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A Non-Fragile H_∞ Output Feedback Controller for Uncertain Fuzzy Dynamical Systems with Multiple Time-Scales

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Abstract:
This paper determines the designing of a non-fragile H_∞ output feedback controller for a class of nonlinear uncertain dynamical systems with multiple time-scales described by a Takagi-Sugeno (TS) fuzzy model. Based on a linear matrix inequality (LMI) approach, we develop a non-fragile H_∞ output feedback controller which guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value for this class of uncertain fuzzy dynamical systems with multiple time-scales. A numerical example is provided to illustrate the design developed in this paper.
Keywords: Fuzzy Control, Linear Matrix Inequality (LMI), Non-fragile $\mathcal{H}_\infty$ Output Feedback Control, Multiple Time-Scale Systems.

1 Introduction

In the last few years, the problem of control design for dynamical systems with multiple time-scale has been intensively studied by a number of researchers; see [1]- [12]. This is due not only to theoretical interest but also to the relevance of this topic in control engineering applications. Singularly perturbed systems are dynamical systems with multiple time-scales. Singularly perturbed systems often occur naturally due to the presence of small "parasitic" parameter, typically small time constants, masses, etc. Indeed multiple time-scales phenomena are almost unavoidable in "real-life" systems. Examples of such systems abound and include convection-diffusion systems, diffusion-drift motion systems, power systems, scheduling systems, economic models, telecommunication systems and bifurcations.

Presently, many researchers have studied the H_∞ control design for a general class of linear singularly perturbed systems due to a great practical importance; see [4, 5, 7]. The main purpose of the singular perturbation approach to analysis and design is the alleviation of high dimensionality and ill-conditioning resulting from the interaction of slow and fast dynamics modes. The separation of states into slow and fast ones is a nontrivial modelling task demanding insight and ingenuity on the part of the analyst. In state space, such systems are commonly modelled using the mathematical framework of singular perturbations, with a small parameter, say ε, determining the degree of separation between the "slow" and "fast" modes of the system. Although many researchers have studied linear singularly perturbed systems for many years, the H_∞ control design of nonlinear singularly perturbed systems remains as an open research area. This is because, in general, nonlinear singularly perturbed systems can not be separated into slow and fast subsystems.

Over the past two decades, there has been rapidly growing interest in application of fuzzy logic to control problem. Researches have been focused on its application to industrial processes and a number of successful results have been reported in the literature. In spite of these successes, there are many basic issues remain to be addressed. One of them is how to achieve a systematic design that guarantees closed-loop stability and performance. Recently, a great amount of effort has been devoted to describing a nonlinear system using a Takagi-Sugeno fuzzy model; see [16]-[29]. The Takagi-sugeno (TS) fuzzy model represents a nonlinear system by a family of local linear models which smoothly blended together through fuzzy membership functions. Unlike conventional modelling techniques which uses a single model to describe the global behavior of a nonlinear system, fuzzy modelling is essentially a multi-model approach in which simple sub-models (typically linear models) are fuzzily combined to described the global behavior of a nonlinear system. Based on this fuzzy model, a number of systematic model-based fuzzy control design methodologies have been developed.

The aim of this paper is to design a non-fragile $\mathcal{H}_\infty$ output feedback controller for a uncertain nonlinear dynamical system with multuple time-scales. Based on an LMI approach, we develop the fuzzy non-fragile $\mathcal{H}_\infty$ output feedback controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than or equal to a prescribed value for this class of fuzzy dynamical systems. In order to alleviate the ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes, the ill-conditioned LMIs are decomposed into ε-independent and ε-dependent LMIs. The ε-independent LMIs are not ill-conditioned and the ε-dependent LMIs tend to zero when ε approaches to zero. It can be shown that when ε is sufficiently small, the original ill-conditioned LMIs are solvable if and only if the ε-independent LMIs are solvable. The proposed approach does not involve the separation of states into slow and fast ones, and it can be applied not only to standard, but also to nonstandard singularly perturbed systems.

This paper is organized as follows. In Section 2, system descriptions and definition are presented. In Section 3, based on an LMI approach, we respectively develop a technique for designing a non-fragile $\mathcal{H}_\infty$ output feedback controllers such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value for the system described in Section 2. The validity of this approach is demonstrated by an example from a literature in Section 4. Finally, conclusions are given in Section 5.

2 System Descriptions and Definitions

In this section, we consider the TS fuzzy system with multiple time-scales to represent a TS fuzzy multiple time-scale system with parametric uncertainties as follows:

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= \textstyle\sum_{i=1}^r \mu_i(\nu(t))\Big[[A_i + \Delta A_i]x(t) + [B_{1_i} + \Delta B_{1_i}]w(t) + [B_{2_i} + \Delta B_{2_i}]u(t)\Big] \\ z(t) &= \textstyle\sum_{i=1}^r \mu_i(\nu(t))\Big[[C_{1_i} + \Delta C_{1_i}]x(t) + [D_{12_i} + \Delta D_{12_i}]u(t)\Big] \\ y(t) &= \textstyle\sum_{i=1}^r \mu_i(\nu(t))\Big[[C_{2_i} + \Delta C_{2_i}]x(t) + [D_{21_i} + \Delta D_{21_i}]w(t)\Big] \end{aligned} \tag{1}$$

where $E_\varepsilon = \begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}$, $\nu(t) = [\nu_1(t) \ \cdots \ \nu_\vartheta(t)]$ is the premise variable vector that may depend on states in many cases, $\varepsilon > 0$ is the singular perturbation parameter, $\mu_i(\nu(t))$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e., $\mu_i(\nu(t)) \geq 0$ and $\sum_{i=1}^r \mu_i(\nu(t)) = 1$), ϑ is the number of fuzzy sets, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $w(t) \in \Re^p$ is the disturbance which belongs to $\mathcal{L}_2[0,\infty)$, $y(t) \in \Re^\ell$ is the measurement, $z(t) \in \Re^s$ is the controlled output, the matrices $A_i, B_{1_i}, B_{2_i}, C_{1_i}, C_{2_i}, D_{12_i}$ and D_{21_i} are of appropriate dimensions, and r is the number of IF-THEN rules. The matrices $\Delta A_i, \Delta B_{1_i}, \Delta B_{2_i}, \Delta C_{1_i}, \Delta C_{2_i}, \Delta D_{12_i}$ and ΔD_{21_i} represent the uncertainties in the system and satisfy the following assumption.

