

Speed Control of a Permanent Magnet Synchronous Machine Powered by an Inverter Voltage Moment Approach

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Abstract: In this paper, the method of moments is presented in order to synthesize controllers for current and speed of a Magnet Permanent Synchronous Machine (PMSM). The controller's dynamics are chosen in respect with the time domain constraints, which have equality relationships between the moments of the closed-loop system and those of the reference model. The Linear Matrix Inequality (LMI) formalism is used for the controller synthesis. The proposed controller is applied for the speed tracking of a PMSM to illustrate the performances of the method.

Keywords: Times Specifications, method of moments, Reference Model, LMI, PMSM.

1 Introduction

New industrial applications require variable speed drives with high dynamic performance. In recent years several techniques of control have been developed, allowing PMSM with variable speed to achieve these performances. However, vector control, which allows decoupling between control variables remains the most widely used [1–3]. The main advantages of this configuration are that the regulatory cascade is a very industrial responded [4, 5]. It gives high dynamic performance for a wide range of applications. The complexity of the dynamic model of the permanent magnet synchronous variable speed and the presence of external disturbances and parametric variations limit the performance of the control law [6]. Thus, our main contribution in this paper consist in introducing a time constraint with the help of a reference model, simple to operate and which characterizes the dynamics of the closed loop system. This time constraint is formulated by the equality of the first time moments of the closed-loop system and those of the reference model [7–11].

Practically, a first LMI is used to insure closed-loop stability, a second one permits, by minimizing the norm 2 of time moments cost, to identify time specifications (overshoot, response time ...).

Without loss of generality, the methodology has been restricted in this paper to the case of square, invertible, MIMO (multi inputs, multi outputs) systems.

The paper is organised as follows: in section 2, we define the state space representations of the systems and those of the controller; moreover, the problem of the design of a controller verifying stability and time performances is stated. In section 3, we propose a solution of this problem, owing to the demonstration of a specific theorem. Experimental results applied to a permanent magnet synchronous machine (PMSM) are presented in section 4.

2 Problem statement

Consider a Linear Time Invariant (LTI) square invertible system $H(s)$ with m inputs and m outputs:

$$\begin{cases} \dot{x}_{sys}(t) = A_{sys}x_c(t) + B_{sys}u(t) \\ y(t) = C_{sys}x(t) \end{cases} \quad (1)$$

The controller is modeled in the state space by:

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c e(t) \\ u(t) = C_c x_c(t) + D_c e(t) \end{cases} \quad (2)$$

with

$$e(t) = r(t) - y(t) \quad (3)$$

where: $x_{sys}(t) \in \mathbb{R}^l$ is the state vector of the system, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^m$ is the output, $r(t) \in \mathbb{R}^m$ is the reference of closed-loop system, Figure 1. Matrices A_{sys} , B_{sys} , C_{sys} are supposed to be known with appropriate dimensions. The controller model matrices A_c , B_c , C_c and D_c are to be computed.

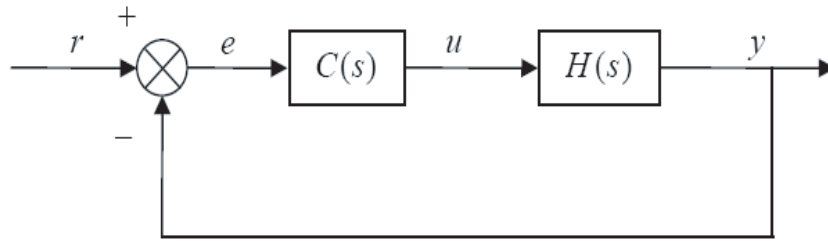


Figure 1: Closed-Loop System

The synthesis of controller requires the construction of the augmented model which regroups the system and the controller model. Consider the augmented state vector $z(t)$ defined as:

$$z(t) = \begin{bmatrix} x_{sys}(t) \\ x_c(t) \end{bmatrix}. \quad (4)$$

Then, the closed-loop system, Figure 1, can be represented by the following model:

$$\begin{cases} \dot{z}(t) = A_{bf}z(t) + B_{bf}r(t) \\ y(t) = C_{bf}z(t) \end{cases} \quad (5)$$

with:

$$A_{bf} = \begin{bmatrix} A_{sys} - B_{sys}D_cC_{sys} & B_{sys}C_c \\ -B_cC_{sys} & A_c \end{bmatrix}; \quad B_{bf} = \begin{bmatrix} B_{sys}D_c \\ B_c \end{bmatrix}; \quad C_{bf} = [C_{sys} \quad 0] \quad (6)$$

