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Stability of Discrete-Time Systems with Time-Varying Delay: Delay Decomposition Approach

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Abstract:
This article deals with the problem of obtaining delay-dependent stability conditions for a class of discrete-time systems with interval time-varying delay. Using the decomposition the delay interval into two unequal subintervals by tuning parameter α, a new interval delay-dependent Lyapunov-Krasovskii functional is constructed to derive novel delay-dependent stability conditions which are expressed in terms of linear matrix inequalities. This leads to reduction of conservatism in terms of the upper bounds of the maximum time-delay. The numerical examples show that the obtained result is less conservative than some existing ones in the literature.
Keywords: time-delay systems, interval time-varying delay, asymptotic stability, delay-dependent stability, Lyapunov-Krasovskii methods.

1 Introduction

Time-delay frequently occurs in many practical systems, such as manufacturing systems, telecommunication and economic systems etc. Since time-delay is an important source of instability and poor performance, considerable attention has been paid to the problem of stability analysis and controller synthesis for continuous time-delay systems (see e.g. [3-5, 10, 11, 17-21, 23-26] and the reference therein). Inversely, less attention has been drawn to the corresponding results for discrete-time delay systems (see e.g. [1, 2, 6-9, 12-15, 22, 24]). This is mainly due to the fact that such systems can be transformed into augmented systems without delay. This augmentation of the system is, however, inappropriate for systems with unknown delays and for systems with time-varying delay which are the subject analysis in this work.

Recently, increasing attention has been devoted to the problem of delay-dependent stability of linear systems with time-varying delay, including continuous-time (see e.g. [5, 10, 11, 18-21, 23], 25, 26]) and discrete-time systems (see e.g. [1, 2, 7-9, 12, 14, 15, 24]) and a great number of delay-dependent stability criteria were derived. The key point for deriving the delay-dependent stability criterions is the choice of an appropriate Lyapunov–Krasovskii functional (LKF). It is known that the existence of a complete quadratic Lyapunov-Krasovskii functional (CQLKF) is a sufficient and necessary condition for asymptotic stability of the time-delay system. Using the CQLKF, one can obtain the maximum allowable upper bound (MAUB) of delay which is very close to the analytical delay limit for stability. However, the CQLKF leads to a complicated system of partial differential equations, yielding infinite dimensional linear matrix inequalities (LMIs). Therefore, to develop simpler stability criteria, many authors have used special forms of LKF rather than CQLKF, which give LMIs with finite order and a reduced value of MAUB.

Further, to reduce the conservativeness of the existing results, some new analysis methods have been proposed, such as descriptor system transformation method [3-5], free weighting matrix method [8, 11, 23], matrix inequality method [10, 17, 18] and input–output approach [19]. Using these methods, many stability criteria were derived by checking a variation of LKF in a *whole interval* of the time-delay. Contrary to this approach, in [24, 25], in order to obtain some less conservative stability conditions, the interval of the time delay is divided into *multiple equidistant subintervals* and interval delay-dependent LKF (ID-D LKF) is constructed. By checking the variation of the ID-D LKF defined on the subintervals, some new delay-dependent stability criteria are derived. It is worth pointing out that the main difference between LKF and ID-D LKF lies in that the former allows taking different weighing matrices on different subintervals. Therefore, ID-D LKF, as expected, will yield less conservative delay-dependent stability criteria.

Inspired by the idea of Zhu and Yang [26] on splitting the delay of continuous-time systems into *two unequal subintervals*, a new method is developed in this paper for stability analysis for discrete-time systems with time-varying delay. The delay interval $[k-h_M,\ k-1]$ in the ID-D LKF is divided into two unequal subintervals: $[k-h_M,\ k-\alpha-1]$ and $[k-\alpha,\ k-1]$, where $0<\alpha<h_M$ is a tuning parameter. The new ID-D LKF is constructed with different weighing matrices in various subintervals. Free-weighting matrices and model transformation are not used in order to derive delay dependent criterion. It is shown that the presented stability condition is much less conservative than the existing ones [1, 2, 6-9, 13-15, 22, 24], because it has a lower value of MAUB. The derived condition can be seen as an extension of the methods in [24, 25], wherein the whole delay range is divided to $n \geq 2$ equal subintervals. As the number of subintervals in [24, 25] is greater than two, the decomposition approach is more complex and the resulting stability conditions are more conservative and difficult to implement. To demonstrate the effectiveness of the proposed method, numerical examples are given in section 3.

