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Fuzzy Linear Physical Programming for Multiple Criteria Decision-Making Under Uncertainty

A. ElSayed, E. Kongar, S.M. Gupta

Ahmed ElSayed
Department of Computer Science and Engineering
University of Bridgeport
221 University Avenue,
Bridgeport, CT 06604, USA
aelsayed@my.bridgeport.edu

Elif Kongar*
Departments of Mechanical Engineering and Technology Management
University of Bridgeport
221 University Avenue,
Bridgeport, CT 06604, USA
*Corresponding author: kongar@bridgeport.edu

Surendra M. Gupta
Department of Mechanical and Industrial Engineering
Northeastern University
360 Huntington Avenue,
Boston, MA 02115, USA
gupta@neu.edu

Abstract: This paper presents a newly developed fuzzy linear physical programming (FLPP) model that allows the decision maker to introduce his/her preferences for multiple criteria decision making in a fuzzy environment. The major contribution of this research is to generalize the current models by accommodating an environment that is conducive to fuzzy problem solving. An example is used to evaluate, compare and discuss the results of the proposed model.
Keywords: Decision Support Systems, Fuzzy Goal Programming, Fuzzy Sets and Systems, Heuristics, Linear Physical Programming, Logistics, Multi-criteria Decision Making, Supply Chain.

1 Introduction

Many real life decisions are often complex and typically involve several parties with different goals and objectives. Today, as also stated by Filip et al. [1], technological and organizational improvements have led to more complex and complicated decision making environments. As a result, current decision models are stretched to the limit to handle these changes as the data availability, number of participants in the decision making process, size of markets and several other factors grow. That is, decision making has become a cumbersome task requiring models that are able to consider various multiple goals, constraints that are somewhat uncertain and flexible in nature.

For instance, the success of supply chains heavily depends on the joint effort focusing on the overall value creation as opposed to optimizing segregated objectives. Furthermore, some of the supply chain outcomes, viz., responsiveness, sustainability, innovation, are less likely to be expressed with crisp objective values [2]. In many cases desired business outcomes are fuzzy in nature and contain a considerable degree of flexibility. The reason for this is twofold. First, the outcome measures are difficult to calculate and quantify. In addition, the conflicting objectives among different parties in the supply chain are likely to mandate varying levels of preference levels even for an identical performance measure. Therefore, it is reasonable to model supply chain performance measurement problems via fuzzy and multiple objective approaches which are, in fact, more suitable for such problem environments.

With this motivation, this paper presents a decision support model which would accommodate the uncertainty involved in the problem environment caused by the flexibility in the goals, information, perspective and/or different visions. The model, based on Linear Physical Programming (LPP), is suitable for individualized decisions as well as decisions where multiple participants are involved and has the ability to integrate and reflect varying preferences of the decision maker.

Linear Physical Programming (LPP) is a preference based optimization method, which operates in the environment of multiple criteria and uses a utility function to represent the decision maker's (DM) preference levels. The key distinguishing feature of LPP is that the decision maker is excluded from the process of choosing appropriate weights [3] which is one of the major challenges when defining a utility function.

Furthermore, similar to other decision support systems [4], the fuzzy LPP formulation can assist the decision maker in overcoming individual limitations and constraint by (i) handling the computational complexity which would otherwise be practically prohibitive and by (ii) introducing vagueness that is naturally embedded in real life decisions or introduced due to multi-participant nature of the decision environment.

Due to these advantages, physical programming has been the topic of various theoretical and practical studies. The technique has been extensively used in various industrial and mechanical engineering applications as well as in other disciplines.

This paper presents a newly developed fuzzy Linear Physical Programming (FLPP) model to accommodate the vagueness of the problem environment while eliminating the need for precision in decision makers' preference levels. The model presented significantly improves the basic methodology and generalizes it to assist in overcoming the decision maker's vagueness.

