the potential function, which possesses all properties of a navigation function (see [13]), is finite (but its gradient with respect to the system states can be unbounded) when a collision occurs. This complicates analysis of collision avoidance. Moreover, the formation is stabilized to any point in workspace instead of being "tied" to a fixed coordinate frame. In [13], [11] and [12], the tuning constants, which are crucial to guarantee that the only desired equilibrium points are asymptotic stable and that the other critical points are unstable, are extremely difficult to obtain. This problem has been removed in [9] where new potential functions were introduced. It is however noted that in [9] each robot requires knowledge of position of all other robots in the group. Moreover, the control design methods (e.g. [3], [14], [15], [18]) based on the potential functions that are equal to infinity when a collision occurs exhibit very large control efforts if the robots are close to each other. Hence, a bounded control is called for. In addition, switching control theory [21] is often used to design a decentralized formation control system (e.g. [2], where a case by case basis is proposed), especially when the vehicles have limited sensing ranges and collision avoidance between vehicles must be considered. Clearly, it is more desirable if we are able to design a non-switching formation control system that can handle the above decentralized and collision avoidance requirements.

In this paper, bounded cooperative controllers are designed for formation stabilization of a group of mobile robots with limited sensing ranges. New general potential functions are constructed to design the controllers that yield (almost) global asymptotic convergence of a group of mobile robots to a desired formation, and guarantee no collisions among the robots. Smooth and p times differential jump functions are introduced and embedded into the potential functions to deal with the robot limited sensing ranges. Moreover, the controlled system exhibits multiple equilibria due to collision avoidance taken into account. We therefore investigate the behavior of equilibrium points by linearizing the closed loop system around those points, and show that critical points, other than the desired point for an robot, are unstable. The proposed formation stabilization solution is then extended to solve a formation tracking problem.

2 Problem statement

We consider a group of N mobile robots, of which each has the following dynamics

$$\dot{q}_i = u_i, i = 1, ..., N \tag{1}$$

where $q_i \in \mathbb{R}^n$ and $u_i \in D \subset \mathbb{R}^n$ are the state and control input of the robot i. We assume that $n > 1$ and $N > 1$. Here, we treat each robot as an autonomous point.

Control objective: Assume that at the initial time $t_0 \geq 0$ each robot starts at a different location, and that each robot has a different desired location, i.e. there exist strictly positive constants ε_1 and ε_2 such that for all $(i, j) \in \{1, 2, ..., N\}, i \neq j$

$$\|q_i(t_0) - q_j(t_0)\| \geq \varepsilon_1, \|q_{if} - q_{jf}\| \geq \varepsilon_2 \tag{2}$$

where $q_{if}, i = 1, .., N$, is the desired location of the robot i. Moreover, the robot i can only measure its own state and can only detect the other group members if these members are in a sphere, which is centered at the robot and has a radius of R_i larger than a strictly positive constant. Design the bounded control input u_i for each robot i such that each robot asymptotically approaches its desired location while avoids collisions with all other robots in the group, i.e. for all $(i, j) \in \{1, 2, ..., N\}, i \neq j, t \geq t_0 \geq 0$

$$\|u_i(t)\| \leq \delta, \lim_{t \to \infty}(q_i(t) - q_{if}) = 0, \|q_i(t) - q_j(t)\| \geq \varepsilon_3 \tag{3}$$

where δ is a strictly positive constant and ε_3 is a positive constant.

3 Preliminaries

We present one definition and one lemma to be used in the control design and stability analysis in the next section.

Definition 1. A scalar function $h(x,a,b)$ is called a p times differential jump function if it enjoys the properties:

$$
\begin{aligned}
&1)\; h(x,a,b) = 0 \quad \text{if} \quad 0 \le x \le a,\\
&2)\; h(x,a,b) = 1 \quad \text{if} \quad x \ge b,\\
&3)\; 0 < h(x,a,b) < 1 \text{ if } a < x < b,\\
&4)\; h(x,a,b) \text{ is } p \text{ times differentiable with respect to } x
\end{aligned} \tag{4}
$$

where p is a positive integer, $x \in \mathbb{R}_+$, and a and b are constants such that $0 \le a < b$. Moreover, if $p = \infty$ then the function $h(x,a,b)$ is called a smooth jump function.

Lemma 1. Let the scalar function $h(x,a,b)$ be defined as

$$h(x,a,b) = \frac{\int_a^x f(\tau - a) f(b - \tau) d\tau}{\int_a^b f(\tau - a) f(b - \tau) d\tau} \tag{5}$$

with the function $f(y)$ being defined as follows

$$f(y) = 0 \text{ if } y \le 0, \quad f(y) = g(y) \text{ if } y > 0 \tag{6}$$

where the function $g(y)$ enjoys the following properties

$$
\begin{aligned}
&a) \quad g(\tau - a) g(b - \tau) > 0 \quad a < \tau < b,\\
&b) \quad g(y) \text{ is } p \text{ times differentiable with respect to } y,\\
&\text{and } \lim_{y \to 0^+} \frac{\partial^k g(y)}{\partial y^k} = 0,\; k = 1, 2, ..., p-1.
\end{aligned} \tag{7}
$$

Then the function $h(x,a,b)$ is a p times differentiable jump function.

Proof. See Appendix A.

Remark 1. Several examples of the function $g(y)$ are $g(y) = y^p$, $g(y) = \tanh(y)^p$, $g(y) = \arctan(y^p)$ for any positive integer p, and $g(y) = \sin(y)^p$ for any even positive integer p.

Corollary 1. If the function $g(y)$ in (7) is taken as $g(y) = \exp(-\frac{1}{y})$ then the function $h(x,a,b)$ defined in (5) is a smooth jump function.

Proof. See Appendix B.

4 Control design

To achieve the control objective, we design the control u_i for the robot i based on the new potential function φ. This potential function must attain its unique minimum value when all robots are at their desired positions, and must equal infinity when there is a collision between any robots. The potential function φ should also be chosen such that the gradient based control u_i for the robot i can handle the limited sensing range of the robot. As such, we propose the following potential function

$$\varphi = \frac{e^\gamma}{\beta^\kappa} - 1 \tag{8}$$

where κ is a positive constant, γ and β are the goal and collision avoidance functions. These functions are specified as follows:
-The goal function is designed such that it puts penalty on stabilization errors for all robots, and is equal to zero when the robots are at their final positions. A simple choice of this function is

$$\gamma = \frac{1}{2}\sum_{i=1}^{N}\|q_i - q_{if}\|^2. \tag{9}$$

-The collision function β is chosen such that it equals zero when there is a collision between any robots, and equals 1 when the robots are at their desired positions. We choose this function as follows:

$$\beta = \prod_{i,j}\beta_{ij}, i = 1...N-1, j = i+1,...,N. \tag{10}$$

The function $\beta_{ij} = \beta_{ji}$ is designed as

$$\beta_{ij} = h(0, b_{ij}^2/2, \|q_{ij}\|^2/2) \tag{11}$$

where $h(0, b_{ij}^2/2, \|q_{ij}\|^2/2)$ is a smooth or $p > 2$ times differentiable jump function presented in the previous section, $q_{ij} = q_i - q_j$, b_{ij} is a strictly positive constant such that $b_{ij} \leq \min(R_i, R_j, \varepsilon_2)$ with ε_2 given in (2).

