

A New Approach to Nonlinear Tracking Control Based on Fuzzy Approximation

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Abstract: The problem of tracking control is addressed for a class of nonlinear systems with uncertainties. The original nonlinear systems are approximated by a fuzzy T-S model based on which a state-feedback controller is constructed by using the linear matrix inequalities. The approximating error is eliminated by an adaptive compensator based on fuzzy logic systems. The effectiveness of the proposed control scheme is demonstrated by a simulation example. The main advantage is that the designer makes milder constraint assumption for the approximation error and the uncertainties in nonlinear systems.

Keywords: fuzzy T-S model; fuzzy logic systems; nonlinear systems; uncertainties; tracking control.

1 Introduction

The problem of controller design for the nonlinear systems with uncertainties is a challenging work. One of effective tools used to solve the problem is fuzzy method. There are two frequently used fuzzy models: fuzzy T-S model [1-13] and fuzzy logic systems [16-17]. Fuzzy T-S model is usually used to approximate nonlinear systems, and it has been widely applied to analyze the stability of nonlinear systems [1-4]. The contributions of these works are very important, but these works could be further improved by a simpler and more practical control scheme. The approximating error is neglected in [1-4], which impacts the stability of the system. Therefore, the designed controller can't always guarantee the stability of the original system. To overcome the effect of the approximating error, some relaxed stability methods are developed in [5-8]. These methods improve the approximation accuracy for nonlinear system. However, the original nonlinear system is still neglected in [5-8]. In order to further relax the effect of the approximating error, it is assumed to satisfy the matching condition in [9-11] and have an upper bound in [12-13]. However, the matching condition and the upper bound are not easy to be found in practice, which adds some difficulties to the controller design. The uncertainties in nonlinear systems are assumed to satisfy the constraint of the upper bound in [14-15]. However, the upper bound may be too large or doesn't exist. On the other hand, fuzzy logic systems have been proved to have

the universal approximation property. By constructing a set of fuzzy "IF-THEN" rules, fuzzy logic system is used to model uncertain nonlinear systems [16-17].

There exist some conservatism of pure fuzzy T-S model in dealing with the approximating error. The pure adaptive fuzzy control also has some shortages of excessively depending on the chosen membership functions. Therefore, it is more interesting to combine both fuzzy models to overcome their shortages each other.

Based on the above discussions, fuzzy T-S model and fuzzy logic systems are combined to design a new tracking-control scheme for a class of nonlinear systems with uncertainties in this paper. A fuzzy T-S model is used to approximate the nonlinear system based on which a state-feedback controller is constructed by use of the linear matrix inequalities. The approximating error and the uncertain nonlinear parts are eliminated by a compensator based on fuzzy logic systems. The main advantages are summarized as follows:

Firstly, fuzzy T-S model and fuzzy logic systems are combined to develop a controller. Compared with the existing works based on fuzzy T-S model [1-13], the proposed method in this paper makes milder constraint assumptions for the approximating error. Secondly, the dimension of the matrix inequalities is reduced, thus, the difficulty of solving the matrix equalities is relaxed. Thirdly, the existing works based on fuzzy logic systems [16-17] excessively depend on the chosen membership functions which are improved in the proposed method of this paper. Finally, the developed controller makes full use of the advantages of two fuzzy models. As a result, it is more convenient to implement the controller in practice.

The rest of the paper is organized as follows. Section 2 provides preliminaries and the formulation of the problem. Section 3 develops a procedure of the controller design. Section 4 presents a simulation example of 2-link manipulator to illustrate the effectiveness of the proposed method. These are followed by conclusions in Section 5.