Notation: $\Re^n$ and Z^+ denote the n-dimensional Euclidean space and positive integers. Notation $P>0 \quad (P\geq 0)$ means that matrix P is real symmetric and positive definite (semi-definite). For real symmetric matrices P and Q, the notation $P>Q$ $(P\geq Q)$ means that matrix $P-Q$ is positive definite (positive semi-definite). I is an identity matrix with an appropriate dimension. Superscript "T" represents the transpose. In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term which is induced by symmetry. If dimensions of matrices are not explicitly given, then they are assumed to be compatible for algebraic operations.

2 Main results

Consider the following system with an interval time-varying delay:

$$x(k+1)=Ax(k)+Bx(k-h(k)) \tag{1}$$

where $x(k)\in\Re^n$ is the state at instant k, matrices $A\in\Re^{n\times n}$and $B\in\Re^{n\times n}$ are constant matrices and $h(k)$is the positive integer representing the time delay of the system that we assume to be time dependent and satisfies the following:

$$0\leq h(k)\leq h_M \tag{2}$$

where h_M is known to be a positive and finite integer.

The aim of this article is to establish the sufficient condition that guarantee the delay-dependent stability of the system (1), which is less conservative than the existing results in literature.

We first introduce the following result, which will be used in the proof of our main results.

Lemma 1. *Let $y(k) = x(k+1) - x(k)$. For any matrix $R > 0$ [24]*

$$-\left(h_M - h_m\right) \sum_{m=k-h_M}^{k-1-h_m} y^T(m)Ry(m) \le \begin{bmatrix} x(k-h_m) \\ x(k-h_M) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(k-h_m) \\ x(k-h_M) \end{bmatrix} = -\left[x(k-h_m) - x(k-h_M)\right]^T R \left[x(k-h_m) - x(k-h_M)\right] \quad (3)$$

Theorem 2. *For given scalars $h_M (h_M > 0)$ and $\alpha (0 < \alpha < h_M)$, the system described by (1)-(2) is asymptotically stable if there exists matrices $P = P^T > 0$, $Q_i = Q_i^T \ge 0$ and $Z_i = Z_i^T \ge 0$ $(i = 1, 2, 3)$, such that the following LMIs hold:*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & 0 & \Phi_{15} \\ * & \Phi_{22} & \Phi_{23} & 0 & \Phi_{25} \\ * & * & \Phi_{33} & \Phi_{34} & 0 \\ * & * & * & \Phi_{44} & 0 \\ * & * & * & * & \Phi_{55} \end{bmatrix} < 0 \quad (4)$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & 0 & \Psi_{15} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} \\ * & * & \Psi_{33} & 0 & 0 \\ * & * & * & \Psi_{44} & 0 \\ * & * & * & * & \Psi_{55} \end{bmatrix} < 0 \quad (5)$$

where

$$\begin{gathered}
\Phi_{11} = A^T PA - P + Q_1 + Q_3 - \tfrac{1}{\alpha}\left(Z_1 + Z_3\right), \\
\Phi_{12} = A^T PB + \tfrac{1}{\alpha}\left(Z_1 + Z_3\right),\ \Phi_{15} = \left(A - I\right)^T U_1, \\
\Phi_{22} = B^T PB - Q_3 - \tfrac{1}{\alpha}\left(2Z_1 + Z_3\right),\ \Phi_{23} = \tfrac{1}{\alpha}Z_1,\ \Phi_{25} = B^T U_1, \\
\Phi_{33} = -Q_1 + Q_2 - \tfrac{1}{\alpha}Z_1 - \tfrac{1}{h_M - \alpha}Z_2,\ \Phi_{34} = \tfrac{1}{h_M - \alpha}Z_2, \\
\Phi_{44} = -Q_2 - \tfrac{1}{h_M - \alpha}Z_2,\ \Phi_{55} = -U_1, \\
\Psi_{11} = \Phi_{11},\ \Psi_{12} = A^T PB,\ \Psi_{13} = \tfrac{1}{\alpha}\left(Z_1 + Z_3\right),\ \Psi_{15} = \left(A - I\right)^T U_2, \\
\Psi_{22} = B^T PB - Q_3 - \tfrac{1}{h_M - \alpha}\left(2Z_2 + Z_3\right),\ \ \Psi_{23} = \tfrac{1}{h_M - \alpha}\left(Z_2 + Z_3\right), \\
\Psi_{24} = \tfrac{1}{h_M - \alpha}Z_2,\ \Psi_{25} = B^T U_2, \\
\Psi_{33} = -Q_1 + Q_2 - \tfrac{1}{\alpha}(Z_1 + Z_3) - \tfrac{1}{h_M - \alpha}\left(Z_2 + Z_3\right), \\
\Psi_{44} = -Q_2 - \tfrac{1}{h_M - \alpha}Z_2,\ \Psi_{55} = -U_2, \\
U_1 = \alpha Z_1 + \left(h_M - \alpha\right) Z_2 + \alpha Z_3,\ U_2 = \alpha Z_1 + \left(h_M - \alpha\right) Z_2 + h_M Z_3
\end{gathered}$$