Remark 2. Thanks to properties of the smooth or $p > 2$ differentiable jump function (see Definition 1), the collision function β equals zero when a collision between any robots occurs, i.e. $\|q_{ij}\| = 0$ for any $i \neq j$. The function β equals 1 when all robots are at their desired locations, i.e. $q_i = q_{if}$ for $i = 1,...,N$. The function β is at least twice differentiable with respect to q_{ij}. Hence, the choice of the goal function γ in (9) and the collision function β in (10) with its components given in (11) ensures that the potential function φ in (8) attains the (unique) minimum value of zero when all the robots are at their desired positions, and equals infinity whenever a collision between any robots occurs. Moreover, the potential function φ is at least twice differentiable.

The derivative of φ along the solutions of (1) satisfies

$$\begin{aligned}\dot{\varphi} &= \frac{e^{\gamma}\dot{\gamma}\beta^{\kappa} - e^{\gamma}\kappa\beta^{\kappa-1}\dot{\beta}}{\beta^{2\kappa}} = \frac{e^{\gamma}}{\beta^{\kappa}}\left(\dot{\gamma} - \kappa\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\dot{\beta}_{ij}}{\beta_{ij}}\right) \\ &= \frac{e^{\gamma}}{\beta^{\kappa}}\sum_{i=1}^{N}\Omega_i^T u_i \end{aligned} \tag{12}$$

where we have used $\dot{\beta} = \beta\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\dot{\beta}_{ij}}{\beta_{ij}}$, and

$$\Omega_i = q_i - q_{if} - \kappa\sum_{j\in\mathbb{N}_i}\bar{\beta}'_{ij}q_{ij} \tag{13}$$

with $\bar{\beta}'_{ij} = \frac{\beta'_{ij}}{\beta_{ij}}$, $\beta'_{ij} = \frac{\partial\beta_{ij}}{\partial(\|q_{ij}\|^2/2)}$, and $\mathbb{N}_i$ the set of all robots, denoted by $\mathbb{N}$, in the group except for the robot i. From (12), a bounded control u_i for the robot i is simply designed as follows:

$$u_i = -c\Psi(\Omega_i) \tag{14}$$

where c is a positive constant, and $\Psi(\Omega_i)$ denotes a vector of bounded functions of elements of Ω_i in the sense that $\Psi(\Omega_i) = \left[\psi(\Omega_i^1)\ \psi(\Omega_i^2),...,\psi(\Omega_i^l),\ ....,\psi(\Omega_i^n)\right]^T$ with Ω_i^l the l^{th} element of Ω_i, i.e.

$\Omega_i = [\Omega_i^1\ \Omega_i^2 ... \Omega_i^l ... \Omega_i^n]^T$. The function $\psi(x)$ is a scalar, differentiable and bounded function, and satisfies

$$\begin{array}{ll} 1) & |\psi(x)| \leq M_1, \\ 2) & \psi(x) = 0 \quad \text{if } x = 0, \quad x\psi(x) > 0 \text{ if } x \neq 0, \\ 3) & \psi(-x) = -\psi(x), (x-y)[\psi(x) - \psi(y)] \geq 0, \\ 4) & |\frac{\psi(x)}{x}| \leq M_2, |\frac{\partial \psi(x)}{\partial x}| \leq M_3, \frac{\partial \psi(x)}{\partial x}|_{x=0} = 1 \end{array} \tag{15}$$

for all $x \in \mathbb{R}, y \in \mathbb{R}$, where M_1, M_2, M_3 are strictly positive constants. Some functions that satisfy the above properties are $\arctan(x)$ and $\tanh(x)$. Indeed, the control u_i is bounded, i.e. $\|u_i(t)\| \leq c\sqrt{n}M_1 := \delta, \forall t \geq t_0 \geq 0$.

Remark 3. When Ω_i defined in (13) is substituted into (14), the l^{th} element of the control u_i can be written as $u_i^l = c\psi\big(-(q_i^l - q_{if}^l) - \sum_{j\in\mathbb{N}_i} \bar{\beta}'_{ij} q_{ij}^l\big)$ with q_i^l, q_{if}^l and q_{ij}^l being the l^{th} elements of q_i, q_{if}, and q_{ij}. The argument of ψ consists of two parts: $-(q_i^l - q_{if}^l)$ and $-\sum_{j\in\mathbb{N}_i} \bar{\beta}'_{ij} q_{ij}^l$. The first part, $-(q_i^l - q_{if}^l)$, referred to as the attractive force plays the role of forcing the robot to its desired location. The second part, $-\sum_{j\in\mathbb{N}_i} \bar{\beta}'_{ij} q_{ij}^l$, referred to as the repulsive force, takes care of collision avoidance for the robot i with the other robots. Moreover, the control u_i of the robot i given in (14) depends on only its own state, and the states of other neighbor robots j if these robots are in a sphere, which is centered at the robot and has a radius no greater than R_i because outside this sphere $\bar{\beta}'_{ij} = 0$.

Now substituting (14) into (12) results in

$$\dot{\varphi} = -c\frac{e^\gamma}{\beta^\kappa} \sum_{i=1}^{N} \Omega_i^T \Psi(\Omega_i). \tag{16}$$

Substituting (14) into (1) results in the closed loop system

$$\dot{q}_i = -c\Psi(\Omega_i), i = 1, ..., N. \tag{17}$$

Theorem 1. Assume that at the initial time $t_0 \geq 0$ each robot starts at a different location, and that each robot has a different desired location, i.e. the conditions given in (2) hold, the bounded controls given in (14) guarantee that no collisions between any robots can occur, the solutions of the closed loop system (17) exist and the robots asymptotically approach their desired positions (a set of equilibria) defined by $q_{if}, i = 1, ..., N$.

Proof. See Appendix C.

5 Simulations

We carry out a simulation with $n = 2, N = 10$. The robots are initialized randomly in a circle, which is centered at the origin and has a radius of 1. The desired formation is specified in shape, location and orientation as $q_{if} = R_f[\sin((i-1)2\pi/N); \cos((i-1)2\pi/N)]$, $i = 1, ..., N$ with $R_f = 8$, i.e. the desired formation is a polygon whose vertices are uniformly distributed on a circle, which is centered at the origin and has a radius of R_f. All robots have the same sensing range: $R_i = 2$. The parameters of the p times differential jump functions are $p = 2, b_{ij} = 1, g(y) = y^p$. The function ψ is taken as arctan. The control gains are chosen as $\kappa = 1, c = 2$. Simulation results are plotted in Fig. 1. It is seen that all robots nicely approach their desired locations. Since the robots initialize pretty close to each other, they quickly move away from each other then approach their desired locations, see Sub-figure A) of Fig. 1, where the trajectory of the robot 1 is plotted in the thick line, and robots 1 and 2 are indicated by 1 and 2. For clarity, only the control input $u_1 = [u_{1x}\ u_{1y}]^T$ is plotted in Sub-figure B) of Fig. 1. Noticing that the values of the continuous controls u_{1x} and u_{1y} are in the range $\pm\pi$. Sub-figure C) of Fig.1 plot the functions $\beta_{1j}, j = 2,...,N$. It is seen that these functions are always greater than zero and approach 1. Sub-figure D) of Fig. 1 plots a 'mean-product' distance, $\text{dist}_{\text{all}} = \left(\prod_{(i,j)\in\mathbb{N}} \|q_{ij}\|\right)^{N(N-1)/2}$, see the thick line, and the distances between the robot 1 and other robots in the group. Clearly, no collisions between any robots occurred since dist_{all} is larger than zero for all simulation time. The bottom figure in Fig. 1 plots the functions $\beta_{1j}, j = 2,...,N$. It is noted that all β_{1j} equal 1 when $\|q_{1j}\|$ are larger or equal to 1.