Assumption 1.

$$\Delta A_i = F(x(t),t)H_{1_i},\ \ \Delta B_{1_i} = F(x(t),t)H_{2_i},\ \ \Delta B_{2_i} = F(x(t),t)H_{3_i},\ \ \Delta C_{1_i} = F(x(t),t)H_{4_i},$$

$$\Delta C_{2_i} = F(x(t),t)H_{5_i},\ \ \Delta D_{12_i} = F(x(t),t)H_{6_i}\ \ \textit{and}\ \ \Delta D_{21_i} = F(x(t),t)H_{7_i}$$

where H_{j_i}, $j = 1, 2, \cdots, 7$ are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t),t)\| \leq \rho \tag{2}$$

for any known positive constant ρ. Next, let us recall the following definition.

Definition 1. *Suppose γ is a given positive number. A system (1) is said to have an $\mathcal{L}_2$-gain less than or equal to γ if*

$$\int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2 \left[\int_0^{T_f} w^T(t)w(t)dt\right], \tag{3}$$

for all $T_f \geq 0$, $x(0) = 0$ and $w(t) \in \mathcal{L}_2[0, T_f]$.

Note that for the symmetric block matrices, we use $(*)$ as an ellipsis for terms that are induced by symmetry.

3 Non-fragile $\mathcal{H}_\infty$ Output Feedback Controller

The nature of the information of the state available to the controller has a major effect on the complexity of the designing problem and of the resulting controller. The state-feedback control design problem is an easier problem in which all information are available. However, in most real physical systems, the state is not perfectly known, and so we must estimate it. The process of estimating the system state from the measurement output that are available is called the estimator design. By utilizing the state estimator, the output feedback problem is converted to the state-feedback problem for a new problem. This new problem employs the estimated state as its own state variable and the solution of the new state-feedback problem leads to the solution of the dynamic output feedback control problem. Basically, the dynamic output feedback is a coupling of control and estimation.

This section aims at designing a full order dynamic non-fragile $\mathcal{H}_\infty$ fuzzy output feedback controller of the form

$$E_\varepsilon \dot{\hat{x}}(t) \ = \ \sum_{i=1}^r \sum_{j=1}^r \hat{\mu}_i \hat{\mu}_j \left[\hat{A}_{ij}(\varepsilon)\hat{x}(t) + \hat{B}_i y(t)\right], \qquad u(t) = \sum_{i=1}^r \hat{\mu}_i \hat{C}_i \hat{x}(t) \tag{4}$$

where $\hat{x}(t) \in \Re^n$ is the controller's state vector, $\hat{A}_{ij}$, $\hat{B}_i$ and $\hat{C}_i$ are parameters of the controller which are to be determined, and $\hat{\mu}_i$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e., $\hat{\mu}_i \geq 0$ and $\sum_{i=1}^r \hat{\mu}_i = 1$), such that the inequality (3) holds.

Clearly, in real control problems, all of the premise variables are not necessarily measurable. Thus, in this section, we consider the designing of the non-fragile $\mathcal{H}_\infty$ output feedback control into two cases as follows. In Subsection 3.1, we consider the case where the premise variable of the fuzzy model μ_i is measurable, while in Subsection 3.2, the premise variable which is assumed to be unmeasurable is considered.

3.1 Case I–$\nu(t)$ is available for feedback

The premise variable of the fuzzy model $\nu(t)$ is available for feedback which implies that μ_i is available for feedback. Thus, we can select our controller that depends on μ_i as follows:

$$E_\varepsilon \dot{\hat{x}}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \Big[\hat{A}_{ij}(\varepsilon)\hat{x}(t) + \hat{B}_i y(t)\Big], \qquad u(t) = \sum_{i=1}^r \mu_i \hat{C}_i \hat{x}(t). \tag{5}$$

Before presenting our next results, the following lemma is recalled.

Lemma 1. *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance γ and a positive constant δ, if there exist matrices $X_\varepsilon = X_\varepsilon^T$, $Y_\varepsilon = Y_\varepsilon^T$, $\mathcal{B}_i(\varepsilon)$ and $\mathcal{C}_i(\varepsilon)$, $i = 1, 2, \cdots, r$, satisfying the following ε-dependent linear matrix inequalities:*

$$\begin{aligned}
\begin{bmatrix} X_\varepsilon & I \\ I & Y_\varepsilon \end{bmatrix} &> 0 && (6)\\
X_\varepsilon &> 0 \quad \textit{and} \quad Y_\varepsilon > 0 && (7)\\
\Psi_{11_{ii}}(\varepsilon) \quad \textit{and} \quad \Psi_{22_{ii}}(\varepsilon) &< 0, \quad i = 1, 2, \cdots, r && (8)\\
\Psi_{11_{ij}}(\varepsilon) + \Psi_{11_{ji}}(\varepsilon) \quad \textit{and} \quad \Psi_{22_{ij}}(\varepsilon) + \Psi_{22_{ji}}(\varepsilon) &< 0, \quad i < j \le r && (9)
\end{aligned}$$