2 Problem formulation

Consider the following nonlinear systems with uncertainties

$$\begin{aligned}
 \dot{x}_1 &= x_2, \\
 &\dots \\
 \dot{x}_{(\beta_1-1)} &= x_{\beta_1}, \\
 \dot{x}_{\beta_1} &= f_1(x, u) + \tilde{f}_1(x, u) + d_1, \\
 \dot{x}_{(\beta_1+1)} &= x_{(\beta_1+2)}, \\
 &\dots \\
 \dot{x}_n &= f_m(x, u) + \tilde{f}_m(x, u) + d_m,
 \end{aligned} \tag{1}$$

where x, u are the system state vector, control input vector, respectively;

$x = [x_1, \dots, x_1^{(\beta_1-1)}, \dots, x_{(n-\beta_m+1)}, \dots, x_{(n-\beta_m+1)}^{(\beta_m-1)}]^T \in R^n$, $\beta_1 + \beta_2 + \dots + \beta_m = n$, $u = [u_1, \dots, u_m]^T \in R^m$, $f_i (i = 1, \dots, m)$ are known smooth nonlinear functions, $\tilde{f}_i (i = 1, \dots, m)$ are unknown uncertain nonlinearities of the system, and $d_i (i=1,2,\dots,m)$ denote the external disturbances.

Remark 1: There are many practical physical systems which can be described by the model (1), for example, the mass-spring-damper [18], the rotated inverted pendulum [19] and the n-link manipulator [20].

A reference model is as follows:

$$\dot{x}_r(t) = A_r x_r(t) + r(t), \tag{2}$$

where $x_r(t)$ is a reference state, $r(t)$ is a bounded reference input, and A_r is an asymptotically stable matrix.

Control objective: Design a controller to guarantee that the nonlinear system (1) is stable and the state can track the reference state $x_r(t)$.

The known part of the system (1) can be approximated by a fuzzy T-S model composed of L rules. For convenience of research, fuzzy T-S model includes the external disturbance d . The i th rule of the fuzzy model is as follows:

IF $z_1(t)$ is F_1^i and,...,and $z_s(t)$ is F_s^i , THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) + d, \quad i = 1, 2, \dots, L, \quad (3)$$

where $z_1(t), \dots, z_s(t)$ are the premise variables, $F_j^i (j=1,2,\dots,s)$ are the fuzzy sets, L is the number of IF-THEN rules, A_i and B_i are some constant matrices with compatible dimensions, $B_i = [0, \dots, b_{i1}^T, \dots, 0, \dots, b_{im}^T]^T \in R^{n \times m}$ with $b_{i1} \in R^m, \dots, b_{im} \in R^m$, and $d = [0, \dots, d_1, \dots, 0, \dots, d_m]^T$. The final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^L \mu_i A_i x(t) + \sum_{i=1}^L \mu_i B_i u(t) + d, \quad (4)$$

where

$$\mu_i = \nu_i(z(t)) \left/ \sum_{i=1}^L \nu_i(z(t)) \right., \quad \nu_i(z(t)) = \prod_{j=1}^S F_j^i(z_j(t)), \quad (5)$$

and $F_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in F_j^i . Therefore, the approximating error for the nonlinear system (1) and the uncertainties of the nonlinear system (1) can be expressed as $B\Delta(x)$, where $B = \text{diag}[B^1, \dots, B^m]$, $B^i = [0, \dots, 0, 1]^T \in R^{\beta_i}$ and $\Delta(x) = [\Delta_1, \dots, \Delta_m]^T$.

Therefore, the nonlinear system (1) could be rearranged as

$$\dot{x}(t) = \sum_{i=1}^L \mu_i A_i x(t) + \sum_{i=1}^L \mu_i B_i u(t) + B\Delta(x) + d. \quad (6)$$

3 Design of controller and stability analysis

3.1 Design of controller

The controller is chosen as

$$u(t) = u_l(t) - u_f(t), \quad (7)$$

where $u_l(t)$ denotes the state-feedback controller based on fuzzy T-S model, $u_f(t)$ is the adaptive compensator based on fuzzy logic systems.