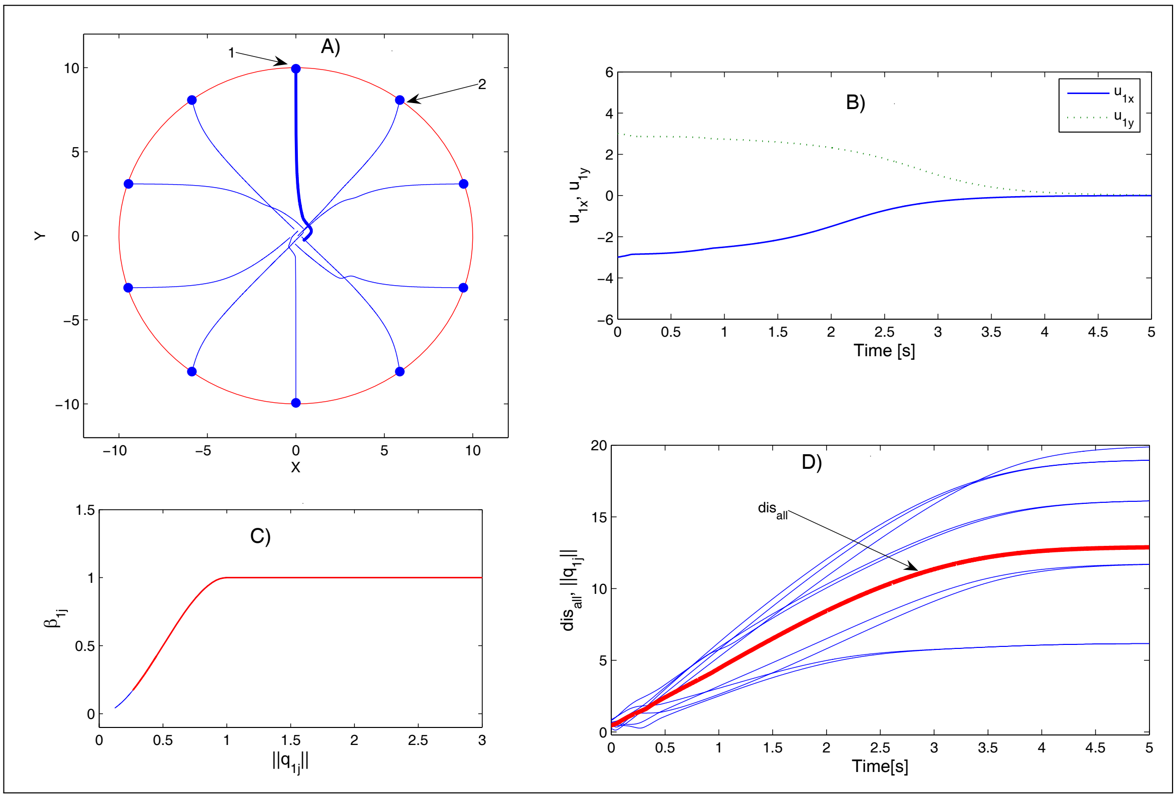

Figure 1: Simulation results.

6 Extension to formation tracking

This section extends the results developed in the previous sections to solve the problem of designing a control input u_i for each robot i that forces the group of N mobile robots whose dynamics are given in (1) to track a moving Desired Formation Graph (DFG), in the sense that the DFG is allowed to move on a common desired trajectory $q_{od}(s)$ with s being the common reference trajectory parameter defined in the fixed coordinate system Π_F, see Fig. 2 and Fig. 3. We consider the DFG whose center moves along the common reference trajectory $q_{od}(s)$. We assume that $q_{od}(s)$ is regular in the sense that it is single valued and its first derivative exists and is bounded. Since the DFG under consideration is only representative, the center does not have to be the "true" center of the DFG but can be any convenient point. When the DFG moves along the trajectory $q_{od}(s)$, the vertex i of DFG generates the reference trajectory $q_{id}(s)$ for the robot i to track. We limit our consideration to two- and three-dimensional (2D and 3D) spaces, which are most common in practice. The control objective is now stated as follows.

Control objective. Assume that at the initial time $t_0 \geq 0$, for all $(i, j) \in \{1,2,...N\}, i \neq j, s \in \mathbb{R}$ there exist strictly positive constants ε_1, ε_2 and ε_3 such that

$$\|q_i(t_0)-q_j(t_0)\| \geq \varepsilon_1,\ \|q_{id}(s)-q_{jd}(s)\| \geq \varepsilon_2, \\ \|\frac{\partial q_{id}(s)}{\partial s}\| \leq \varepsilon_3 \tag{18}$$

Moreover, the robot i can only measure its own state and can only detect the other group members if these members are in a sphere with a radius of R_i larger than a strictly positive constant, and centered at the robot i. Design the control input u_i for each robot i such that

$$\lim_{t\to\infty}(q_i(t)-q_{id}(s))=0, \quad \|q_i(t)-q_j(t)\| \geq \varepsilon_4 \tag{19}$$

for all $(i,j)\in\{1,2,...N\}, i\neq j, s\in\mathbb{R}$, where ε_4 is a positive constant.