where

$$\Psi_{11_{ij}}(\varepsilon) = \begin{pmatrix} \begin{pmatrix} E_\varepsilon^{-1}A_iY_\varepsilon + Y_\varepsilon A_i^T E_\varepsilon^{-1} + E_\varepsilon^{-1}B_{2_i}\mathcal{C}_j(\varepsilon)E_\varepsilon^{-1} \\ +E_\varepsilon^{-1}\mathcal{C}_i^T(\varepsilon)B_{2_j}^T E_\varepsilon^{-1} + \gamma^{-2}E_\varepsilon^{-1}\tilde{B}_{1_i}\tilde{B}_{1_j}^T E_\varepsilon^{-1} \end{pmatrix} & (*)^T \\ \Big[Y_\varepsilon \tilde{C}_{1_i}^T + E_\varepsilon^{-1}\mathcal{C}_i^T(\varepsilon)\tilde{D}_{12_j}^T\Big]^T & -I \end{pmatrix} \tag{10}$$

$$\Psi_{22_{ij}}(\varepsilon) = \begin{pmatrix} \begin{pmatrix} A_i^T E_\varepsilon^{-1} X_\varepsilon + X_\varepsilon E_\varepsilon^{-1} A_i \\ +\mathcal{B}_i(\varepsilon)C_{2_j} + C_{2_i}^T\mathcal{B}_j^T(\varepsilon) + \tilde{C}_{1_i}^T\tilde{C}_{1_j} \end{pmatrix} & (*)^T \\ \Big[X_\varepsilon E_\varepsilon^{-1}\tilde{B}_{1_i} + \mathcal{B}_i(\varepsilon)\tilde{D}_{21_j}\Big]^T & -\gamma^2 I \end{pmatrix} \tag{11}$$

with

$$\tilde{B}_{1_i} = \begin{bmatrix} \delta I & I & \delta I & 0 & B_{1_i} & 0 \end{bmatrix}, \quad \tilde{C}_{1_i} = \begin{bmatrix} \frac{\gamma\rho}{\delta}H_{1_i}^T & 0 & \frac{\gamma\rho}{\delta}H_{5_i}^T & \sqrt{2}\lambda\rho H_{4_i}^T & \sqrt{2}\lambda C_{1_i}^T \end{bmatrix}^T,$$

$$\tilde{D}_{12_i} = \begin{bmatrix} 0 & \frac{\gamma\rho}{\delta}H_{3_i}^T & 0 & \sqrt{2}\lambda\rho H_{6_i}^T & \sqrt{2}\lambda D_{12_i}^T \end{bmatrix}^T, \quad \tilde{D}_{21_i} = \begin{bmatrix} 0 & 0 & 0 & \delta I & D_{21_i} & I \end{bmatrix}$$

$$\textit{and} \ \ \lambda = \left(1 + \rho^2 \sum_{i=1}^r \sum_{j=1}^r \Big[\|H_{2_i}^T H_{2_j}\| + \|H_{7_i}^T H_{7_j}\|\Big]\right)^{\frac{1}{2}},$$

then the system (1) has the prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$. Furthermore, a suitable controller is of the form (5) with

$$\begin{aligned}
\hat{A}_{ij}(\varepsilon) &= E_\varepsilon\Big[Y_\varepsilon^{-1} - X_\varepsilon\Big]^{-1}\mathcal{M}_{ij}(\varepsilon)Y_\varepsilon^{-1} \\
\hat{B}_i &= E_\varepsilon\Big[Y_\varepsilon^{-1} - X_\varepsilon\Big]^{-1}\mathcal{B}_i(\varepsilon) \quad \textit{and} \quad \hat{C}_i = \mathcal{C}_i(\varepsilon)E_\varepsilon^{-1}Y_\varepsilon^{-1}
\end{aligned} \tag{12}$$

$$\begin{aligned}
\textit{where} \ \ \mathcal{M}_{ij}(\varepsilon) = {} & -A_i^T E_\varepsilon^{-1} - X_\varepsilon E_\varepsilon^{-1}A_iY_\varepsilon - X_\varepsilon E_\varepsilon^{-1}B_{2_i}\hat{C}_jY_\varepsilon \\
& -\big[Y_\varepsilon^{-1} - X_\varepsilon\big]E_\varepsilon^{-1}\hat{B}_iC_{2_j}Y_\varepsilon - \tilde{C}_{1_i}^T\big[\tilde{C}_{1_j}Y_\varepsilon + \tilde{D}_{12_j}\hat{C}_jY_\varepsilon\big] \\
& -\gamma^{-2}\Big\{X_\varepsilon E_\varepsilon^{-1}\tilde{B}_{1_i} + \big[Y_\varepsilon^{-1} - X_\varepsilon\big]E_\varepsilon^{-1}\hat{B}_i\tilde{D}_{21_i}\Big\}\tilde{B}_{1_j}^T E_\varepsilon^{-1}.
\end{aligned} \tag{13}$$

Proof: The proof can be carried out the same technique used in Lemma 1. □

Remark 1. *The LMIs given in Lemma 3.1 may become ill-conditioned when ε is sufficiently small, which is always the case for the multiple time-scale systems. In general, these ill-conditioned LMIs are very difficult to solve. Thus, to alleviate these ill-conditioned LMIs, we have the following ε-independent well-posed LMI-based sufficient conditions for the uncertain fuzzy multiple time-scale systems to obtain the prescribed $\mathcal{H}_\infty$ performance.*