Let's note that the closed-loop stability is ensured if and only if there exist a positive definite matrix P with $P = P^T$, such as the LMI

$$PA_{bf} + A_{bf}^T P < 0 \quad (7)$$

is satisfied.

To facilitate the study, our work is restricted to the case where the controller $C(s)$ is to be synthesised. The controller design can be considered as a static feedback gain K for the augmented system.

The controller synthesis is ensured by the fact that matrices A_c and B_c are defined a priori and only the matrices C_c and D_c are to be computed. For this purpose, we consider the following hypothesis:

The matrix A_{bf} of augmented system can be written as follows:

$$A_{bf} = \tilde{A} + \tilde{B}K\tilde{C} \quad (8)$$

where:

$$\tilde{A} = \begin{bmatrix} A_{sys} & 0 \\ -B_c C_{sys} & A_c \end{bmatrix}; \quad \tilde{B} = \begin{bmatrix} B_{sys} \\ 0 \end{bmatrix}; \quad \tilde{C} = \begin{bmatrix} -C_{sys} & 0 \\ 0 & I \end{bmatrix} \quad (9)$$

It is to notice that the gain K is defined as:

$$K = [D_c \quad C_c] \quad (10)$$

The extended system is then characterized by $(\tilde{A}, \tilde{B}, \tilde{C})$. The objective of this work is to ensure closed-loop stability and to specify transient performances which are characterized by a reference model $T_{ref}(S)$. The Dynamics of the reference model are determined by the use of the time moments approach. In this case, the time moments of transfer $W(S)$, which are represented in state space by (A_W, B_W, C_W, D_W) (see [4]), are given by the following relations:

$$\begin{aligned} M_{W,0} &= -C_W A_W^{-1} B_W + D_W \\ M_{W,j} &= (-1)^{j+1} C_W A_W^{-(j+1)} B_W + D_W \end{aligned} \quad (11)$$

$$(j = 1 \dots \infty)$$

$M_{W,0}$: is the 0^{th} order moment.

The exposing j refers to the order of moments. For applying the proposed moment approach, we will define briefly the first three time moments.

The 0^{th} order moment M_0 is equal to the static gain of the system and represents the area of its impulse response. The first order moment M_1 characterizes the average time t_m , the same as the average value of a probability density: $tm = \frac{M_1(g)}{M_0(g)}$

$g(t)$ is the impulse response of the system, represented by Figure 2.

For a first order system, we have $M_1 = \tau M_0$ where represents the time constant of the system. More generally, M_1 is used to characterize the time response of the considered system.

The second order moment M_2 , characterizes the variance σ (or dispersion $\Delta\tau$) of the impulse response around its mean time tm .

In practice, the time moments M_0, M_1, M_2 carry sufficient information on the impulse response: static gain, mean time and dispersion.