The state-feedback controller $u_l(t)$ based on fuzzy T-S model is designed as

$$u_l(t) = \sum_{i=1}^L \mu_i K_i (x(t) - x_r(t)), \quad (8)$$

where $u_l(t)$ is used to stabilize the linear part of the system (1), and $K_i (i=1,2,\dots,L)$ are matrices with proper dimensions and satisfy

$$\bar{A}_{ij}^T + P\bar{A}_{ij} + \frac{1}{\rho^2} PP + \bar{Q} < 0, \quad i, j = 1, 2, \dots, L, \quad (9)$$

where $\bar{A}_{ij} = \begin{bmatrix} A_i + B_i K_j & -B_i K_j \\ 0 & A_r \end{bmatrix}$, $\bar{Q} = \text{diag}\{2Q, 2Q\}$, P and Q are some symmetric and positive definite matrices, and ρ is a positive constant.

The adaptive compensator based on fuzzy logic systems is given by

$$u_f(t) = \begin{cases} E^{-1}\hat{u}(x|\Theta), & \text{if } E \text{ is nonsingular} \\ E^T(I + EE^T)^{-1}\hat{u}(x|\Theta) & \text{if } E \text{ is singular,} \end{cases} \quad (10)$$

which is used to compensate the approximating error and the uncertainties. In(10),

$$E_i = [b_{i1}^T, \dots, b_{im}^T]^T \in R^{m \times m}, E = \sum_{i=1}^L \mu_i E_i,$$

and $\hat{u}(x|\Theta)$ is constructed by fuzzy logic systems. The updating law of Θ is as follows:

$$\dot{\Theta} = \eta_1 \Psi^T(x) \bar{B}^T P x, \quad (11)$$

where η_1 is a positive constant, $\Psi(x)$ is a fuzzy basis-function matrix, and the definition of $\Psi(x)$ is given in (16).

3.2 Stability analysis

Note that

$$\sum_{i=1}^L \mu_i B_i u_f(t) - B \Delta(x, x(t - \tau)) = B(E u_f(t) - \Delta(x)) \quad (12)$$

$$= \begin{cases} B(\hat{u}(x|\Theta) - \Delta(x)) \\ B(\hat{u}(x|\Theta) - (I + EE^T)^{-1}\hat{u}(x|\Theta) - \Delta(x)) \end{cases}$$

$$\underline{\underline{=}} B(\hat{u}(x|\Theta) - \Delta(x)). \quad (13)$$

Substituting (7) into (6) yields

$$\dot{x}(t) = \sum_{i=1}^L \mu_i A_i x(t) + \sum_{i=1}^L \sum_{j=1}^L \mu_i \mu_j B_i K_j (x(t) - x_r(t)) - B(\hat{u}(x|\Theta) - \Delta(x)) + d. \quad (14)$$

Denote $\tilde{x}(t) = [x^T(t), x_r^T(t)]^T$, and $\bar{B} = [B^T \ 0]^T$. From (2) and (14), a new extended closed-loop system is as follows:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^L \sum_{j=1}^L \mu_i \mu_j \bar{A}_{ij} \tilde{x}(t) + \bar{B}(-(\hat{u}(x|\Theta) - \Delta(x))) + d, \quad (15)$$

where $d = [d^T, r^T(t)]^T$. When fuzzy logic systems $\hat{u}(x|\Theta)$ eliminate $\Delta(x)$, then the closed-loop system (15) is stable. Thus, fuzzy logic systems are constructed to approximate the vector function $\Delta(x)$ as follows:

$$\hat{\Delta}(x|\Theta) = \Psi(x)\Theta, \quad (16)$$

where

$$\Psi(x) = \text{diag}[\xi_1^T(x), \dots, \xi_m^T(x)], \Theta = [\theta_1^T, \theta_2^T, \dots, \theta_m^T]^T$$

in which $\theta_i(i=1,2,\dots,m)$ are the column vectors, and the weight Θ is an adaptive parameter. Define the optimal parameter estimation Θ^* as follows:

$$\Theta^* \triangleq \arg \min_{\Theta \in \Omega} [\sup_{x \in U} \|\hat{\Delta}(x|\Theta) - \Delta(x)\|], \quad (17)$$