Control design. Let us construct $q_{id}(s)$. Let the moving coordinate system Π_M, of which the origin $\widehat{O}$ coincides with the center of the desired formation graph, move along $q_{od}(s)$. Let $\widehat{q}_{id}$ be the coordinate vector of the vertex i of the DFG in the moving coordinate system. We then have

$$\widehat{q}_{id}= J(\bullet)(q_{id}(s)-q_{od}(s)) \tag{20}$$

where $J(\bullet)$ whose elements depend on $\partial q_{od}(s)/\partial s$ and is the rotational (invertible) matrix, which describes the rotation of Π_M with respect to Π_F, and is such that $\delta_1 \leq \|J(\bullet)\| \leq \delta_2$, $\delta_3 \leq \|J(\bullet)^{-1}\| \leq \delta_4$ with $\delta_i, i=1,..,4$ strictly positive constants. Therefore, by specifying $\widehat{q}_{id}$ in Π_M, $q_{id}(s)$ in Π_F for the robot i can be calculated from (20). Similarly, in Π_M the coordinate vector of each robot i satisfies

$$\widehat{q}_i= J(\bullet)(q_i-q_{od}(s)). \tag{21}$$

From (20) and (21), we can see that the first two conditions in (18) imply the following condition

$$\|\widehat{q}_i(t_0)-\widehat{q}_j(t_0)\| \geq \widehat{\varepsilon}_1,\ \|\widehat{q}_{id}(s)-\widehat{q}_{jd}(s)\| \geq \widehat{\varepsilon}_2 \tag{22}$$

where $\widehat{\varepsilon}_1$ and $\widehat{\varepsilon}_2$ are some strictly positive constants. Moreover, the tracking control goal specified in (19) is achieved by designing the control u_i for each robot i such that

$$\lim_{t\to\infty}(\widehat{q}_i(t)-\widehat{q}_{id}(s))=0, \quad \|\widehat{q}_i(t)-\widehat{q}_j(t)\| \geq \widehat{\varepsilon}_4 \tag{23}$$

where $\widehat{\varepsilon}_4$ is a positive constant, and by letting the DFG move along the common reference trajectory via giving $\dot{s}$ some desired value.

Now differentiating both sides of (21) gives

$$\dot{\widehat{q}}_i=\widehat{u}_i \tag{24}$$

where $\widehat{u}_i$ is the new control, and we have chosen the control u_i as

$$u_i=\dot{q}_{od}(s)+J(\bullet)^{-1}(\widehat{u}_i-\dot{J}(\bullet)(q_i-q_{od}(s))). \tag{25}$$

The problem of designing $\widehat{u}_i$ for (24) to achieve (23) under (22) is exactly the same as the control objective in Section 2. Therefore, the control design in Section 4 can be used directly to design a bounded control $\widehat{u}_i$ to achieve the goal (23). After $\widehat{u}_i$ is designed, the actual tracking control u_i is calculated from (25). Let us give the expression of the rotational matrix $J(\bullet)$ in 2D and 3D spaces.

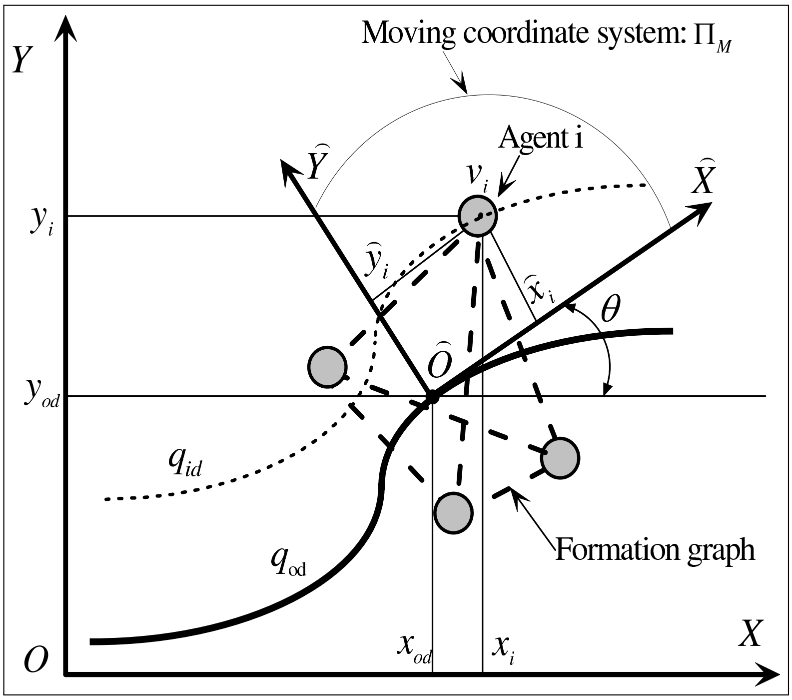

Figure 2: Formation coordinates in 2D.

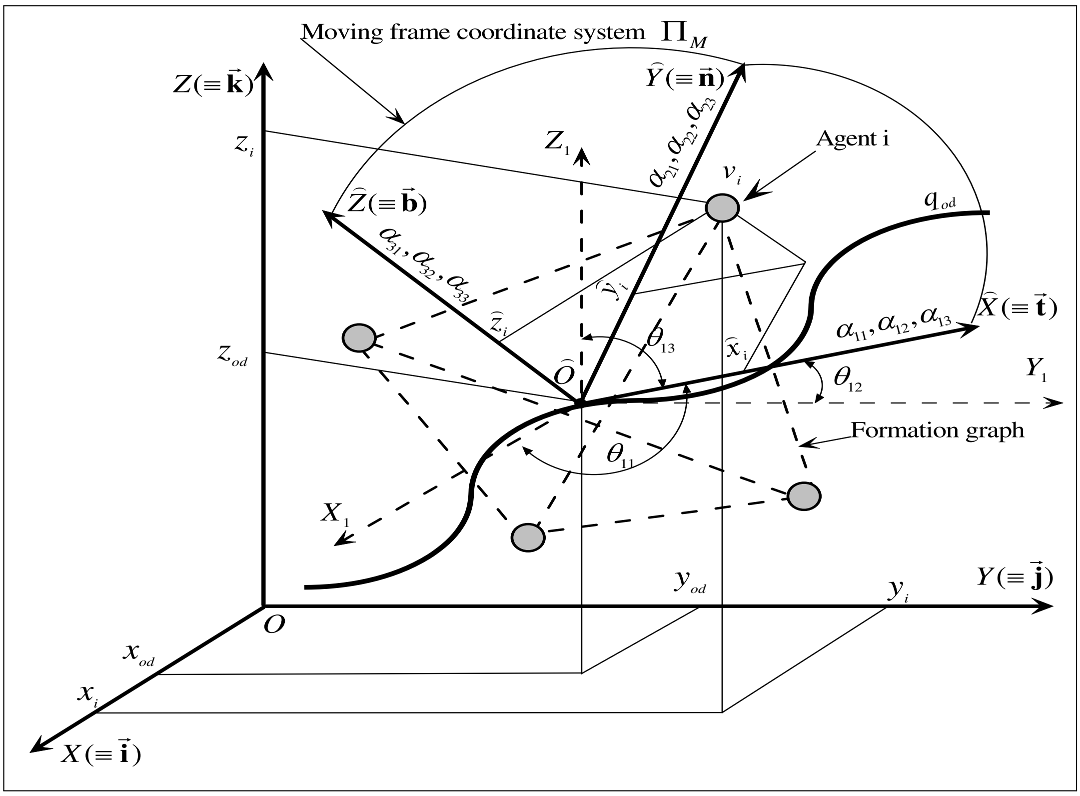

Figure 3: Formation coordinates in 3D.