2 Literature Review

Related body of knowledge offers a large variety of models where the decision maker's preference levels are integrated into an objective function. Most techniques require that the decision maker construct an aggregate objective function using the weights determined as a result of an exasperating trial and error process. In LPP, however, the decision maker specifies the ranges of different degrees of desirability instead of defining the weights. LPP then uses these ranges of desirability to formulate the aggregate objective function, thus eliminating the tedious weight assignment process by providing decision makers with a flexible and more natural problem formulation.

Maria et al. [5] proposed a production-planning model based on Linear Physical Programming (LPP) that would utilize previous design knowledge when available.

Messac et al. [6] applied LPP to reconfigure a distribution network including various quantitative and qualitative factors such as costs of relocation/consolidation, inbound and outbound transportation, relocation, customer delivery time, labor quality, labor-management relations and tax incentives. The proposed model is then utilized to determine the demand allocations of plants to warehouses and warehouses to customers.

Ondemir and Gupta [7] a applied Linear Physical Programming to a multi-criteria advanced repair-to-order and disassembly-to-order (ARTODTO) system for sensor embedded products with RFID tags. The proposed approach aimed at determining how to process EOL products to satisfy life expectancy based demand. Various types of demand are met by a combination of disassembly, repair, and recycling operations. The model allowed third party procurement to prohibit backorders.

A study that introduces heuristics into Physical Programming (PP) was proposed by Sanchis et al. [8]. In this work, the authors replaced PP optimizer by a genetic algorithm to avoid potential local minima issue in addition to simplifying the algorithm that constructs the PP preference functions.

Relevant research combining disassembly sequencing and scheduling via Linear Physical Programming have previously been conducted by Kongar and Gupta [9] and Lambert and Gupta [10].

Another multi-objective method that has been extensively used in problem environments with high uncertainty is fuzzy programming. The method has been widely used due to its ability to respond to the imprecision associated with the input data in multi-objective problems and hence its ability to handle vagueness. In fuzzy programming each objective function value is interpreted as a fuzzy number bounded by the best and worst values of the particular objective. The corresponding fuzzy membership function value of the objective is the deviation from the goal divided by that objective's range, thus a value between 0 and 1. Fuzzy programming aims at finding the solution that minimizes the largest fuzzy membership function. In addition to the goals, other input parameters can be represented by fuzzy numbers and fuzzy arithmetic can be used to combine fuzzy numbers based on fuzzy set theory first introduced by Zadeh [11].

One of the earlier studies in fuzzy was published by Narasimhan [12] who solved a goal programming problem in a fuzzy environment. In order to illustrate the capability of the proposed model, the author provided solutions for the typical goal programming model that aims at a single optimum solution. A similar study has also been proposed by Hannan [13] demonstrating the incorporation of fuzzy goals into typical goal programming formulation via numerical examples. The study also provides a discussion on the advantages and shortcomings of fuzzy and standard goal programming modeling.

Tian et al. [14,15] developed a fuzzy physical programming model for multidisciplinary design optimization. In their study, the authors utilized fuzzy PP for the optimization modeling of through passenger train plan and solved it via genetic algorithm.

Zhang et al. [16] utilized a fuzzy physical programming approach in a linear environment. However, a major drawback in this approach was that it limited the fuzziness only to the boundaries of the problem whereas fuzziness can occur in the variables of both the goals and the classes resulting in more complicated fuzzy relationship between the two.

Another relevant work in the area of fuzzy physical programming (FPP) was published by Huang et al. [17]. In that study, the authors proposed a fuzzy aggregate objective function with fuzzy constraints for each soft and hard class. These studies utilized a single membership function for each goal, and calculated the class function as a function of the membership values as opposed to as a function of the goal values as proposed by the original physical programming methodology. Class functions are considered to be aggregation of goal values and its membership function. The problem is then converted back to its crisp form and solved by genetic algorithm.

Ilgin and Gupta [18] provide a state of the art review of Physical Programming (PP) by classifying the PP related studies into four areas, viz., (1) methodological papers, (2) industrial engineering applications, (3) mechanical engineering applications, and (4) other applications.