where $U = \{x \in R^n\}, \Omega = \{\Theta \in R^{pm \times 1}\}$. U, Ω denote the sets of suitable bounds on x, Θ , respectively. Then the estimation error for the vector function $\Delta(x)$ can be expressed as

$$\hat{\Delta}(x|\Theta) - \Delta(x) = \Psi(x)\tilde{\Theta} + w, \quad (18)$$

where $w = [w_1, \dots, w_m]^T$ is a residual term, $\tilde{\Theta} = \Theta - \Theta^* = [(\theta_1 - \theta_1^*)^T, \dots, (\theta_m - \theta_m^*)^T]^T$.

Denote $w^t = [\bar{w}^T, r^T(t)]^t$, and $\bar{w} = [0, \dots, d_1 - w_1, \dots, 0, \dots, d_m - w_m]^T$. Substituting (18) into (15), (15) is rearranged as

$$\dot{\tilde{x}}(t) = \sum_{i=1}^L \sum_{j=1}^L \mu_i \mu_j \bar{A}_{ij} \tilde{x}(t) + \bar{B}(-\Psi(x)\tilde{\Theta}) + w^t. \quad (19)$$

Theorem 1. For the nonlinear system (1), if the controller is chosen as (7) composed of the fuzzy state-feedback controller (8) and the adaptive compensator(10), and the updating law for the weight is chosen as (11), then the closed-loop system (15) is uniformly ultimately bounded (UUB) and the following tracking performance is achieved as

$$\int_0^T (x(t) - x_r(t))^T Q(x(t) - x_r(t)) dt \leq \tilde{x}^T(0) P \tilde{x}(0) + \frac{1}{\eta_1} \tilde{\Theta}^T(0) \tilde{\Theta}(0) + \rho^2 \int_0^T (w^t w^t) dt, \quad (20)$$

where $\rho > 0, P, Q$ are some symmetric and positive definite matrices.

Proof. Consider the following functional

$$V = \frac{1}{2} \tilde{x}^T P \tilde{x} + \frac{1}{2\eta_1} \tilde{\Theta}^T \tilde{\Theta} \quad (21)$$

whose derivative can be computed as follows:

$$\dot{V} = \frac{1}{2} \dot{\tilde{x}}^T(t) P \tilde{x}(t) + \frac{1}{2} \tilde{x}^T(t) P \dot{\tilde{x}}(t) + \frac{1}{\eta_1} \tilde{\Theta}^T \dot{\tilde{\Theta}} = \dot{V}_1 + \dot{V}_2, \quad (22)$$

where

$$\dot{V}_1 = \left(\sum_{i=1}^L \sum_{j=1}^L \mu_i \mu_j \bar{A}_{ij} \tilde{x}(t) \right)^T P \tilde{x}(t) + \tilde{x}^T(t) P \left(\sum_{i=1}^L \sum_{j=1}^L \mu_i \mu_j \bar{A}_{ij} \tilde{x}(t) \right) + \frac{1}{2} w^t P x(t) + \frac{1}{2} x^T(t) P w^t \quad (23)$$

$$V_2 = [\tilde{x}^T P \bar{B}(-\Psi(x)\tilde{\Theta}) + \frac{1}{\eta_1} \tilde{\Theta}^T \dot{\tilde{\Theta}}]. \quad (24)$$

$$\dot{V}_1 \leq \frac{1}{2} \sum_{i=1}^L \sum_{j=1}^L \mu_i \mu_j \tilde{x}^T(t) (\bar{A}_{ij}^T P + P \bar{A}_{ij} + \frac{1}{\rho^2} P P) \tilde{x}(t) + \frac{1}{2} \rho^2 w^t w^t \quad (25)$$

Substituting (9) into (25) yields

$$\dot{V}_1 \leq -\frac{1}{2}\tilde{x}^T(t)\bar{Q}\tilde{x}(t) + \frac{1}{2}\rho^2 w^T w. \quad (26)$$