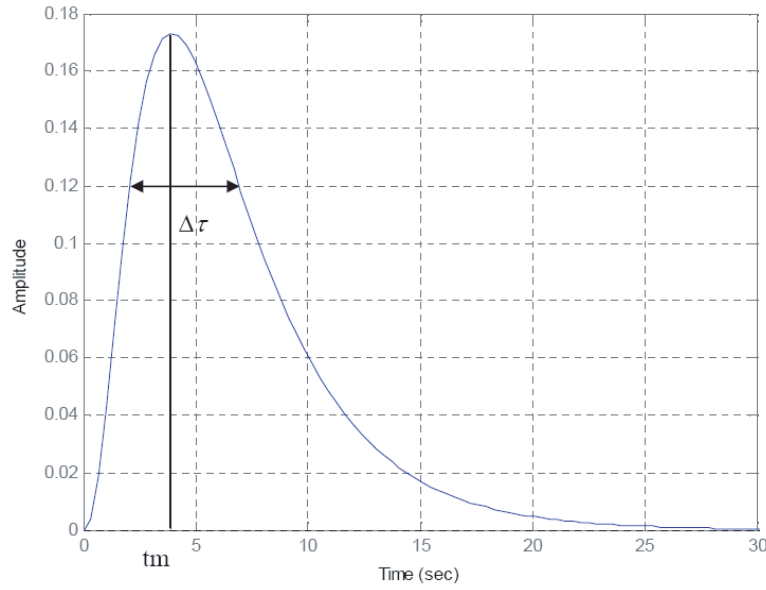


Figure 2: Characterization of an impulse response

The controller has to be designed such that the closed-loop transfer $T(S)$ is equal to the reference model $T_{ref}(s)$.

$$(I + H(s)C(s))^{-1}H(s)C(S) = T_{ref}(s) \quad (12)$$

or

$$C(s)(I - T_{ref}(s)) = H^{-1}(s)T_{ref}(s) \quad (13)$$

It is known that such control law $u(t)$ is in the form: $u(t) = Ke(t)$ Using state space representation of $C(s)$, we can write:

$$u(s) = [C_c(sI - A_c)^{-1}B_c + D_c]e(s) \quad (14)$$

While referring to considered hypothesis given by relation (9), equation (13) can be written as:

$$K \begin{bmatrix} I \\ (sI - A_c)^{-1}B_c \end{bmatrix} (I - T_{ref}(s)) = H^{-1}(s)T_{ref}(s) \quad (15)$$

If we set:

$$F(s) = H^{-1}(s)T_{ref}(s) \quad (16)$$

$$W(s) = \begin{bmatrix} I \\ (sI - A_c)^{-1}B_c \end{bmatrix} (I - T_{ref}(s)) \quad (17)$$

Then, the time constraint (15) can be expressed by:

$$KW(s) = F(s) \quad (18)$$

Notice that the last relation (18) is linear, then the formalism LMI can be applied. In fact, an ideal equality in the form (18) between the closed-loop $T(s)$ and $T_{ref}(s)$ is not possible to obtain.

Therefore, it is recommended to replace an ideal equality by an approximation between $T(s)$ and $T_{ref}(s)$. In order to achieve this condition, we use the time moments approach, as mentioned previously.

Therefore, when the objective is to characterize the time response, the equality (18) is transformed to a minimization of the quadratic distance between the first $(n + 1)$ time moments of the closed-loop system and its reference model, where the minimization criterion is defined as:

$$(KM_{W,j} - M_{F,j})^T (KM_{W,j} - M_{F,j}) < \gamma I \quad (19)$$

$j = 0, 1, \dots, n$.

Using this criterion, we can notice that the transient responses of $T(s)$ and $T_{ref}(s)$ become approximately similar (they have approximately the same overshoot, the same response time, etc.)

The overall objective is achieved if there exist a symmetric and positive defined matrix P and a gain K given by relation (10). In such case, the closed-loop stability condition (7) and the minimization criterion condition (19) are satisfied. In the next section, we present a solution to this problem, which can be formulated by the theorem 1.

3 Mains results

The objective of this section is to give solution of the previous problem.

3.1 Theorem 1.

Consider a LTI system $(A_{sys}, B_{sys}, C_{sys})$ (1) and the corresponding augmented system $(\tilde{A}, \tilde{B}, \tilde{C})$ defined in (9). Consider a static state feedback K_e stabilizing the pair (\tilde{A}, \tilde{B}) .

The stability of the closed-loop system and the time constraints imposed by the reference model $T_{ref}(s)$ are guaranteed, if the optimisation problem:

Minimum γ

(L, G, P) with the constraints

$P > 0$

$$\begin{bmatrix} \text{Sym}\{P(\tilde{A} + \tilde{B}K_e)\} & (L\tilde{C} - GK_e)^T + P\tilde{B} \\ (L\tilde{C} - GK_e) + \tilde{B}^T P & -(G + G^T) \end{bmatrix} < 0 \quad (20)$$

and

$$\begin{bmatrix} -I & (LM_{W,j} - GM_{F,j}) \\ (LM_{W,j} - GM_{F,j})^T & -\gamma I \end{bmatrix} < 0 \quad j = 0, 1, \dots, n. \quad (21)$$

has a solution, then the gain of dynamic controller $C(s)$ is given by :

$$K = G_{opt}^{-1} L_{opt} \quad (22)$$

Notice that L_{opt} , G_{opt} correspond to the minimal value of γ denoted γ_{opt} . Before demonstration of Theorem 1, let us recall the following result [6, 7].