The methodology described in this paper combines Linear Physical Programming (LPP) and Fuzzy Sets and proposes a multi-objective model for disassembly sequencing under uncertainty.

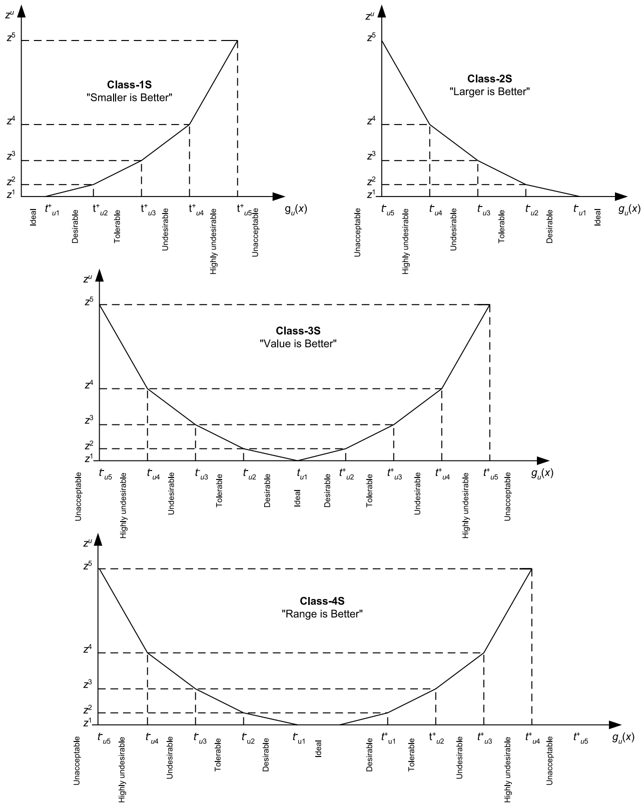

Figure 1: LPP soft class functions for a generic u^{th} objective

3 Theoretical Background and Methodology

3.1 Linear Physical Programming

The physical programming algorithm requires that the decision maker expresses his/her preferences with respect to each criterion using one of the eight different *classes*. The first four classes are *"Soft class functions"*, and represent minimization (Class-1S), maximization (Class-2S), value (Class 3S), and range (Class 4S) optimization. The remaining four are *"Hard class functions"* and are used to introduce inequality and range restrictions into the problem environment. In this regard, Class 1H and Class 2H define upper and lower bounds, respectively, while Class 3H imposes equality and Class 4H imposes range related restrictions to the problem environment. The qualitative and quantitative depiction of each *class* is provided in Figures 1 and 2.

The *soft* class functions allow the DM to express varying levels of preferences for each criterion. This is done by introducing corresponding constraints for each preference level in each of the classes. To provide better understanding, consider Class 1-S and Class 2-S, depicted in Figure 1, which are used for "Smaller is Better" and "Larger is Better" cases respectively. Table 1 demonstrates the ranges and corresponding constraints for the problem. Note that all the *soft* class functions will be embedded in the aggregate objective function to be minimized.

In Figure 1, the u^{th} generic criterion is indicated as $g_u(x)$ where x is the decision variable vector. The goal value, g_u is represented on the horizontal axis while, z_u, the *class function* that is subject to minimization is represented on the vertical axis. Since LPP algorithm considers the lower values of the class functions as "better" values but prohibits negative values, the class function that corresponds to the ideal value is set to zero.