Theorem 1. *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and a positive constant δ, if there exist matrices X_0, Y_0, $\mathcal{B}_{0_i}$ and $\mathcal{C}_{0_i}$, $i = 1, 2, \cdots, r$, satisfying the following ε-independent linear matrix inequalities:*

$$\begin{bmatrix} X_0E + DX_0 & I \\ I & Y_0E + DY_0 \end{bmatrix} > 0 \tag{14}$$

$$EX_0^T = X_0E, \ X_0^TD = DX_0, \ X_0E + DX_0 > 0 \tag{15}$$

$$EY_0^T = Y_0E, \ Y_0^TD = DY_0, \ Y_0E + DY_0 > 0 \tag{16}$$

$$\Psi_{11_{ii}} \quad and \quad \Psi_{22_{ii}} < 0, \quad i = 1, 2, \cdots, r \tag{17}$$

$$\Psi_{11_{ij}} + \Psi_{11_{ji}} \quad and \quad \Psi_{22_{ij}} + \Psi_{22_{ji}} < 0, \quad i < j \leq r \tag{18}$$

where $E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$,

$$\Psi_{11_{ij}} = \begin{pmatrix} A_iY_0^T + Y_0A_i^T + B_{2_i}\mathcal{C}_{0_j} + \mathcal{C}_{0_i}^TB_{2_j}^T + \gamma^{-2}\tilde{B}_{1_i}\tilde{B}_{1_j}^T & (*)^T \\ \left[Y_0\tilde{C}_{1_i}^T + \mathcal{C}_{0_i}^T\tilde{D}_{12_j}^T\right]^T & -I \end{pmatrix} \tag{19}$$

$$\Psi_{22_{ij}} = \begin{pmatrix} A_i^TX_0^T + X_0A_i + \mathcal{B}_{0_i}C_{2_j} + C_{2_i}^T\mathcal{B}_{0_j}^T + \tilde{C}_{1_i}^T\tilde{C}_{1_j} & (*)^T \\ \left[X_0\tilde{B}_{1_i} + \mathcal{B}_{0_i}\tilde{D}_{21_j}\right]^T & -\gamma^2I \end{pmatrix} \tag{20}$$

with

$$\tilde{B}_{1_i} = \begin{bmatrix} \delta I & I & \delta I & 0 & B_{1_i} & 0 \end{bmatrix}, \ \tilde{C}_{1_i} = \begin{bmatrix} \frac{\gamma\rho}{\delta}H_{1_i}^T & 0 & \frac{\gamma\rho}{\delta}H_{5_i}^T & \sqrt{2}\lambda\rho H_{4_i}^T & \sqrt{2}\lambda C_{1_i}^T \end{bmatrix}^T$$

$$\tilde{D}_{12_i} = \begin{bmatrix} 0 & \frac{\gamma\rho}{\delta}H_{3_i}^T & 0 & \sqrt{2}\lambda\rho H_{6_i}^T & \sqrt{2}\lambda D_{12_i}^T \end{bmatrix}^T, \ \tilde{D}_{21_i} = \begin{bmatrix} 0 & 0 & 0 & \delta I & D_{21_i} & I \end{bmatrix}$$

$$and \ \lambda = \left(1 + \rho^2\sum_{i=1}^{r}\sum_{j=1}^{r}\left[\|H_{2_i}^TH_{2_j}\| + \|H_{7_i}^TH_{7_j}\|\right]\right)^{\frac{1}{2}},$$

then there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, the prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ is guaranteed. Furthermore, a suitable controller is of the form (5) with

$$\hat{A}_{ij}(\varepsilon) = \left[Y_\varepsilon^{-1} - X_\varepsilon\right]^{-1}\mathcal{M}_{0_{ij}}(\varepsilon)Y_\varepsilon^{-1}, \quad \hat{B}_i = \left[Y_0^{-1} - X_0\right]^{-1}\mathcal{B}_{0_i}, \quad and \quad \hat{C}_i = \mathcal{C}_{0_i}Y_0^{-1} \tag{21}$$

$$\begin{aligned} where \quad \mathcal{M}_{0_{ij}}(\varepsilon) = & -A_i^T - X_\varepsilon A_iY_\varepsilon - X_\varepsilon B_{2_i}\hat{C}_jY_\varepsilon - \left[Y_\varepsilon^{-1} - X_\varepsilon\right]\hat{B}_iC_{2_j}Y_\varepsilon \\ & -\tilde{C}_{1_i}^T\left[\tilde{C}_{1_j}Y_\varepsilon + \tilde{D}_{12_j}\hat{C}_jY_\varepsilon\right] - \gamma^{-2}\left\{X_\varepsilon\tilde{B}_{1_i} + \left[Y_\varepsilon^{-1} - X_\varepsilon\right]\hat{B}_i\tilde{D}_{21_i}\right\}\tilde{B}_{1_j}^T \end{aligned} \tag{22}$$

$$X_\varepsilon = \left\{X_0 + \varepsilon\tilde{X}\right\}E_\varepsilon \quad and \quad Y_\varepsilon^{-1} = \left\{Y_0^{-1} + \varepsilon N_\varepsilon\right\}E_\varepsilon \tag{23}$$

with $\tilde{X} = D\Big(X_0^T - X_0\Big)$ *and* $N_\varepsilon = D\Big((Y_0^{-1})^T - Y_0^{-1}\Big)$.