3.2 Lemma 1.

Consider the augmented system (5), where the state matrix A_{bf} is defined by (8). The following statements are equivalent:

1) There exist a symmetric and positive defined matrix $P = P^T > 0$ and two matrices K_e and K such that:

$$P(\tilde{A} + \tilde{B}K_e) + (\tilde{A} + \tilde{B}K_e)^T P < 0 \quad (23)$$

$$P(\tilde{A} + \tilde{B}K\tilde{C}) + (\tilde{A} + \tilde{B}K\tilde{C})^T P < 0 \quad (24)$$

2) There exist a symmetric and positive defined matrix $P = P^T > 0$, a non singular matrix G and two matrices K_e and K such that:

$$\begin{bmatrix} \text{Sym}\{(\tilde{A} + \tilde{B}K_e)^T P\} & P\tilde{B} \\ \tilde{B}^T P & 0 \end{bmatrix} + \text{sym}\left\{ \begin{bmatrix} 0 \\ I \end{bmatrix} G[(K\tilde{C} - K_e) - I] \right\} \quad (25)$$

3.3 Proof of lemma 1

First of all, notice that the orthogonal complements of the second term of (25) are:
 $V = [I \ 0]$, the orthogonal complement of V , such that $VV^\perp = 0$, is $V^\perp = \begin{bmatrix} 0 \\ I \end{bmatrix}$;
 and $U = [I \ (K\tilde{C} - K_e)^T]$, the orthogonal complement of U , such that $UU^\perp = 0$, is:
 $U^\perp = \begin{bmatrix} (K\tilde{C} - K_e)^T \\ -I \end{bmatrix}$

Applications of elimination lemma [9] lead to left and right multiplication of (25) by V and V^\perp respectively, which gives inequality (23):

$$[I \ 0] \text{sym}\left\{ \begin{bmatrix} 0 \\ I \end{bmatrix} G[(K\tilde{C} - K_e) - I] \right\} \begin{bmatrix} I \\ 0 \end{bmatrix} = P(\tilde{A} + \tilde{B}K_e) + (\tilde{A} + \tilde{B}K_e)^T P < 0$$

Even, if (25) is left and right multiplied respectively by U and U^\perp , we get inequality (24).

3.4 Proof of Theorem 1

By using Shur complement, the condition (21) can be expressed as:

$$(LM_{W,j} - GM_{F,j})(LM_{W,j} - GM_{F,j})^T < \gamma I \quad (26)$$

$j = 0, 1, \dots, n$.

Besides, if one multiplies equation (18) by G :

$$LW(s) = GF(s) \quad (27)$$

equation (19) becomes:

$$(LM_{W,j} - GM_{F,j})(LM_{W,j} - GM_{F,j})^T < \gamma I \quad (28)$$

$j = 0, 1, \dots, n$.

One demonstrates that (26) and then the condition (21) correspond to the minimization of the 2 norm of the error between the $(n + 1)$ first moments of the closed-loop and those of the reference model. If the minimization of γ_{opt} is satisfied, then the two transfers $T(s)$ and $T_{ref}(s)$ are very close in low frequencies, which implies the desired transients for the closed-loop system. The stability is also insured due to the fact that condition (20) implies condition (24), equivalent to condition (7) which expresses closed-loop stability.