From (11),(24)

$$V_2 = [\tilde{x}^t P \bar{B}(-(\Psi(x)\tilde{\Theta}) + \frac{1}{\eta_1}\tilde{\Theta}^T \dot{\tilde{\Theta}})] = 0. \quad (27)$$

Thus

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \leq -\frac{1}{2}\tilde{x}^T(t)\bar{Q}\tilde{x}(t) + \frac{1}{2}\rho^2 w^T w. \quad (28)$$

When $\|e\| > \frac{\rho}{\lambda_{\min}(\bar{Q})}\|\bar{w}\|$, $\dot{V} < 0$. Thus, the closed-loop system (15) is UUB.

Note that

$$\begin{aligned} \int_0^T (x(t) - x_r(t))^T Q (x(t) - x_r(t)) dt &= \int_0^T [x^T(t) \ x_r^T(t)] \begin{bmatrix} Q & -Q \\ -Q & Q \end{bmatrix} [x^T(t) \ x_r^T(t)]^T dt \\ &\leq \int_0^T [x^T(t) \ x_r^T(t)] \text{diag}\{2Q, 2Q\} [x^T(t) \ x_r^T(t)]^T dt = \int_0^T \tilde{x}^T(t)\bar{Q}\tilde{x}(t) dt \end{aligned} \quad (29)$$

Integrating the above inequality (28) from $t=0$ to T yields (20).

By Schur complements, the inequalities (9) are transformed into the linear matrix inequalities. Therefore, the common solution P and $K_j (j=1,2,\dots,L)$ are required to be found. P is chosen as the form $P = \text{diag}\{P_1, P_2\}$, where P_1, P_2 are some symmetric and positive definite matrices. The inequalities (9) are equivalent to the following matrix inequalities

$$\begin{bmatrix} S_{11} & -P_1 B_i K_j & 0 \\ -(B_i K_j)^T P_1 & S_{22} & P_2 \\ 0 & P_2 & -\rho^2 I \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, L, \quad (30)$$

where $S_{11} = P_1(A_i + B_i K_j) + (A_i + B_i K_j)^T P_1 + \frac{1}{\rho^2} P_1 P_1 + 2Q$, and $S_{22} = P_2 A_r + A_r^T P_2 + 2Q$.

The matrix inequalities (30) imply $S_{11} < 0$. Denote $W = P_1^{-1}$ and $Y_j = K_j W$. $S_{11} < 0$ is equivalent to the linear matrix inequalities

$$\begin{bmatrix} S & W \\ W & -(2Q)^{-1} \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, L, \quad (31)$$

where $S = A_i W + W A_i^T + B_i Y_j + (B_i Y_j)^T + (\rho^2)^{-1} I$.

P_1 and $K_j (j = 1, 2, \dots, L)$ are obtained by (31). And then, substituting P_1 and $K_j (j = 1, 2, \dots, L)$ into (30), P_2 is obtained.

Remark 2: If a controlled system is fourth-order, the dimension of the matrix inequalities (30) is 12. By use of the method in [12], the dimension of the matrix inequalities (58) is 20. By use of the method in [9], the dimension of matrix inequalities in Theorem 1 is no less than 20. Thus, the dimension of matrix inequalities is reduced.

4 Simulation example

Consider a 2-link manipulator system in [20]

$$M(q)\ddot{q}(t) + C(q, \dot{q})\dot{q}(t) + G(q) = \tau(t). \tag{32}$$

Consider the existence of the uncertainties and external disturbances in system (32). Thus, the plant is modified as follows:

$$\ddot{q}(t) + C(q, \dot{q})\dot{q}(t) + g(q) = B(q)u(t) + \sum_{i=1}^r \xi_i(t)q(t) + dt, \tag{33}$$

where $C(q, \dot{q}) = H^{-1}(q)C'(q, \dot{q})$, $g(q) = H^{-1}(q)g'(q)$, $B(q) = H^{-1}(q)$, $dt = H^{-1}(q)d$, $q = [q_1, q_2]^T$ and $u(t) = [u_1, u_2]^T = \tau(t)$. $\xi_i(t)$ ($i = 1, 2, \dots, r$) are uncertain and bounded. d is random noise with zero mean and variance 0.05, and d is bounded.