Table 1: Preference levels and constraints for Class-1S and Class 2-S

Class 1-S			Class 2-S		
Range Index	Preference Level	Constraint	Range Index	Preference Level	Constraint
1	Ideal	$g_u \leq t^+_{u_1}$	1	Ideal	$g_u \geq t^-_{u_1}$
2	Desirable	$t^+_{u_1} \leq g_u \leq t^+_{u_2}$	2	Desirable	$t^-_{u_1} \leq g_u \leq t^-_{u_2}$
3	Tolerable	$t^+_{u_2} \leq g_u \leq t^+_{u_3}$	3	Tolerable	$t^-_{u_2} \leq g_u \leq t^-_{u_3}$
4	Undesirable	$t^+_{u_3} \leq g_u \leq t^+_{u_4}$	4	Undesirable	$t^-_{u_3} \leq g_u \leq t^-_{u_4}$
5	Highly Undesirable	$t^+_{u_4} \leq g_u \leq t^+_{u_5}$	5	Highly Undesirable	$t^-_{u_4} \leq g_u \leq t^-_{u_5}$
6	Unacceptable	$t^+_{u_5} \leq g_u$	6	Unacceptable	$t^-_{u_5} \geq g_u$

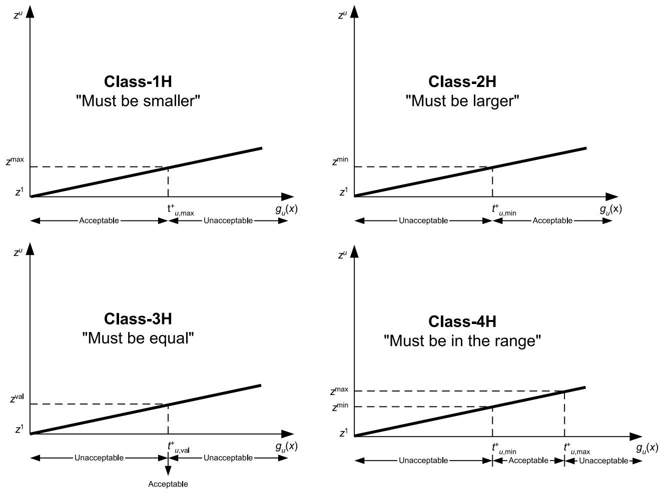

Figure 2: LPP hard class functions for a generic u^{th} objective

As for the hard classes, only two ranges are defined namely, *acceptable* and *unacceptable.* Hard class functions appear as constraints in the LPP model. They can be considered as the hard constraints in Goal Programming and are handled in the same manner. Therefore, they are used for bounding and hence creating the feasible search space (Figure 2).

It is important to note that both soft and hard class functions are piecewise linear and convex. This can be achieved by ensuring that the calculated weights are positive. LPP's built-in-weight calculation algorithm, Linear Physical Programming Weight (LPPW), ensures this condition and guarantees that is each criterion is associated with a piecewise linear class function. Briefly, LPPW, first proposed by Messac [19], uses the decision maker's target values for each criterion to calculate a weight vector. Following this, LPP utilizes these weights to form the objective function to be minimized.

3.2 Fuzzy Linear Physical Programming Model

The methodology proposed in this paper utilizes separate membership functions for each linguistic variable for each goal, and converts the relationship between the class variable and the goals to a fuzzy relationship. Furthermore, the weight for each goal is also calculated based on the fuzzy relationship prior to problem solving via fuzzy goal programming. Therefore, the algorithm detailed in this paper is fuzzy in nature; modeled, initialized and solved in a fuzzy environment throughout.

The concept of creating a fuzzy relationship between the goals and the classes was introduced

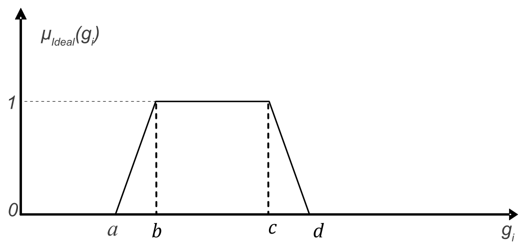

Figure 3: Trapezoidal membership function

by Kosko [20].

The steps of the proposed algorithm are provided below.