Proof: Construct the interval delay-dependent LKF as

$$V(k) = V_1(k) + V_2(k) + V_3(k) \quad (6)$$

where

$$V_1(k) = x^T(k)Px(k) \quad (7)$$

$$V_2(k) = \sum_{i=k-\alpha}^{k-1} x^T(i)Q_1x(i) + \sum_{i=k-h_M}^{k-1-\alpha} x^T(i)Q_2x(i) + \sum_{i=k-h(k)}^{k-1} x^T(i)Q_3x(i) \quad (8)$$

$$V_3(k) = \sum_{i=-\alpha}^{-1} \sum_{j=k+i}^{k-1} y^T(j)Z_1y(j) + \sum_{i=-h_M}^{-1-\alpha} \sum_{j=k+i}^{k-1} y^T(j)Z_2y(j) + \sum_{i=-h(k)}^{-1} \sum_{j=k+i}^{k-1} y^T(j)Z_3y(j) \quad (9)$$

where $P = P^T > 0$, $Q_i = Q_i^T \geq 0$ and $Z_i = Z_i^T > 0$ $(i = 1, 2, 3)$. Note, the delay interval $[k - h_M,\ k - 1]$ in the LKF is divided into two unequal subintervals: $[k - h_M,\ k - \alpha - 1]$ and $[k - \alpha,\ k - 1]$, where $0 < \alpha < h_M$ is a tuning parameter.

Taking the difference of $\Delta V_i(k) = V_i(k+1) - V_i(k)$, we can obtain

$$\begin{aligned}\Delta V_1(k) = {} & x^T(k)\left(A^T PA - P\right)x(k) + 2x^T(k)A^T PBx\left(k - h(k)\right) \\ & + x^T\left(k - h(k)\right)B^T PBx\left(k - h(k)\right)\end{aligned} \tag{10}$$

$$\begin{aligned}\Delta V_2(k) = {} & x^T(k)Q_1x(k) - x^T(k-\alpha)Q_1x(k-\alpha) \\ & + x^T(k-\alpha)Q_2x(k-\alpha) - x^T(k-h_M)Q_2x(k-h_M) \\ & + x^T(k)Q_3x(k) - x^T\left(k-h(k)\right)Q_3x\left(k-h(k)\right)\end{aligned} \tag{11}$$

$$\begin{aligned}\Delta V_3(k) = {} & \sum_{i=-\alpha}^{-1}\left[y^T(k)Z_1y(k) - y^T(k+i)Z_1y(k+i)\right] \\ & + \sum_{i=-h_M}^{-1-\alpha}\left[y^T(k)Z_2y(k) - y^T(k+i)Z_2y(k+i)\right] \\ & + \sum_{i=-h(k)}^{-1}\left[y^T(k)Z_3y(k) - y^T(k+i)Z_3y(k+i)\right] \\ = {} & \alpha y^T(k)Z_1y(k) - \sum_{i=-\alpha}^{-1} y^T(k+i)Z_1y(k+i) \\ & + \left(h_M - \alpha\right)y^T(k)Z_2y(k) - \sum_{i=-h_M}^{-1-\alpha} y^T(k+i)Z_2y(k+i) \\ & + h(k)y^T(k)Z_3y(k) - \sum_{i=-h(k)}^{-1} y^T(k+i)Z_3y(k+i) \\ = {} & y^T(k)\left[\alpha Z_1 + \left(h_M-\alpha\right)Z_2 + h(k)Z_3\right]y(k) \\ & - \sum_{m=k-\alpha}^{k-1} y^T(m)Z_1y(m) - \sum_{m=k-h_M}^{k-1-\alpha} y^T(m)Z_2y(m) \\ & - \sum_{m=k-h(k)}^{k-1} y^T(m)Z_3y(m) \\ = {} & y^T(k)\left[\alpha Z_1 + \left(h_M-\alpha\right)Z_2 + h(k)Z_3\right]y(k) \\ & - \sum_{m=k-\alpha}^{k-1} y^T(m)Z_1y(m) - \sum_{m=k-h_M}^{k-1-\alpha} y^T(m)Z_2y(m) \\ & - \sum_{m=k-h(k)}^{k-1} y^T(m)Z_3y(m)\end{aligned} \tag{12}$$