4 Application to the Permanent Magnet Synchronous Machine (PMSM)

Our Permanent Magnet Synchronous Machine (PMSM) is powered by a two levels voltage inverter. To obtain non reciprocating quantities, our model is done in the Park landmark [10]. Thus we have the following equations in the synchronously d-q reference frame [11, 12]:

$$\left\{ \begin{array}{l} \dot{I}_d = \frac{-R_s}{L_d} I_d + \frac{L_q}{L_d} \omega_r I_q + \frac{1}{L_d} v_d \\ \dot{I}_q = \frac{-R_s}{L_q} I_q - \frac{L_d}{L_q} \omega_r I_d - \frac{\phi}{L_q} \omega_r + \frac{1}{L_q} v_q \\ \dot{\omega}_r = p \frac{(c_e - c_r)}{J} - \frac{f_c}{J} \omega_r \\ c_e = p[(L_d - L_q)I_d + \phi]I_q - \frac{f_c}{J} \omega_r - \frac{f_c}{J} \omega_r \\ \dot{\omega}_r = \frac{p^2}{J} [(L_d - L_q)I_d + \phi]I_q - \frac{f_c}{J} \omega_r - \frac{p}{J} c_r \end{array} \right. \quad (29)$$

where v_d and v_q are the stator voltages, I_d and I_q are the stator currents, L_d and L_q are the inductances, R_s is the stator winding resistance, ϕ is the flux linkage of the permanent magnets, ω_r is the angular velocity of the motor shaft, f_c is the friction coefficient relating to the rotor speed; J is the moment of inertia of the rotor, c_e is the electromagnetic torque, c_r is the load torque and p is the number of pole. In an equivalent manner, these equations can be written in the form of a general transfer function $H(s)$ who contains the open-loop transfer of variables to control namely $H_{id}(s)$ and $H_{wr}(s)$.

$$H(s) = \begin{bmatrix} H_{id}(s) & 0 \\ 0 & H_{wr}(s) \end{bmatrix} \quad (30)$$

where:

$$H_{id}(s) = \frac{\frac{1}{R_s}}{1 + \tau_{id}(s)}, \quad H_{wr}(s) = \frac{H_{bfIq} \frac{p^2 \phi}{f_c}}{1 + \tau(s)} \quad (31)$$

where $\tau_{id} = \frac{L_d}{R_s}$; $\tau = \frac{J}{f_c}$

Let $H_{bfIq}(s)$, is the closed-loop transfer of the current I_q : $H_{bfIq} = \frac{1}{1 + \tau_{bfIq}(s)}$ τ_{bfIq} is chosen equal to $7.0175 \cdot 10^{-4}$ s in order to perform a time response of the closed-loop system ten times faster than open-loop.

Notice that: $\tau_{iq} = \frac{L_q}{R_s}$; $H_{iq}(s) = \frac{1}{1 + \tau_{iq}(s)}$

The objective is to reduce the time response of the system, to keep the overshoot of the closed-loop inferior to 5% while decoupling the two outputs. These specifications can be carried out by a second order reference model with static unity gain for the speed, and a first order one for the current I_d :

$$T_{ref}(s) = \begin{bmatrix} \frac{1}{1 + \tau_{idref}s} & 0 \\ 0 & \frac{1}{1 + \frac{2\zeta}{w}(s) + \frac{s^2}{w^2}} \end{bmatrix} \quad (32)$$

The design choices correspond to $\tau_{idref} = 0.1\tau_{id}$ in order to perform a time response of the closed-loop current ten times faster than the open-loop.

$\zeta = 7.6205$, $w = 93.906rds-1$ in order to obtain a closed-loop speed 3.3 times faster than the open-loop one. Let's note that H_{wr} has two constants time:

$\tau = \frac{J}{f_c} = 0.5333s$, $\tau_1 = 7.047210^{-4}s^{-1}$ The two constants time of the closed-loop speed are $\tau_{bf} = 0.1618s$, $\tau_{1bf} = 7.017510^{-4}s$ Notice that the decoupling specification is insured by a diagonal reference model. The d-axis current I_d is regulated to follow a zero setpoint. The static

state feedback K_e stabilizing the pair (\tilde{A}, \tilde{B}) such as (23) feasible has been computed:

$$K_e = \begin{bmatrix} -79.1667 & 0 & 0 & 722 & 0 \\ 0 & 0 & -2.1433 & 0 & 0.0939 \end{bmatrix} \quad (33)$$