Proof: Suppose the inequalities (14)-(16) hold, then the matrices X_0 and Y_0 are of the following forms:

$$X_0 = \begin{pmatrix} X_1 & X_2 \\ 0 & X_3 \end{pmatrix} \text{ and } Y_0 = \begin{pmatrix} Y_1 & Y_2 \\ 0 & Y_3 \end{pmatrix}$$

with $X_1 = X_1^T > 0$, $X_3 = X_3^T > 0$, $Y_1 = Y_1^T > 0$ and $Y_3 = Y_3^T > 0$. Substituting X_0 and Y_0 into (23), respectively, we have

$$X_\varepsilon = \Big\{X_0 + \varepsilon\tilde{X}\Big\}E_\varepsilon = \begin{pmatrix} X_1 & \varepsilon X_2 \\ \varepsilon X_2^T & \varepsilon X_3 \end{pmatrix} \tag{24}$$

$$Y_\varepsilon^{-1} = \Big\{Y_0^{-1} + \varepsilon N_\varepsilon\Big\}E_\varepsilon = \begin{pmatrix} Y_1^{-1} & -\varepsilon Y^{-1}Y_2Y_3^{-1} \\ -\varepsilon(Y^{-1}Y_2Y_3^{-1})^T & \varepsilon Y_3^{-1} \end{pmatrix}. \tag{25}$$

Clearly, $X_\varepsilon = X_\varepsilon^T$, and $Y_\varepsilon^{-1} = (Y_\varepsilon^{-1})^T$. Knowing the fact that the inverse of a symmetric matrix is a symmetric matrix, we learn that Y_ε is a symmetric matrix. Using the matrix inversion lemma, we can see that

$$Y_\varepsilon = E_\varepsilon^{-1}\Big\{Y_0 + \varepsilon\tilde{Y}\Big\} \tag{26}$$

where $\tilde{Y} = Y_0N_\varepsilon(I + \varepsilon Y_0N_\varepsilon)^{-1}Y_0$. Employing the Schur complement, one can show that there exists a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (7) holds.

Now, we need to show that

$$\begin{pmatrix} X_\varepsilon & I \\ I & Y_\varepsilon \end{pmatrix} > 0. \tag{27}$$

By the Schur complement, it is equivalent to showing that

$$X_\varepsilon - Y_\varepsilon^{-1} > 0. \tag{28}$$

Substituting (24) and (25) into the left hand side of (28), we get

$$\begin{bmatrix} X_1 - Y_1^{-1} & \varepsilon(X_2 + Y_1^{-1}Y_2Y_3^{-1}) \\ \varepsilon(X_2 + Y_1^{-1}Y_2Y_3^{-1})^T & \varepsilon(X_3 - Y_3^{-1}) \end{bmatrix}. \tag{29}$$

The Schur complement of (14) is

$$\begin{bmatrix} X_1 - Y_1^{-1} & 0 \\ 0 & X_3 - Y_3^{-1} \end{bmatrix} > 0. \tag{30}$$

According to (30), we learn that

$$X_1 - Y_1^{-1} > 0 \qquad \text{and} \qquad X_3 - Y_3^{-1} > 0. \tag{31}$$

Using (31) and the Schur complement, it can be shown that there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (6) holds.

Next, employing (24), (25) and (26), the controller's matrices given in (12) can be re-expressed as follows:

$$\begin{aligned} \mathcal{B}_i(\varepsilon) &= \big[Y_0^{-1} - X_0\big]\hat{B}_i + \varepsilon\big[N_\varepsilon - \tilde{X}\big]\hat{B}_i\mathcal{B}_{0_i} + \varepsilon\mathcal{B}_{\varepsilon_i} \\ \mathcal{C}_i(\varepsilon) &= \hat{C}_iY_0^T + \varepsilon\hat{C}_i\tilde{Y}^T\mathcal{C}_{0_i} + \varepsilon\mathcal{C}_{\varepsilon_i}. \end{aligned} \tag{32}$$

Substituting (24), (25), (26) and (32) into (10) and (11), and pre-post multiplying (10) by $\begin{pmatrix} E_\varepsilon & 0 \\ 0 & I \end{pmatrix}$, we, respectively, obtain

$$\Psi_{11_{ij}} + \psi_{11_{ij}} \text{ and } \Psi_{22_{ij}} + \psi_{22_{ij}} \tag{33}$$

where the ε-independent linear matrices $\Psi_{11_{ij}}$ and $\Psi_{22_{ij}}$ are defined in (19) and (20), respectively and the ε-dependent linear matrices are

$$\psi_{11_{ij}} = \varepsilon \begin{pmatrix} A_i\tilde{Y}^T + \tilde{Y}A_i^T + B_{2_i}\mathcal{C}_{\varepsilon_j} + \mathcal{C}_{\varepsilon_i}^T B_{2_j}^T & (*)^T \\ \left[\tilde{Y}\tilde{C}_{1_i}^T + \mathcal{C}_{\varepsilon_i}^T \tilde{D}_{12_j}^T\right]^T & 0 \end{pmatrix} \tag{34}$$

$$\psi_{22_{ij}} = \varepsilon \begin{pmatrix} A_i^T\tilde{X} + \tilde{X}^T A_i + \mathcal{B}_{\varepsilon_i}C_{2_j} + C_{2_i}^T\mathcal{B}_{\varepsilon_j}^T & (*)^T \\ \left[\tilde{X}\tilde{B}_{1_i} + \mathcal{B}_{\varepsilon_i}\tilde{D}_{21_j}\right]^T & 0 \end{pmatrix}. \tag{35}$$

Note that the ε-dependent linear matrices tend to zero when ε approaches zero.

Employing (17)-(18) and knowing the fact that for any given negative definite matrix $\mathcal{W}$, there exists an $\varepsilon > 0$ such that $\mathcal{W} + \varepsilon I < 0$, one can show that there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (8)-(9) hold. Since (6)-(9) hold, using Lemma 2, the inequality (3) holds. □