The reference model is as follows:

$$\dot{x}_r(t) = A_r x_r(t) + r(t),$$

where

$$A_r = \text{diag}\{A_{r1}, A_{r2}\}, A_{r1} = A_{r2} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}, r(t) = [0, r_1(t), 0, r_2(t)]^T.$$

Denote $x_1 = q_1, x_2 = \dot{q}_1, x_3 = q_2, x_4 = \dot{q}_2$. Fuzzy T-S model is used to approximate the nonlinear system at $x_1 = -\frac{\pi}{2}, 0, \frac{\pi}{2}$ and $x_3 = -\frac{\pi}{2}, 0, \frac{\pi}{2}$. The membership functions are adopted as triangle type. Fuzzy T-S model with Nine rules in the form (3) is given, where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 5.927 & -0.001 & -0.315 & -0.0000084 \\ 0 & 0 & 0 & 1 \\ -6.859 & 0.002 & 3.155 & 0.0000062 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3.0428 & -0.0011 & -0.1791 & -0.0002 \\ 0 & 0 & 0 & 1 \\ -3.5436 & 0.0313 & 2.5611 & 0.0000114 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.2728 & 0.003 & 0.4339 & -0.0001 \\ 0 & 0 & 0 & 1 \\ -9.1041 & 0.0158 & -1.0574 & -0.000032 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.5434 & 0.0017 & 1.2427 & -0.0002 \\ 0 & 0 & 0 & 1 \\ -3.1873 & 0.0306 & -5.1911 & -0.000018 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 11.1336 & 0 & -1.8145 & 0 \\ 0 & 0 & 0 & 1 \\ -9.0918 & 0 & 9.1638 & 0 \end{bmatrix}, A_6 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.1702 & -0.001 & 1.687 & -0.0002 \\ 0 & 0 & 0 & 1 \\ -2.3559 & 0.0314 & 4.5298 & -0.000011 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.1206 & 0.0041 & 0.6205 & 0.0001 \\ 0 & 0 & 0 & 1 \\ 8.8794 & 0.0193 & -1.0119 & 0.000044 \end{bmatrix}, A_8 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3.6421 & -0.0018 & 0.0721 & 0.0002 \\ 0 & 0 & 0 & 1 \\ 2.429 & -0.0305 & 2.9832 & -0.000019 \end{bmatrix},$$

$$A_9 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.2933 & -0.0009 & 0.2188 & -0.000012 \\ 0 & 0 & 0 & 1 \\ -7.4649 & 0.0024 & 3.2693 & -0.0000092 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 2 \end{bmatrix}^T, B_2 = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$B_3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}^T, B_4 = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T, B_5 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 2 \end{bmatrix}^T, B_6 = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$B_7 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}^T, B_8 = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T, B_9 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 2 \end{bmatrix}^T.$$

Using the LMI box in Matlab, $K_j (j = 1, 2, \dots, L)$ are obtained

$$K_1 = \begin{bmatrix} -75.5707 & -41.2895 & -19.7728 & -8.9886 \\ 4.5163 & -0.5944 & -49.7511 & -24.5727 \end{bmatrix}, K_2 = \begin{bmatrix} -76.4364 & -41.3132 & -13.7595 & -5.9976 \\ 7.1834 & 1.0290 & -49.0587 & -24.2578 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -75.4503 & -41.2461 & -19.8614 & -9.0371 \\ 4.3907 & -0.6474 & -49.6965 & -24.54837 \end{bmatrix}, K_4 = \begin{bmatrix} -76.4364 & -41.3132 & -13.7595 & -5.9976 \\ 7.1834 & 1.0290 & -49.0587 & -24.2578 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} -76.4364 & -41.3132 & -13.7595 & -5.9976 \\ 7.1834 & 1.0290 & -49.0587 & -24.2578 \end{bmatrix}, K_6 = \begin{bmatrix} -76.4364 & -41.3132 & -13.7595 & -5.9976 \\ 7.1834 & 1.0290 & -49.0587 & -24.2578 \end{bmatrix},$$