1. Select a class type for every goal S1, S2, S3, S4 or H1, H2, H3, H4

2. Decision maker should define the membership function of each linguistic variable for each goal. Assuming that the membership function is trapezoidal, then the function can be expressed mathematically as follows:

$$\mu_{Ideal}\left(g_i\right) = \begin{cases} 0, & \left(g_i < a\right) or \left(g_i > d\right) \\ \frac{g_i - a}{b-a}, & a \leq g_i \leq b \\ 1, & b \leq g_i \leq c \\ \frac{d-g_i}{d-c}, & c \leq g_i \leq d \end{cases} \tag{1}$$

where g_i is the goal number i to be used in the decision process, and, a, b, c, and d are the parameters of the membership function of the Ideal linguistic variable as shown in Figure 3.

3. The class function z^i will also turn into a set of linguistic variables, each member of the set corresponding to the respective linguistic variable for the goal, *i.e.*, Ideal, Desirable, etc., with a trapezoidal assumption the membership function will be as demonstrated follows:

$$\mu_{z^i}\left(z\right) = \begin{cases} 0, & \left(z < a\right) or \left(z > d\right) \\ \frac{z-a}{b-a}, & a \leq z \leq b \\ 1, & b \leq z \leq c \\ \frac{d-z}{d-c}, & c \leq z \leq d \end{cases} \tag{2}$$

For instance $\mu_{z^{Ideal}}\left(z\right)$ value for $\tilde{z}^{Ideal}$ will be as follows (Figure 4):

$$\mu_{z^{Ideal}}\left(z\right) = \begin{cases} 0, & \left(z > c\right) \\ 1, & a \leq z \leq b \\ \frac{c-z}{c-b}, & b \leq z \leq c \end{cases} \tag{3}$$

The values of a, b, c, and d of each membership function will be determined according to the LPPW algorithm to guarantee that the fuzzy relationship between the class function and each goal is a convex fuzzy relationship.

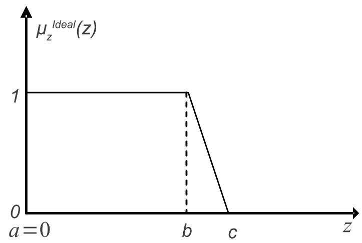

Figure 4: Sample trapezoidal membership function for the Ideal linguistic variable for Class 1-S

4. After determining the values of membership function limits of z_i sets, the fuzzy arithmetic is utilized to obtain the value of the fuzzy weight of each linguistic variable for each goal. Through a fuzzy division, the Ideal set will be calculated as follows:

$$\tilde{w}_{Ideal} = \frac{\tilde{z}^{Ideal}}{\tilde{g}_{Ideal}} \tag{4}$$

which will generate a membership function for the fuzzy weight.

Please note that equation 4 can easily be solved using a computer program based on the fuzzy division operation described in [21, 22].

5. The weight of each linguistic variable can be obtained by defuzzification of the fuzzy weights. In this paper the center of gravity (centroid) *defuzzification* technique is utilized to defuzzify the weights in order to compute the corresponding crisp values. Following *defuzzification*, each crisp weight is multiplied by the membership function of the corresponding goal's linguistic variable. This way each linguistic goal is shaped according to its priority. The resulting modified membership function becomes:

$$\mu_{Ideal}(g_i) = \begin{cases} 0, & (g_i < a)\, or\, (g_i > d) \\ \frac{w_{Ideal}(g_i - a)}{b-a}, & a \le g_i \le b \\ w_{Ideal}, & b \le g_i \le c \\ \frac{w_{Ideal}(d - g_i)}{d-c}, & c \le g_i \le d \end{cases} \tag{5}$$

6. Calculate the union of all the membership functions of the linguistic variables of each goal to obtain the final membership function for the corresponding goal values:

$$\mu_{Goal_i}(g_i) = \mu_{Ideal}(g_i) \vee \mu_{Desiarble}(g_i) \vee \mu_{Tolerable}(g_i) \vee \mu_{Undesirable}(g_i) \vee \mu_{HUndesirable}(g_i) \vee \mu_{Unacceptable}(g_i) \tag{6}$$

7. Define the membership function of the constrains $\mu_{C_i}(g_1, g_2, g_3, \ldots)$ where C_i is the constraint number i defined by the decision maker (can be considered as one of the Hard classes).