3.2 Case II–$\nu(t)$ is unavailable for feedback

The output feedback fuzzy controller is assumed to be the same as the premise variables of the fuzzy system model. This actually means that the premise variables of fuzzy system model are assumed to be measurable. However, in general, it is extremely difficult to derive an accurate fuzzy system model by imposing that all premise variables are measurable. In this subsection, we do not impose that condition, we choose the premise variables of the controller to be different from the premise variables of fuzzy system model of the plant. In here, the premise variables of the controller are selected to be the estimated premise variables of the plant. In the other words, the premise variable of the fuzzy model $\nu(t)$ is unavailable for feedback which implies μ_i is unavailable for feedback. Hence, we cannot select our controller which depends on μ_i. Thus, we select our controller as (4) where $\hat{\mu}_i$ depends on the premise variable of the controller which is different from μ_i. Let us re-express the system (1) in terms of $\hat{\mu}_i$, thus the plant's premise variable becomes the same as the controller's premise variable. By doing so, the result given in the previous case can then be applied here. Note that it can be done by using the same technique as in subsection. After some manipulation, we get

$$\begin{array}{lcl} E_\varepsilon \dot{x}(t) & = & \sum_{i=1}^r \hat{\mu}_i \Big[[A_i + \Delta\bar{A}_i]x(t) + [B_{1_i} + \Delta\bar{B}_{1_i}]w(t) + [B_{2_i} + \Delta\bar{B}_{2_i}]\Big]u(t) \\ z(t) & = & \sum_{i=1}^r \hat{\mu}_i \Big[[C_{1_i} + \Delta\bar{C}_{1_i}]x(t) + [D_{12_i} + \Delta\bar{D}_{12_i}]u(t)\Big] \\ y(t) & = & \sum_{i=1}^r \hat{\mu}_i \Big[[C_{2_i} + \Delta\bar{C}_{2_i}]x(t) + [D_{21_i} + \Delta\bar{D}_{21_i}]w(t)\Big] \end{array} \tag{36}$$

where

$$\Delta\bar{A}_i = \bar{F}(x(t),\hat{x}(t),t)\bar{H}_{1_i},\ \ \Delta\bar{B}_{1_i} = \bar{F}(x(t),\hat{x}(t),t)\bar{H}_{2_i},\ \ \Delta\bar{B}_{2_i} = \bar{F}(x(t),\hat{x}(t),t)\bar{H}_{3_i},$$

$$\Delta\bar{C}_{1_i} = \bar{F}(x(t),\hat{x}(t),t)\bar{H}_{4_i},\ \ \Delta\bar{C}_{2_i} = \bar{F}(x(t),\hat{x}(t),t)\bar{H}_{5_i},\ \ \Delta\bar{D}_{12_i} = \bar{F}(x(t),\hat{x}(t),t)\bar{H}_{6_i}$$

$$\text{and } \Delta\bar{D}_{21_i} = \bar{F}(x(t),\hat{x}(t),t)\bar{H}_{7_i}$$

with

$$\bar{H}_{1_i} = \left[H_{1_i}^T\; A_1^T\; \cdots A_r^T\; H_{1_1}^T \cdots H_{1_r}^T\right]^T,\;\; \bar{H}_{2_i} = \left[H_{2_i}^T\; B_{1_1}^T \cdots B_{1_r}^T\; H_{2_1}^T \cdots H_{2_r}^T\right]^T,$$

$$\bar{H}_{3_i} = \left[H_{3_i}^T\; B_{2_1}^T \cdots B_{2_r}^T\; H_{3_1}^T \cdots H_{3_r}^T\right]^T,\;\; \bar{H}_{4_i} = \left[H_{4_i}^T\; C_{1_1}^T \cdots C_{1_r}^T\; H_{4_1}^T \cdots H_{4_r}^T\right]^T,$$

$$\bar{H}_{5_i} = \left[H_{5_i}^T\; C_{2_1}^T \cdots C_{2_r}^T\; H_{5_1}^T \cdots H_{5_r}^T\right]^T,\;\; \bar{H}_{6_i} = \left[H_{6_i}^T\; D_{12_1}^T \cdots D_{12_r}^T\; H_{6_1}^T \cdots H_{6_r}^T\right]^T$$

$$\bar{H}_{7_i} = \left[H_{7_i}^T\; D_{21_1}^T \cdots\; D_{21_r}^T\; H_{7_1}^T \cdots H_{7_r}^T\right]^T \;\text{ and}$$

$$\bar{F}(x(t),\hat{x}(t),t) = \Big[F(x(t),t)\;\; (\mu_1-\hat{\mu}_1)\;\; \cdots\;\; (\mu_r-\hat{\mu}_r)\;\; F(x(t),t)(\mu_1-\hat{\mu}_1)\;\; \cdots\;\; F(x(t),t)(\mu_r-\hat{\mu}_r)\Big].$$

Note that $\|\bar{F}(x(t),\hat{x}(t),t)\| \leq \bar{\rho}$ where $\bar{\rho} = \{3\rho^2+2\}^{\frac{1}{2}}$. $\bar{\rho}$ is derived by utilizing the concept of vector norm in the basic system control theory and the fact that $\mu_i \geq 0$, $\hat{\mu}_i \geq 0$, $\sum_{i=1}^r \mu_i = 1$ and $\sum_{i=1}^r \hat{\mu}_i = 1$.

Note that the above technique is basically employed in order to obtain the plant's premise variable to be the same as the controller's premise variable; e.g. [28]. Now, the premise variable of the system is the same as the premise variable of the controller, thus we can apply the result given in Case I.