Solving LMIs (20) and (21) by LMI optimization algorithm, the matrices P , G and L can be obtained:

$$G = 10^8 \begin{bmatrix} 0.0065 & 0 \\ 0 & 4.5881 \end{bmatrix}; \gamma_{opt} = 6.609610^{-14} \quad (34)$$

The obtained static gain is:

$$K = \begin{bmatrix} 5.7 & 0 & 722 & 0 \\ 0 & 0.5006 & 0 & 0.09386 \end{bmatrix} \quad (35)$$

5 Experimental results and interpretations

The implementation of our model was performed on a test benchmark consisting of two Permanent Magnet Synchronous Machines (figure 3) fitted by the same encoder and powered by a voltage inverter; one was used as a motor and the other one as a load. The proposed control algorithm was executed by the use of Matlab/ Simulink software. Then it was compiled and implemented on Dspace 1104. The digital sampling period was taken equal to $0.1ms$.

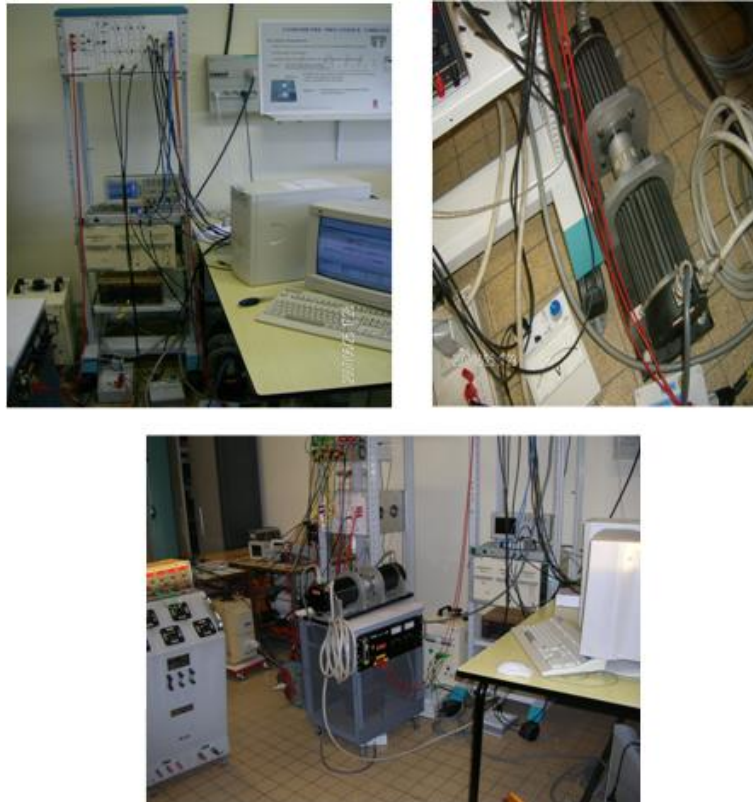


Figure 3: Photo of the test bench.

the synthesized controller have a good behaviour: indeed, the measured speed and dq-axis currents track well the trajectory of reference one with good accuracy over the whole speed range, moreover, the time constraint imposed with the help of time moments has permitted to keep the closed loop outputs very close to those of the reference model.

6 Conclusion

In this paper, a controller design, for current and speed tracking of a PMSM has been developed by using the method of moments. The controller's dynamics are chosen in respect with the time domain constraints. These constraints have equality relationships between the moments of the closed-loop system and those of the reference model. The proposed controller has been achieved in order to ensure the closed-loop stability and to specify transient performances in regard with a reference model. In this study, the LMI formalism has been used for the controller synthesis. Finally, we have applied the proposed controller for the speed tracking of a PMSM which has given satisfactory results.

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8 Appendix : Motor parameters

Motor rated power	1 KW
Rated current	6.5 A
Pole pair number (p)	2
d-axis inductance Ld	4.5mH
Stator resistance	0.56 Ω
Motor inertia J	$2.08.10^{-3} Kg.m^2$
Friction coefficient fc	$3.9.10^{-3} Nm.s.rad^{-1}$
Magnet flux constant ϕ	0.064 wb

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