Two-dimensional space. Consider the moving coordinate frame, $\overset{\frown}{O}\overset{\frown}{X}\overset{\frown}{Y}$ attached to the DFG, as shown in Fig. 2. The origin $\overset{\frown}{O}$ coincides with the center of the graph, and is on the common reference trajectory $q_{od}(s) = [x_{od}(s)\ y_{od}(s)]^T$. The $\overset{\frown}{O}\overset{\frown}{X}$ and $\overset{\frown}{O}\overset{\frown}{Y}$ axes are tangential and perpendicular to the reference trajectory $q_{od}(s)$. Therefore the angle θ between $\overset{\frown}{O}\overset{\frown}{X}$ and OX is calculated as $\theta = \arctan(y_d^{'}/x_d^{'})$, where $\bullet' \triangleq \partial \bullet / \partial s$. Hence the rotational matrix $J(\bullet)$ is given by $J(\bullet) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$, which is indeed invertible for all $\theta \in \mathbb{R}$, and $\|J(\bullet)\| = 1$.

Three-dimensional space. Consider the moving coordinate frame, $\overset{\frown}{O}\overset{\frown}{X}\overset{\frown}{Y}\overset{\frown}{Z}$, attached to the DFG as shown in Fig. 3. The coordinate frame $\overset{\frown}{O}\,X_1Y_1Z_1$ is parallel to $OXYZ$. The origin $\overset{\frown}{O}$ coincides with the center of the graph, and is on the reference trajectory $q_{od}(s) = [x_{od}(s)\ y_{od}(s)\ z_{od}(s)]^T$. The $\overset{\frown}{O}\overset{\frown}{X}$, $\overset{\frown}{O}\overset{\frown}{Y}$ and $\overset{\frown}{O}\overset{\frown}{Z}$ axes coincide with the unit tangent vector $\vec{t}$, the unit principal vector $\vec{n}$, and the unit binormal vector $\vec{b}$ of the trajectory $q_{od}(s)$ at the point $\overset{\frown}{O}$. These unit vectors form a positively oriented triple of vectors called the moving triad, and are given by $\vec{t} = \vec{q}^{\,'}_{od}/\|\vec{q}^{\,'}_{od}\|$, $\vec{n} = \vec{t}^{\,'}/\|\vec{t}^{\,'}\|$, $\vec{b} = \vec{t} \times \vec{n}$, where $\times$ stands for the vector cross product operation, $(\vec{i},\ \vec{j},\ \vec{k})$ are the unit vectors of the $OXYZ$ coordinate frame. Let $(\xi_{i1},\ \xi_{i2}, \xi_{i3}), i = 1,2,3$ be the directional cosines of $\overset{\frown}{O}\overset{\frown}{X}$, $\overset{\frown}{O}\overset{\frown}{Y}$ and $\overset{\frown}{O}\overset{\frown}{Z}$ with respect to the fixed axes OX, OY and OZ, respectively. This notation means that if we denote $(\theta_{i1},\ \theta_{i2}, \theta_{i3}), i = 1,2,3$ as the angles between the axes $\overset{\frown}{O}\overset{\frown}{X}$, $\overset{\frown}{O}\overset{\frown}{Y}$ and $\overset{\frown}{O}\overset{\frown}{Z}$, and the axes OX, OY and OZ (see Fig. 3 for representation of θ_{11}, θ_{12} and θ_{13}), we have $\xi_{ij} = \cos(\theta_{ij})$, $\forall i, j \in \{1,2,3\}$. The rotational matrix $J(\bullet)$ is given by $J(\bullet) = \begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} \\ \xi_{21} & \xi_{22} & \xi_{23} \\ \xi_{31} & \xi_{32} & \xi_{33} \end{bmatrix}$. It is shown in [20] that the determinant of $J(\bullet)$ is equal to 1, i.e. $J(\bullet)$ is globally invertible, and $\|J(\bullet)\| = 1$.

7 Summary and Conclusions

This paper has presented a method to design bounded and continuous (even smooth) controllers to force a group of mobile robots with limited sensing ranges to achieve a desired formation while avoiding collisions among themselves. The control development was based on construction of new potential functions, and guaranteed that all critical points, except for the desired points in formation, are unstable points. These potential functions with embedded smooth or p times differential jump functions are attractive parts of the paper since they do not require the use of switching control theory despite the robot limited sensing ranges. These functions can certainly be modified to solve other cooperative control problems such as flocking and consensus of mobile robots. The problem of formation tracking was also addressed. Future work is to extend the proposed techniques in this paper and those for centralized formation control of nonholonomic mobile robots in [10] to design a decentralized formation control system for a group of nonholonomic mobile robots.

8 Appendix A: Proof of Lemma 1

We need to verify that the function $h(x,a,b)$ given in (5) satisfy all properties defined in (4). Property 1) holds because by (6) for all $0 \le x \le a$, we have $\int_a^x f(\tau - a) f(b - \tau) d\tau = 0$. Property 2) holds since by (6) we have $\int_a^x f(\tau - a) f(b - \tau) d\tau = \int_a^b f(\tau - a) f(b - \tau) d\tau$ for all $x \ge b$. To prove Property 3), we first note from Property a) of the function $g(y)$ given in (7) that $\int_a^x f(\tau - a) f(b - \tau) d\tau > 0$ for all $a < x < b$. Therefore, $0 < \frac{\int_a^x f(\tau-a)f(b-\tau)d\tau}{\int_a^b f(\tau-a)f(b-\tau)d\tau} < 1$, which means that Property 3) of the function $h(x,a,b)$ holds. To prove Property 4), we just need to show that $f(y)$ is $p-1$ times differentiable. We first note that $f(y)$ is

p times differentiable except at $y = 0$. Hence, we only need to verify that $f^{(k)}(0) = \frac{\partial^k f(y)}{\partial y^k}\Big|_{y=0} = 0$ for any positive integer $k < p$. Clearly, $\lim_{y\to 0^-} f^{(k)}(y) = 0$ since $f(y) = 0,\ \forall y \le 0$. On the other hand, since $f(y) = g(y), y > 0$, from Property b) of the function $g(y)$ we have $\lim_{y\to 0^+} f^{(k)}(y) = \lim_{y\to 0^+} g^{(k)}(y) = 0$, where $g^{(k)} = \frac{\partial^k g(y)}{\partial y^k}$. Since both left- and right-hand limits are equal to 0, we have $f^{(k)}(0) = 0$. Hence Property 4) holds. □