Theorem 2 *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and a positive constant δ, if there exist matrices X_0, Y_0, $\mathcal{B}_{0_i}$ and $\mathcal{C}_{0_i}$, $i = 1,2,\cdots,r$, satisfying the following ε-independent linear matrix inequalities:*

$$\begin{bmatrix} X_0E+DX_0 & I \\ I & Y_0E+DY_0 \end{bmatrix} > 0 \tag{37}$$

$$EX_0^T = X_0E,\; X_0^TD = DX_0,\; X_0E+DX_0 > 0 \tag{38}$$

$$EY_0^T = Y_0E,\; Y_0^TD = DY_0,\; Y_0E+DY_0 > 0 \tag{39}$$

$$\Psi_{11_{ii}} \quad \text{and} \quad \Psi_{22_{ii}} < 0,\; i = 1,2,\cdots,r \tag{40}$$

$$\Psi_{11_{ij}} + \Psi_{11_{ji}} \quad \text{and} \quad \Psi_{22_{ij}} + \Psi_{22_{ji}} < 0,\; i < j \leq r \tag{41}$$

where $E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$,

$$\Psi_{11_{ij}} = \begin{pmatrix} A_iY_0^T + Y_0A_i^T + B_{2_i}\mathcal{C}_{0_j} + \mathcal{C}_{0_i}^TB_{2_j}^T + \gamma^{-2}\tilde{\bar{B}}_{1_i}\tilde{\bar{B}}_{1_j}^T & (*)^T \\ \left[Y_0\tilde{\bar{C}}_{1_i}^T + \mathcal{C}_{0_i}^T\tilde{\bar{D}}_{12_j}^T\right]^T & -I \end{pmatrix} \tag{42}$$

$$\Psi_{22_{ij}} = \begin{pmatrix} A_i^TX_0^T + X_0A_i + \mathcal{B}_{0_i}C_{2_j} + C_{2_i}^T\mathcal{B}_{0_j}^T + \tilde{\bar{C}}_{1_i}^T\tilde{\bar{C}}_{1_j} & (*)^T \\ \left[X_0\tilde{\bar{B}}_{1_i} + \mathcal{B}_{0_i}\tilde{\bar{D}}_{21_j}\right]^T & -\gamma^2I \end{pmatrix} \tag{43}$$

with

$$\tilde{\bar{B}}_{1_i} = \begin{bmatrix} \delta I & I & \delta I & 0 & B_{1_i} & 0 \end{bmatrix},\;\; \tilde{\bar{C}}_{1_i} = \begin{bmatrix} \frac{\gamma\bar{\rho}}{\delta}\bar{H}_{1_i}^T & 0 & \frac{\gamma\bar{\rho}}{\delta}\bar{H}_{5_i}^T & \sqrt{2}\bar{\lambda}\bar{\rho}\bar{H}_{4_i}^T & \sqrt{2}\bar{\lambda}C_{1_i}^T \end{bmatrix}^T,$$

$$\tilde{\bar{D}}_{12_i} = \begin{bmatrix} 0 & \frac{\gamma\bar{\rho}}{\delta}\bar{H}_{3_i}^T & 0 & \sqrt{2}\bar{\lambda}\bar{\rho}\bar{H}_{6_i}^T & \sqrt{2}\bar{\lambda}D_{12_i}^T \end{bmatrix}^T,\;\; \tilde{\bar{D}}_{21_i} = \begin{bmatrix} 0 & 0 & 0 & \delta I & D_{21_i} & I \end{bmatrix}$$

$$\text{and}\;\; \bar{\lambda} = \left(1 + \bar{\rho}^2\sum_{i=1}^r\sum_{j=1}^r\left[\|\bar{H}_{2_i}^T\bar{H}_{2_j}\| + \|\bar{H}_{7_i}^T\bar{H}_{7_j}\|\right]\right)^{\frac{1}{2}},$$

then there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0,\hat{\varepsilon}]$, the prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ is guaranteed. Furthermore, a suitable controller is of the form (4) with

$$\hat{A}_{ij}(\varepsilon) = \left[Y_\varepsilon^{-1} - X_\varepsilon\right]^{-1}\mathcal{M}_{0_{ij}}(\varepsilon)Y_\varepsilon^{-1},\;\; \hat{B}_i = \left[Y_0^{-1} - X_0\right]^{-1}\mathcal{B}_{0_i},\;\; \text{and}\;\; \hat{C}_i = \mathcal{C}_{0_i}Y_0^{-1} \tag{44}$$

$$\begin{aligned} \textit{where} \quad \mathcal{M}_{0_{ij}}(\varepsilon) &= -A_i^T - X_\varepsilon A_i Y_\varepsilon - X_\varepsilon B_{2_i}\hat{C}_j Y_\varepsilon - \left[Y_\varepsilon^{-1} - X_\varepsilon\right]\hat{B}_i C_{2_j} Y_\varepsilon \\ &\quad -\tilde{\tilde{C}}_{1_i}^T\left[\tilde{\tilde{C}}_{1_j} Y_\varepsilon + \tilde{\tilde{D}}_{12_j}\hat{C}_j Y_\varepsilon\right] - \gamma^{-2}\Big\{X_\varepsilon \tilde{\tilde{B}}_{1_i} + \left[Y_\varepsilon^{-1} - X_\varepsilon\right]\hat{B}_i \tilde{\tilde{D}}_{21_i}\Big\}\tilde{\tilde{B}}_{1_j}^T \quad (45) \end{aligned}$$

$$X_\varepsilon = \Big\{X_0 + \varepsilon\tilde{X}\Big\}E_\varepsilon \quad \textit{and} \quad Y_\varepsilon^{-1} = \Big\{Y_0^{-1} + \varepsilon N_\varepsilon\Big\}E_\varepsilon \tag{46}$$

with $\tilde{X} = D\Big(X_0^T - X_0\Big)$ *and* $N_\varepsilon = D\Big((Y_0^{-1})^T - Y_0^{-1}\Big)$.