$$K_7 = \begin{bmatrix} -76.4364 & -41.9511 & -6.2666 & -2.2808 \\ 11.1565 & 3.4005 & -48.7116 & -24.1209 \end{bmatrix}, K_8 = \begin{bmatrix} -76.4364 & -41.3132 & -13.7595 & -5.9976 \\ 7.1834 & 1.0290 & -49.0587 & -24.2578 \end{bmatrix},$$

$$K_9 = \begin{bmatrix} lcl - 76.7763 & -42.0400 & -6.0715 & -2.1821 \\ -11.3875 & 3.4986 & -48.8204 & -24.1694 \end{bmatrix},$$

Then, the controller is given by

$$u(t) = u_l(t) - u_f(t),$$

where

$$u_l(t) = \sum_{i=1}^9 \mu_i K_i (x(t) - x_r(t)) \text{ and } u_f = \left(\sum_{i=1}^9 \mu_i E_i \right)^{-1} \hat{u}(x|\Theta)$$

with the updating law (11). In (11), the symmetric and positive definite matrix

$$P = \begin{bmatrix} 0.0070 & 0.0035 & -0.0004 & -0.0002 \\ 0.0035 & 0.0021 & 0.0004 & 0.0001 \\ -0.0004 & 0.0004 & 0.0073 & 0.0032 \\ -0.0002 & 0.0001 & 0.0032 & 0.0017 \end{bmatrix}.$$

Seven fuzzy rules are defined in adaptive fuzzy logic systems.

$R^{(j)}$: if x_1 is F_1^j, \dots, x_4 is F_4^j , then y is G^j ($j=1,2,\dots,7$),

$$\mu_{F_i^1}(x_i) = \frac{1}{1 + \exp[5(x_i + 0.8)]} (i = 1, 2, \dots, 4), \quad \mu_{F_i^1}(x_i) = \exp[-(x_i + 0.6)^2] (i = 1, 2, \dots, 4),$$

$$\mu_{F_i^3}(x_i) = \exp[-(x_i + 0.4)^2] (i = 1, 2, \dots, 4), \quad \mu_{F_i^4}(x_i) = \exp[-(x_i)^2] (i = 1, 2, \dots, 4),$$

$$\mu_{F_i^5}(x_i) = \exp[-(x_i - 0.4)^2] (i = 1, 2, \dots, 4), \quad \mu_{F_i^6}(x_i) = \exp[-(x_i - 0.6)^2] (i = 1, 2, \dots, 4),$$

$$\mu_{F_i^7}(x_i) = \frac{1}{1 + \exp[5(x_i - 0.8)]} (i = 1, 2, \dots, 4).$$

Denote $S_1 = \sum_{j=1}^7 \sum_{i=1}^4 \mu_{F_i^j}(x_i)$, then

$$\xi(x) = \left[\prod_{i=1}^4 \mu_{F_i^1}(x_i)/S_1, \dots, \prod_{i=1}^4 \mu_{F_i^7}(x_i)/S_1 \right] = [\xi_1, \dots, \xi_7], \Psi(x) = \text{diag}[\xi^T(x), \xi^T(x)].$$

The initial condition is set to be

$$(x_1(0), x_2(0), x_3(0), x_4(0), x_{r1}(0), x_{r2}(0), x_{r3}(0), x_{r4}(0)) = (0.4, 0, -0.4, 0, 0, 0, 0, 0).$$

Choose $r_1(t) = r_2(t) = 4\sin(t), r = 2, \xi_1(t) = 1 + 20 \sin(t), \xi_2(t) = 2(1 - \exp(-t))/(1 + \exp(-t))$ and the parameter $\eta_1 = 20$. Simulation results are shown in Fig.1- Fig.3.