9 Appendix B: Proof of Corollary 1

We first note that Property a) of the function $g(y)$ in (7) can be proven without a difficulty. We focus on proof of Property b). We note that $g^{(k)}(y) = \frac{\partial^k g(y)}{\partial y^k} = Q_k(\frac{1}{y})e^{-\frac{1}{y}}$ where $Q_k(\frac{1}{y})$ is a polynomial function of $\frac{1}{y}$, and k is any positive integer. We will prove Property b) of the function $g(y)$ in (7) by induction. It is clear that $\lim_{y\to 0^+} g^{(1)}(y) = \lim_{y\to 0^+} \frac{g(y)-g(0)}{y-0} = \lim_{y\to 0^+} \frac{e^{-\frac{1}{y}}}{y} = \lim_{\xi\to\infty} \frac{1}{e^{\xi}} = 0$ where $\xi = \frac{1}{y}$ and we have used l'Hopital's rule. This means that Property b) of the function $g(y)$ holds for $k = 1$. Assuming that $\lim_{y\to 0^+} g^{(k)}(y) = 0$, we now compute $\lim_{y\to 0^+} g^{(k+1)}(y)$ as follows:

$$\lim_{y\to 0^+} g^{(k+1)}(y) = \lim_{y\to 0^+} \frac{g^{(k)}(y) - g^{(k)}(0)}{y-0} =$$
$$\lim_{y\to 0^+} \tilde{Q}_k(\frac{1}{y})e^{-\frac{1}{y}} = \lim_{\xi\to\infty} \frac{\tilde{Q}_k(\xi)}{e^{\xi}} = 0 \tag{26}$$

where l'Hopital's rule has been used, $\xi = \frac{1}{y}$, and $\tilde{Q}_k(\xi) = \xi Q_k(\xi)$ is another polynomial of ξ. Therefore we have proved that $\lim_{y\to 0^+} g^{(k)}(y) = 0$ for any k, which means Property b) of the function $g(y)$ holds for any positive integer p, i.e. p can be equal to infinity. By Definition 1, the function $h(x,a,b)$ is a smooth jump function.□

10 Appendix C: Proof of Theorem 1

We prove Theorem 1 in two steps. In the first step, we prove that no collisions between the robots can occur and that the robots asymptotically approach their target points or some critical points. In the second step, we investigate stability of the closed loop system (17) at these points, we linearize the closed loop system at these points. We then prove that only desired target points are unique asymptotic stable and that other critical points are unstable.

Step 1. Proof of no collision and existence of solutions. From (16) and properties of the function ψ , see (15), we have $\dot{\varphi} \le 0$, which implies that $\varphi(t) \le \varphi(t_0), \forall t \ge t_0$. With definition of the function φ in (8) and its components in (9) and (10), we have

$$\frac{e^{0.5\sum_{i=1}^{N}\|q_i(t)-q_{if}\|^2}}{\prod h(0, b_{ij}^2/2, \|q_{ij}(t)\|^2/2)} \le \frac{e^{0.5\sum_{i=1}^{N}\|q_i(t_0)-q_{if}\|^2}}{\prod h(0, b_{ij}^2/2, \|q_{ij}(t_0)\|^2/2)} \tag{27}$$

for all $t \ge t_0 \ge 0, i = 1,...,N-1, j = i+1,...N$. From the first condition in (2) and properties of the jump function $h(0, b_{ij}^2/2, \|q_{ij}\|^2/2)$, we have $\prod h(0, b_{ij}^2/2, \|q_{ij}(t_0)\|^2/2)$ is larger than a strictly positive constant. Therefore the right hand side of (27) is bounded by a positive constant depending on the initial conditions. Boundedness of the right hand side of (27) implies that the left hand side of (27) must be also bounded. As a result, $\prod h(0, b_{ij}^2/2, \|q_{ij}(t)\|^2/2)$ must be larger than some positive constant depending on the initial conditions for all $t \geq t_0 \geq 0$. From properties of $h(0, b_{ij}^2/2, \|q_{ij}\|^2/2)$, $\|q_{ij}(t)\|$ must be larger than some positive constant depending on the initial conditions denoted by ε_3, i.e. there are no collisions for all $t \geq t_0 \geq 0, i = 1, ..., N-1, j = i+1, ...N$. Boundedness of the left hand side of (27) also implies that of $\|q_{ij}(t)\|$ and $\|q_i(t)\|$ for all $t \geq t_0 \geq 0$, i.e. the solutions of the closed loop system (17) exist.

+*Equilibrium points*. Since we have already proved that there are no collisions between any robots and that the solutions of the closed loop system (17) exist, an application of Theorem 8.4 in [19] to (16) yields

$$\lim_{t\to\infty} \Omega_i^T(t)\Psi(\Omega_i(t)) = 0, \forall i = 1, 2, ..., N. \tag{28}$$

Thanks to Property 2) of the function ψ, see (15), the limit equation (28) implies that

$$\lim_{t\to\infty} \Omega_i(t) = \lim_{t\to\infty} \Big[q_i(t) - q_{if} - \kappa \sum_{j\in\mathbb{N}_i} \bar{\beta}'_{ij}(t) q_{ij}(t) \Big] = 0 \tag{29}$$

for all $i = 1, 2, ..., N$. The limit equation (29) implies that the state $q(t) = [q_1^T(t)\ q_2^T(t), ..., q_N^T(t)]^T$ converges to the manifold $\mathbb{M}$ of (17) contained in $\mathbb{E} = \{q \in \mathbb{R}^{n\times N}|_{\Omega=0}\}$ with $\Omega = [\Omega_1^T\ \Omega_2^T, ..., \Omega_N^T]^T$, i.e. on the surface where $\dot{\varphi} = 0$. This surface is continuous because we have already proved that $\|q_{ij}\| > 0, \forall (i, j) \in \{1, 2, ..., N\}, i \neq j$, i.e. $\bar{\beta}'_{ij}$ is continuous. As the time t goes to infinity, it can be verified that one solution of (29) is $q_f = [q_{1f}^T\ q_{2f}^T, ..., q_{Nf}^T]^T$ since $\beta'_{ij}|_{\|q_{ij}\|=\|q_{ijf}\|} = 0 \Rightarrow \bar{\beta}'_{ij}|_{\|q_{ij}\|=\|q_{ijf}\|} = 0$ (we have chosen $b_{ij} \leq \min(R_i, R_j, \varepsilon_2)$, see (11)), and other solutions are denoted by $q_c = [q_{1c}^T\ q_{2c}^T, ..., q_{Nc}^T]^T$. It is noted that some elements of q_c can be equal to that of q_f. However, for simplicity we abuse the notation, i.e. we still denote that vector as q_c. Indeed, the vector q_c is such that

$$\Omega_i|_{q=q_c} = \Big[q_i - q_{if} - \kappa \sum_{j\in\mathbb{N}_i} \bar{\beta}'_{ij} q_{ij} \Big]\Big|_{q=q_c} = 0 \tag{30}$$

for all $i = 1, ..., N$. Next, we will show that q_f is stable and q_c is unstable, by linearizing (17) at these points.