Proof: Since (36) is of the form of (1), it can be shown by employing the proof for Theorem 3.1. □

4 Example

Consider the tunnel diode circuit where the tunnel diode is characterized by $i_D(t) = -0.2v_D(t) - 0.01v_D^3(t)$. Assume that ε is a "parasitic" inductance in the network. Let $x_1(t) = v_C(t)$ be the capacitor voltage and $x_2(t) = i_L(t)$ be the inductor current. Then, the circuit can be modelled by the following state equations:

$$\begin{aligned} C\dot{x}_1(t) &= 0.2x_1(t) + 0.01x_1^3(t) + x_2(t) \\ \varepsilon\dot{x}_2(t) &= -x_1(t) - Rx_2(t) + u(t) + 0.1w_2(t) \\ y(t) &= Jx(t) + 0.1w_1(t), \qquad z(t) = [x_1(t) \quad x_2(t)]^T \end{aligned} \tag{47}$$

where $u(t)$ is the control input, $w_1(t)$ is the measurement noise, $w_2(t)$ are is the process noise which may represent un-modelled dynamics, y(t) is the measured output, $z(t)$ is the controlled output, J is the sensor matrix, $x(t) = [x_1^T(t) \;\; x_2^T(t)]^T$ and $w(t) = [w_1^T(t) \;\; w_2^T(t)]^T$. Note that the variables $x_1(t)$ and $x_2(t)$ are treated as the deviation variables (variables deviate from its desired trajectories). The parameters in the circuit are given by $C = 100\ mF$ and $R = 1 \pm 0.3\%\ \Omega$, with these parameters (47) can be rewritten as

$$\begin{aligned} \dot{x}_1(t) &= 2x_1(t) + (0.1x_1^2(t)) \cdot x_1(t) + 10x_2(t) \\ \varepsilon\dot{x}_2(t) &= -x_1(t) - (1 \pm \Delta R)x_2(t) + u(t) + 0.1w_2(t) \\ y(t) &= Jx(t) + 0.1w_1(t), \qquad z(t) = [x_1(t) \quad x_2(t)]^T . \end{aligned} \tag{48}$$

For the sake of simplicity, we will use as few rules as possible. Assuming that $|x_1(t)| \leq 3$, the nonlinear network system (48) can be approximated by the following TS fuzzy model:

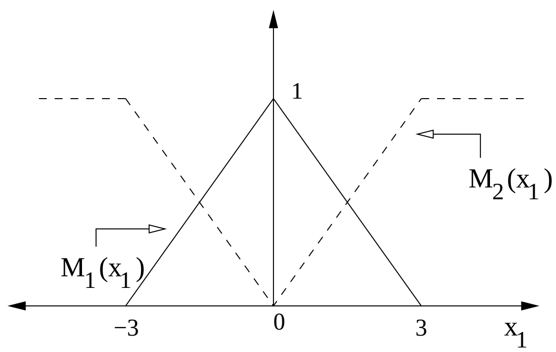

Figure 1: Membership functions for the two fuzzy set.

Plant Rule 1: IF $x_1(t)$ is $M_1(x_1(t))$ THEN

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= [A_1 + \Delta A_1]x(t) + B_1 w(t) + B_{2_1} u(t), \\ z(t) &= C_1 x(t), \qquad y(t) = C_{2_1} x(t) + D_{21} w(t). \end{aligned}$$

Plant Rule 2: IF $x_1(t)$ is $M_2(x_1(t))$ THEN

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= [A_2 + \Delta A_2]x(t) + B_1 w(t) + B_{2_2} u(t), \\ z(t) &= C_1 x(t), \qquad y(t) = C_{2_2} x(t) + D_{21} w(t) \end{aligned}$$

where $x(0) = 0$, $x(t) = [x_1^T(t) \;\; x_2^T(t)]^T$, $w(t) = [w_1^T(t) \;\; w_2^T(t)]^T$,

$$A_1 = \begin{bmatrix} 2 & 10 \\ -1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} 2.9 & 10 \\ -1 & -1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},\ B_{2_1} = B_{2_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C_{2_1} = C_{2_2} = J,\ D_{21} = \begin{bmatrix} 0.1 & 0 \end{bmatrix},\ \Delta A_1 = F(x(t),t)H_{1_1},; \Delta A_2 = F(x(t),t)H_{1_2} \text{ and}$$

$E_\varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}$. Note that, the plot of the membership functions is the same as in Figure 1. Now, by assuming that in (2), $\|F(x(t),t)\| \le \rho = 1$ and since the values of R are uncertain but bounded within 30% of their nominal values given in (47), we have $H_{1_1} = H_{1_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0.3 \end{bmatrix}$. Note that by employing the results given in Lemma 1 and the Matlab LMI solver, it is easy to realize that when $\varepsilon < 0.03$, the LMIs become ill-conditioned and the Matlab LMI solver yields the error message, “Rank Deficient". Using the LMI optimization algorithm and Theorems 3.1-3.2 with $\varepsilon = 0.01$, $\gamma = 1$ and $\delta = 1$, we obtain the following results as shown in Table 1.

Table 1: The performance index γ of the system with different values of ε.

ε	The performance index γ Output Feedback Case I	Case II
0.01	0.316	0.346
0.15	0.574	0.922
0.16	0.600	>1
0.28	0.989	>1
0.29	>1	>1

Remark 2. *For a sufficiently small ε, the non-fragile output feedback controllers guarantee that the $\mathcal{L}_2$-gain, γ, is less than the prescribed value. The disturbance input signal, $w(t)$, which was used during the simulation is the rectangular signal with magnitude 0.1 and frequency 1 Hz. For an example, when ε is 0.01, the output feedback controller in Case I where $\gamma = 0.316$ and in Case II where $\gamma = 0.346$, all are less than the prescribed value 1. Thus, Table 1 shows the result of the performance index γ with different values of ε.*

5 Conclusion

This paper has considered the problem of designing a non-fragile output feedback controller for a TS fuzzy system with multiple time-scales. Sufficient conditions for the existence of non-fragile fuzzy controllers are derived in terms of a family of ε-independent linear matrix inequalities. The proposed approach does not involve the separation of states into slow and fast ones, and it can be applied not only to standard, but also to nonstandard multiple time-scale systems. A numerical simulation example has been presented to illustrate the effectiveness of the designs.

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