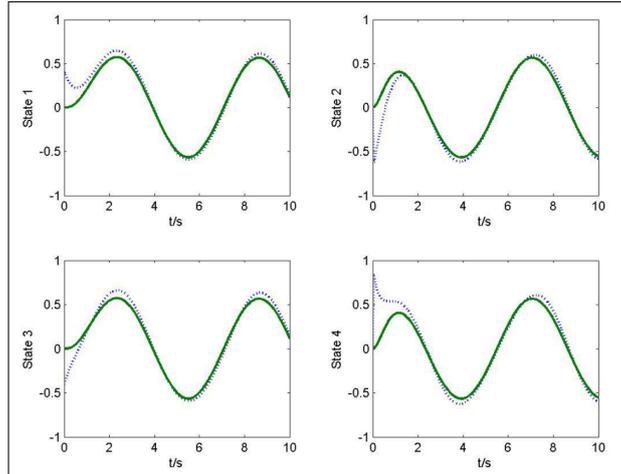


Figure 1: State responses of x_1, x_2, x_3 and x_4 (dotted line), x_{r1}, x_{r2}, x_{r3} and x_{r4} (solid line)

By only using the fuzzy controller based on T-S model [20], simulation result is shown in Fig.4 and the tracking performance comparison between proposed method and approach in [20] is given in table 1.

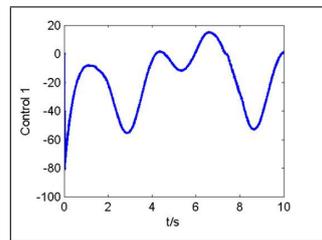
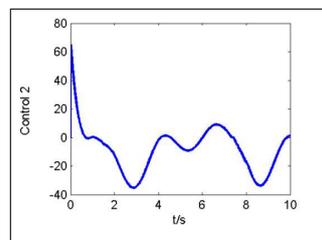
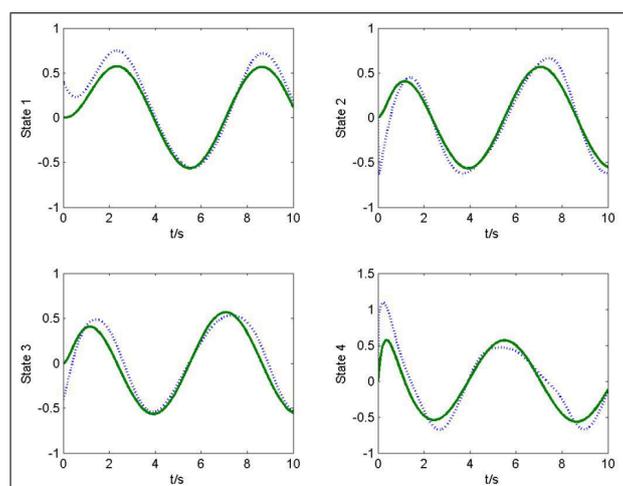
Figure 2: The control input u_1 Figure 3: The control input u_2 Figure 4: State responses of x_1, x_2, x_3 and x_4 (dotted line), x_{r1}, x_{r2}, x_{r3} and x_{r4} (solid line)

Table 1: The tracking performance comparison between proposed method and approach in [20].

$E_i = \int_0^t (x_i - x_{ri})^2 dt$	E_1	E_2	E_3	E_4
The proposed method	0.0677	0.1338	0.0587	0.2177
The method in [20]	0.1380	0.1760	0.1244	0.3343

5 Conclusion

The developed controller makes full use of the advantages of two fuzzy models. Theory analysis verifies the feasibility of the proposed control scheme and simulation results demonstrate the effectiveness of the proposed control scheme.

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