8. Apply Fuzzy Goal Programming by calculating the minimum point that provides the highest value for the membership function of the intersection of every defined membership function :

$$\mu_D(g_1, g_2, g_3, \ldots) = \min_i (\mu_{C_i}(g_1, g_2, g_3, \ldots)) \wedge \min_j (\mu_j(g_j)) \tag{7}$$

where $\mu_D(g_1, g_2, g_3, \ldots)$ will be the membership function of the required objectives. Then the optimum goal values will be:

$$G = \min_{G} (\max (\mu_D (g_1, g_2, g_3, \ldots))) \tag{8}$$

where G is the optimum values of all the goals which satisfy the required constrains.

4 Model Application and Discussion

To demonstrate the applicability of the proposed algorithm, consider the numerical example provided in Messac et al. [3]:

A company manufactures two types of products, product A and product B. Due to factory's physical and human-resource limitations, the company can comfortably manufacture 25 units per month of product A, and 10 of product B. These levels are considered ideal and independent of other considerations. The profits generated per unit from product A and B are respectively \$12k and \$10k. At these levels of manufacturing, the resulting monthly profit becomes \$400k. Investors have decided that if the company is to remain in business, a minimum monthly profit of \$580k must be achieved at the optimal level. After appropriate market research and examination of the situation, the DM decides that, the preferences regarding production rate are as follows:

Preference Level	Product A	Product B
Ideal	<25	<10
Desirable	25-31	10-18
Tolerable	31-36	18-26
Undesirable	36-44	26-33
Highly-Undesirable	44-50	33-40
Unacceptable	>50	>40

The profit (constraint) equation for the factory takes the form: $12g_1 + 10g_2 \geq 580$, where g_1 and g_2 represent two criteria that denote the respective monthly production levels of products A and B.

The following demonstrates the application of the proposed algorithm to the problem described above. Both goals (g_1 and g_2) belong to Class-1S since they are "Smaller Is Better" in nature. Then the membership function of the goals' linguistic variable is assumed as shown in Figure 5 (*i.e.*, '*' for Ideal, '+' for Desirable, '.' for Tolerable, 'o' for Undesirable and '–' for Highly-Undesirable).

Following this assumption, the LPPW algorithm is then employed to calculate the parameters of the class function membership limits to guarantee the convexity of the fuzzy relation between the goals and the class function. Figure 6 shows the resulting membership function for the class function.

Figure 7 shows the fuzzy relation between the class function z and the goal for Product A.

The membership function of the constrains is also assumed as shown in Figure 8,where the constraints are fuzzified so that, for instance, "Less than 25 is Ideal" statement is converted into "Less than 25 is *somewhat* Ideal".

By calculating the weights and applying equation 6, the final membership function for the intersection of all goals become as shown in Figure 9.

The final membership function after applying equation 7 is depicted in Figure 10.

The algorithm results in (g_1 , g_2) = (25.5, 20.2) for Product A and Product B respectively in the case of trapezoid membership functions (shown figures). Product A result (25.5) is in the

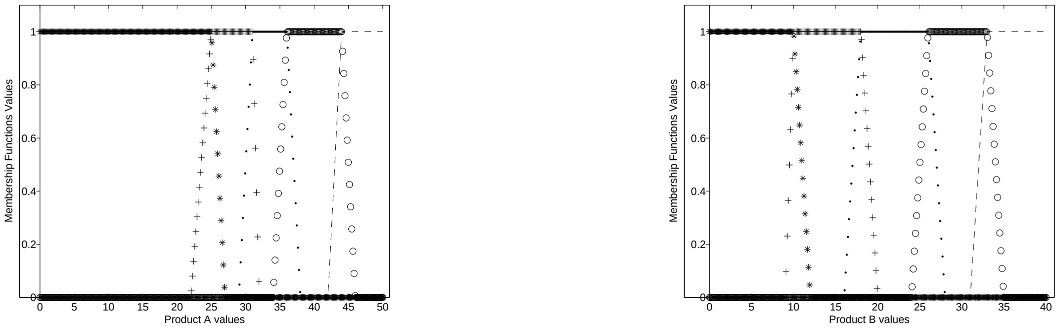

Figure 5: Membership functions of the Product A and Product B linguistic variables