+*Properties of equilibrium points*. The closed loop system (17) can be written in a vector form as $\dot{q} = -c\Psi_q(q, q_f)$, and $\Psi_q(q, q_f) = [\Psi^T(\Omega_1),\ \ ..., \Psi^T(\Omega_i), ..., \Psi^T(\Omega_N)]^T$. Therefore, near an equilibrium point q_o, which can be either q_f or q_c, we have

$$\dot{q} = -c\, \partial\Psi_q(q, q_f)/\partial q\big|_{q=q_o}(q - q_o) \tag{31}$$

where

$$\frac{\partial\Psi_q(q, q_f)}{\partial q} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \cdots & \cdots & \Delta_{1N} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \Delta_{i1} & \cdots & \Delta_{ii} & \cdots & \Delta_{iN} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta_{N1} & \cdots & \cdots & \cdots & \Delta_{NN} \end{bmatrix} \tag{32}$$

with $\Delta_{ij} = \frac{\partial\Psi(\Omega_i)}{\partial\Omega_i}\frac{\partial\Omega_i}{\partial q_i}, (i, j) \in \mathbb{N}$, where $\mathbb{N}$ denotes the set of all robots in the group. A simple calculation shows that for all $i = 1, ..., N, j \in \mathbb{N}_i, j \neq i$

$$\begin{aligned} \frac{\partial\Omega_i}{\partial q_i} &= \Big(1 - \kappa \sum_{i\in\mathbb{N}_i} \bar{\beta}'_{ij}\Big) I_n - \kappa \sum_{j\in\mathbb{N}_i} \bar{\beta}''_{ij} q_{ij} q_{ij}^T, \\ \frac{\partial\Omega_i}{\partial q_j} &= \kappa\bar{\beta}'_{ij} I_{n\times n} + \kappa\bar{\beta}''_{ij} q_{ij} q_{ij}^T \end{aligned} \tag{33}$$

where $\bar{\beta}''_{ij} = \frac{\partial \bar{\beta}'_{ij}}{\partial(\|q_{ij}\|^2/2)}$. Let $\mathbb{N}^*$ be the set of the robots such that if the robots i and j belong to the set $\mathbb{N}^*$ then $\|q_{ij}\| < b_{ij}$. Next we will show that q_f is asymptotically stable and that q_c is unstable.

Step 2. Behavior of equilibrium points. Evaluating (33) at $q = q_f$ gives

$$\frac{\partial \Psi(\Omega_i)}{\partial \Omega_i}\Big|_{q=q_f} = I_n, \frac{\partial \Omega_i}{\partial q_i}\Big|_{q=q_f} = I_n, \frac{\partial \Omega_i}{\partial q_j}\Big|_{q=q_f} = 0 \tag{34}$$

where we have used $\bar{\beta}'_{ij}|_{q_{ij}=q_{ijf}} = 0$ and $\bar{\beta}''_{ij}|_{q_{ij}=q_{ijf}} = 0$ since $\beta'_{ij}|_{q_{ij}=q_{ijf}} = 0$ and $\beta''_{ij}|_{q_{ij}=q_{ijf}} = 0$ (we have chosen $b_{ij} \le \min(R_i, R_j, \varepsilon_2)$, see (11)). We consider the Lyapunov function candidate $V_f = 0.5\|q - q_f\|^2$ whose derivative along the solutions of the linearized closed loop system (31) with q_o replaced by q_f, and using (34) satisfies $\dot{V}_f = -c\sum_{i=1}^{N} \|q_i - q_{if}\|^2 = -2cV_f$, which implies that q_f is asymptotically stable.

- *Proof of q_c being unstable.* Now evaluating (33) at $q = q_c$ give that for all $i = 1, ..., N, i \neq j$:

$$\frac{\partial \Psi(\Omega_i)}{\partial \Omega_i}\Big|_{q=q_c} = I_n, \ \frac{\partial \Omega_i}{\partial q_i}\Big|_{q=q_c} = \big(1 - \kappa \sum_{j\in\mathbb{N}_i} \bar{\beta}'_{ijc}\big) I_n - \kappa \sum_{j\in\mathbb{N}_i} \bar{\beta}''_{ijc} q_{ijc} q_{ijc}^T, \ \frac{\partial \Omega_i}{\partial q_j}\Big|_{q=q_c} = \kappa \bar{\beta}'_{ijc} + \kappa \bar{\beta}''_{ijc} q_{ijc} q_{ijc}^T \tag{35}$$

where $q_{ijc} = q_{ic} - q_{jc}$, $\bar{\beta}'_{ijc} = \bar{\beta}'_{ij}|_{q_{ij}=q_{ijc}}$ and $\bar{\beta}''_{ijc} = \bar{\beta}''_{ij}|_{q_{ij}=q_{ijc}}$. Since the related collision avoidance functions β_{ij}, (hence $\bar{\beta}'_{ij}$ and $\bar{\beta}''_{ij}$), are specified in terms of relative distances between robots and it is extremely difficult to obtain q_c explicitly by solving (30), it is very difficult to use the Lyapunov function candidate $V_c = 0.5\|q - q_c\|$ to investigate stability of (31) at q_c. Therefore, we consider the following Lyapunov function candidate

$$\bar{V}_c = 0.5\|\bar{q} - \bar{q}_c\|^2 \tag{36}$$

where $\bar{q} = [q_{12}^T, q_{13}^T, ...q_{1N}^T, q_{23}^T, ..., q_{2N}^T, ..., q_{N-1,N}^T]^T$ and $\bar{q}_c = [q_{12c}^T, q_{13c}^T, ...q_{1Nc}^T, q_{23c}^T, ..., q_{2Nc}^T, ..., q_{N-1,Nc}^T]^T$. Differentiating both sides of (36) along the solution of (31) with q_o replaced by q_c gives

$$\dot{\bar{V}}_c = -c \sum_{(i,j)\in\mathbb{N}\setminus\mathbb{N}^*} \|q_{ij} - q_{ijc}\|^2 - c \sum_{(i,j)\in\mathbb{N}^*} (1 - \kappa N \bar{\beta}'_{ijc}) \times \|q_{ij} - q_{ijc}\|^2 + \kappa c N \sum_{(i,j)\in\mathbb{N}^*} \bar{\beta}''_{ijc} \big(q_{ijc}^T (q_{ij} - q_{ijc})\big)^2 \tag{37}$$

where $i \neq j$ and (35) has been used. To investigate stability properties of $\bar{q}_c$ based on (37), we will use (30). Define $\Omega_{ijc} = \Omega_{ic} - \Omega_{jc}$, $\forall (i,j) \in \{1, ..., N\}$, $i \neq j$ where $\Omega_{ic} = \Omega_i|_{q=q_c} = 0$, see (30). Therefore $\Omega_{ijc} = 0$. Hence $\sum_{(i,j)\in\mathbb{N}^*} q_{ijc}^T \Omega_{ijc} = 0, i \neq j$, which by using (30) is expanded to

$$\sum_{(i,j)\in\mathbb{N}^*} \big(q_{ijc}^T (q_{ijc} - q_{ijf}) - \kappa N \bar{\beta}'_{ijc} q_{ijc}^T q_{ijc}\big) = 0$$
$$\Rightarrow \sum_{(i,j)\in\mathbb{N}^*} (1 - \kappa N \bar{\beta}'_{ijc}) q_{ijc}^T q_{ijc} = \sum_{(i,j)\in\mathbb{N}^*} q_{ijc}^T q_{ijf} \tag{38}$$

where $i \neq j$. The sum $\sum_{(i,j)\in\mathbb{N}^*} q_{ijc}^T q_{ijf}$ is strictly negative since at the point where $q_{ij} = q_{ijf}$, $\forall (i,j) \in \mathbb{N}^*, i \neq j$ (the point F in Fig. 4) all attractive and repulsive forces are equal to zero while at the point where $q_{ij} = q_{ijc}$ $\forall (i,j) \in \mathbb{N}^*, i \neq j$ (the point C in Fig. 4) the sum of attractive and repulsive forces are equal to zero (but attractive and repulsive forces are nonzero). Therefore the point where $q_{ij} = 0$, $\forall (i,j) \in \mathbb{N}^*, i \neq j$ (the point O in Fig. 4) must locate between the points F and C for all $(i,j) \in \mathbb{N}^*, i \neq j$. That is

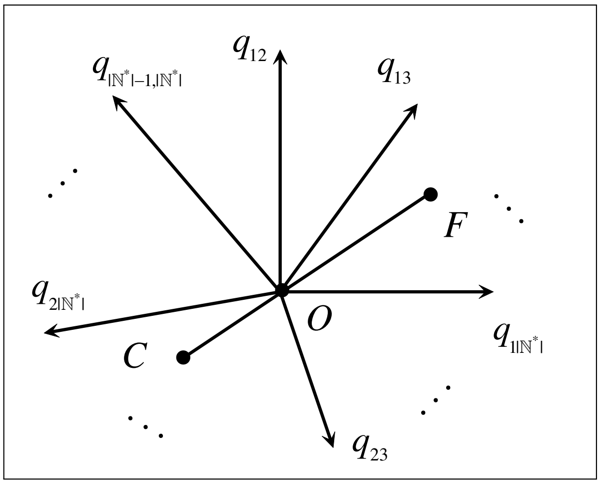

Figure 4: Illustration of equilibrium points.

there exists a strictly positive constant b such that $\sum_{(i,j)\in\mathbb{N}^*} q_{ijc}^T q_{ijf} < -b$, which is substituted into (38) to yield

$$\sum_{(i,j)\in\mathbb{N}^*} (1-\kappa N\bar{\beta}'_{ijc}) q_{ijc}^T q_{ijc} < -b, i \neq j. \tag{39}$$

Since $q_{ijc}^T q_{ijc} > 0, \forall (i,j) \in \mathbb{N}^*, i \neq j$, there exists a nonempty set $\mathbb{N}^{**} \subset \mathbb{N}^*$ such that for all $(i,j) \in \mathbb{N}^{**}, i \neq j$, $(1-\kappa N\bar{\beta}'_{ijc})$ is strictly negative, i.e. there exists a strictly positive constant b^{**} such that $(1-\kappa N\bar{\beta}'_{ijc}) < -b^{**}$, $\forall (i,j) \in \mathbb{N}^{**}, i \neq j$. We now write (37) as

$$\begin{aligned}\dot{\bar{V}}_c = -c\Bigg[&\sum_{(i,j)\in\mathbb{N}\setminus\mathbb{N}^*} \|q_{ij}-q_{ijc}\|^2 + \sum_{(i,j)\in\mathbb{N}^*\setminus\mathbb{N}^{**}} (1-\kappa N\bar{\beta}'_{ijc}) \times \\ &\|q_{ij}-q_{ijc}\|^2 - \kappa N \sum_{(i,j)\in\mathbb{N}^*} \bar{\beta}''_{ijc}\big(q_{ijc}^T(q_{ij}-q_{ijc})\big)^2\Bigg] - \\ &c\sum_{(i,j)\in\mathbb{N}^{**}} (1-\kappa N\bar{\beta}'_{ijc})\|q_{ij}-q_{ijc}\|^2\end{aligned} \tag{40}$$

where $i \neq j$. We now define a subspace such that $q_{ij}-q_{ijc}=0$, $\forall (i,j) \in \mathbb{N}\setminus\mathbb{N}^{**}$ and $q_{ijc}^T(q_{ij}-q_{ijc}) = 0$, $\forall (i,j) \in \mathbb{N}^*, i \neq j$. In this subspace, we have

$$\begin{aligned}\bar{V}_c &= 0.5\sum_{(i,j)\in\mathbb{N}^{**}} \|q_{ij}-q_{ijc}\|^2, \\ \dot{\bar{V}}_c &= -c\sum_{(i,j)\in\mathbb{N}^{**}} (1-\kappa N\bar{\beta}'_{ijc})\|q_{ij}-q_{ijc}\|^2 \geq 2cb^{**}\bar{V}_c\end{aligned} \tag{41}$$

where we have used $(1-\kappa N\bar{\beta}'_{ijc}) < -b^{**}$, $\forall (i,j) \in \mathbb{N}^{**}, i \neq j$. Clearly (41) implies that

$$\sum_{(i,j)\in\mathbb{N}^{**}} \|q_{ij}(t)-q_{ijc}\| \geq \sum_{(i,j)\in\mathbb{N}^{**}} \|q_{ij}(t_0)-q_{ijc}\| e^{cb^{**}(t-t_0)} \tag{42}$$

for all $i \neq j$, $t \geq t_0 \geq 0$. Now assume that q_c is a stable equilibrium point of the closed loop system (17), i.e. $\lim_{t\to\infty}\|q_i(t)-q_{ic}\| = d_i, \forall i \in \mathbb{N}$ with d_i a nonnegative constant. Note that $\mathbb{N}^{**} \subset \mathbb{N}$, we have $\lim_{t\to\infty}\|q_i(t)-q_{ic}\| = d_i, \forall i \in \mathbb{N}^{**}$, which implies that $\lim_{t\to\infty}\sum_{(i,j)\in\mathbb{N}^{**}}\|q_{ij}(t)-q_{ijc}\| = d^{**}, \forall (i,j) \in \mathbb{N}^{**}, i \neq j$ with d^{**} a nonnegative constant, since $q_{ij} = q_i - q_j$ and $q_{ijc} = q_{ic} - q_{jc}$. This contradicts (42) for the case $\sum_{(i,j)\in\mathbb{N}^{**}}\|q_{ij}(t_0)-q_{ijc}\| \neq 0$, since the right hand side of (42) is divergent (so does the left hand side). For the case $\sum_{(i,j)\in\mathbb{N}^{**}} \|q_{ij}(t_0) - q_{ijc}\| = 0$, there would be no contradiction. However this case is never observed in practice since the ever-present physical noise would cause $\|q_{ij}(t^*) - q_{ijc}\|$ for some $(i, j) \in \mathbb{N}^{**}, i \neq j$ to be different from 0 at the time $t^* \geq t_0$. We now write (42) as

$$\sum_{(i,j)\in\mathbb{N}^{**}} \|q_{ij}(t) - q_{ijc}\| \geq \sum_{(i,j)\in\mathbb{N}^{**}} \|q_{ij}(t^*) - q_{ijc}\| e^{cb^{**}(t-t^*)} \tag{43}$$

for all $i \neq j$, $t \geq t^* \geq t_0 \geq 0$. Since $\sum_{(i,j)\in\mathbb{N}^{**}} \|q_{ij}(t^*) - q_{ijc}\| \neq 0$, the right hand side of (43) is divergent (so does the left hand side). This contradicts $\lim_{t\to\infty} \sum_{(i,j)\in\mathbb{N}^{**}} \|q_{ij}(t) - q_{ijc}\| = d^{**}, \forall (i, j) \in \mathbb{N}^{**}, i \neq j$. Therefore q_c must be an unstable equilibrium point of the closed loop system (17). Proof of Theorem 1 is completed.

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Received: December 26, 2006