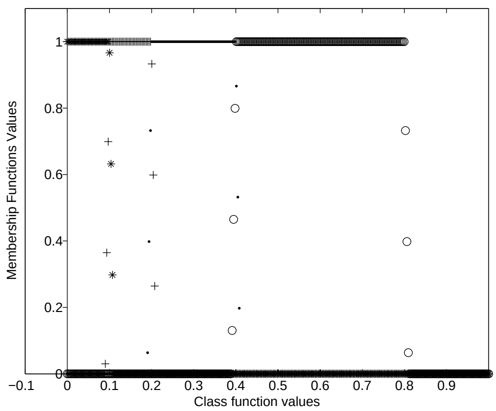

Figure 6: Membership Functions of the Class Function Linguistic Variables

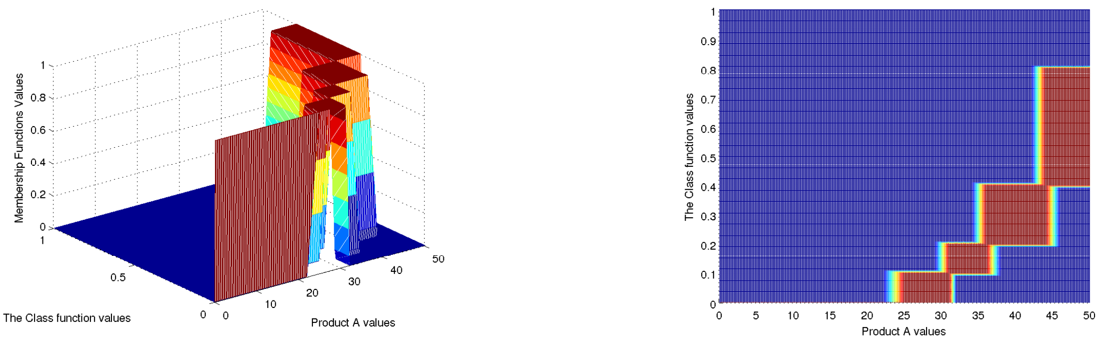

Figure 7: Two Views of the Fuzzy Relation between Product A Values and the Corresponding Class Function

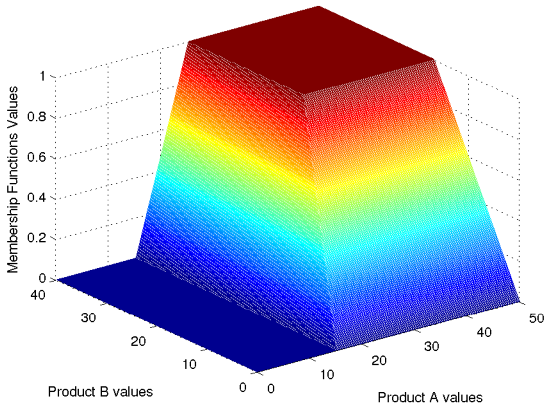

Figure 8: Membership Functions of the Constraints

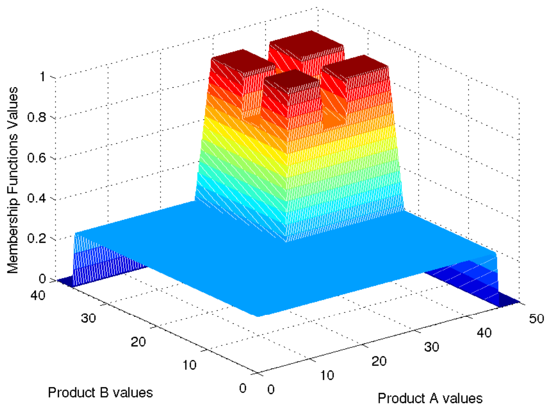

Figure 9: The Intersection Region Between the Membership Functions of Product A and Product B

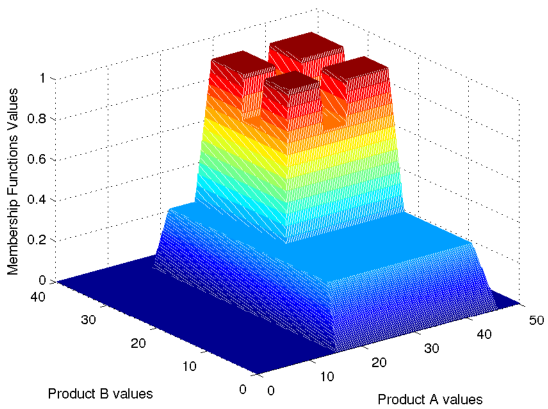

Figure 10: The Intersection Region between the Membership Functions of Product A and Product B and Constrains

Table 2: Number of samples and corresponding execution times

Number of Samples	Execution Time (seconds)
150	1.02
300	2.79
400	4.70
500	7.35
600	10.78

desirable range with close proximity to the *ideal*. As for Product B, the result (20.2) is in the *tolerable* range closer to the *desirable* boundary. When the membership function is defined as triangular, the best goals become (g_1 , g_2) = (28, 22.4) for Product A and Product B, respectively. Here, Product A result (28) is in the *desirable* range, whereas Product B result (22.4) is in the *tolerable* range.

Messac et al. [19] first modeled and solved the problem via Goal Programming (GP) and calculated the (g_1 , g_2) coordinates as (25, 28) and (40, 10). The authors described the solution alternatives as quite restrictive and highly dependent on the choice of weights and on the goals/targets. Furthermore, the authors note that DM's knowledge cannot be implemented in the problem model. Following this, the study proposed the utilization of LPP for the same problem. The resulting value is located at the *desirable/tolerable* boundary for g_1, and within the *tolerable* range for g_2, with (g_1 , g_2) = (31, 20.8).

Compared to both GP and LPP results, the proposed methodology provides better solutions for all goals. In addition, the proposed algorithm eliminates the need for deciding on the weights of goals.

The approach can be easily adapted to general purpose problems as well as specialized problem environments with narrower focus. That is, if a problem can be formulated with an objective function and one or more technological constraints and sign restrictions the presented model will be able to solve the problem in a complete fuzzy environment. The model can also be easily converted into a multiple objective optimization and regardless of the number of objectives in the problem environment, it would still be able to preserve its fuzzy nature. It should also be noted that the only alpha cut used in the computation is in the final step of the optimum value for calculation. Here, the maximum membership function is:

$$\alpha = \max\left(\mu_D\left(g_1, g_2, g_3, \ldots\right)\right),$$

whereas the corresponding optimal value becomes:

$$G = \min\left(A_\alpha\right).$$

.

All the computations in the example are numerical, since they were computed via Von-Neumann architecture computer system and not via a fuzzy processor. For the example shown, the number of samples used for all variables constitutes a domain of 300 samples (*i.e.*, N =300). Increasing the number of samples would adversely affect the overall execution time since calculations and complexity of the fuzzy relations are similar to regular matrix multiplication. Table 2 depicts the relationship between the number of samples and the execution time for a selection of numerical values.

5 Conclusions

Unlike conventional goal programming techniques, fuzzy goal programming, including the proposed fuzzy linear physical programming model, implies a fuzzy system. Therefore, the model's computational complexity on regular digital machines not only depends on the number of input and output variables, but also on the number of discretization levels, viz., samples, from both variables [15]. Using this principle, the computational complexity of the problem under test can be restricted to a number that improves result accuracy while improving the computational time (approximately 300 discretization level for each input and output variable universe of discourse). This simplified fuzzy logic control is proven be suitable for a loop controller with regard to the computation time and the required memory.

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