It is know from (2) that, for any $k \in Z^+$, $h(k) \in [0, \alpha - 1]$ or $h(k) \in [\alpha, h_M]$. Define two sets

$$\Omega_1 = \left\{k:\ h(k) \in [0, \alpha],\ k \in Z^+\right\} \tag{13}$$

$$\Omega_2 = \left\{k:\ h(k) \in [\alpha + 1, h_M],\ k \in Z^+\right\} \tag{14}$$

In the following, we will discuss the variation of $\Delta V(k)$ for two cases ($k \in \Omega_1$ and $k \in \Omega_2$).

Case 1. For $k \in \Omega_1$, i.e. $0 \leq h(k) \leq \alpha$.

$$\sum_{m=k-\alpha}^{k-1} y^T(m)Z_1y(m) = \sum_{m=k-\alpha}^{k-1-h(k)} y^T(m)Z_1y(m) + \sum_{m=k-h(k)}^{k-1} y^T(m)Z_1y(m) \tag{15}$$

$$\begin{aligned}\Delta V_3(k) = y^T(k)\left[\alpha Z_1 + (h_M-\alpha)Z_2 + h(k)Z_3\right]y(k) - \sum_{m=k-\alpha}^{k-1-h(k)} y^T(m)Z_1y(m) \\ - \sum_{m=k-h(k)}^{k-1} y^T(m)\left(Z_1+Z_3\right)y(m) - \sum_{m=k-h_M}^{k-1-\alpha} y^T(m)Z_2y(m)\end{aligned} \tag{16}$$

Because $Z_1 + Z_3 > 0$, $h(k) \le \alpha$ and $\alpha - h(k) \le \alpha$, using Lemma 1, it follows

$$\begin{aligned}&-\sum_{m=k-h(k)}^{k-1} y^T(m)\left(Z_1+Z_3\right)y(m) \\ &\quad \le -\tfrac{1}{h(k)}\left[x(k)-x\left(k-h(k)\right)\right]^T\left(Z_1+Z_3\right)\left[x(k)-x\left(k-h(k)\right)\right] \\ &\quad \le \tfrac{1}{\alpha}x^T(k)\left(-Z_1-Z_3\right)x(k) + \tfrac{1}{\alpha}2x^T(k)\left(Z_1+Z_3\right)x\left(k-h(k)\right) \\ &\qquad +\tfrac{1}{\alpha}x^T\left(k-h(k)\right)\left(-Z_1-Z_3\right)x\left(k-h(k)\right)\end{aligned} \tag{17}$$

$$\begin{aligned}&-\sum_{m=k-\alpha}^{k-1-h(k)} y^T(m)Z_1y(m) \\ &\quad \le -\tfrac{1}{\alpha-h(k)}\left[x\left(k-h(k)\right)-x\left(k-\alpha\right)\right]^T Z_1\left[x\left(k-h(k)\right)-x\left(k-\alpha\right)\right] \\ &\quad \le \tfrac{1}{\alpha}x^T\left(k-h(k)\right)\left(-Z_1\right)x\left(k-h(k)\right) + \tfrac{1}{\alpha}2x^T\left(k-h(k)\right)Z_1x\left(k-\alpha\right) \\ &\qquad +\tfrac{1}{\alpha}x^T\left(k-\alpha\right)\left(-Z_1\right)x\left(k-\alpha\right)\end{aligned} \tag{18}$$

$$\begin{aligned}&-\sum_{m=k-h_M}^{k-1-\alpha} y^T(m)Z_2y(m) \\ &\quad \le -\tfrac{1}{h_M-\alpha}\left[x\left(k-\alpha\right)-x(k-h_M)\right]^T Z_2\left[x\left(k-\alpha\right)-x(k-h_M)\right] \\ &\quad \le \tfrac{1}{h_M-\alpha}x^T\left(k-\alpha\right)\left(-Z_2\right)x\left(k-\alpha\right) + \tfrac{1}{h_M-\alpha}2x^T\left(k-\alpha\right)Z_2x\left(k-h_M\right) \\ &\qquad +\tfrac{1}{h_M-\alpha}x^T\left(k-h_M\right)\left(-Z_2\right)x\left(k-h_M\right)\end{aligned} \tag{19}$$

Combining (10)-(19), it yields

$$\begin{aligned}&\Delta V(k) \le \xi^T(k)\hat{\Phi}\xi(k) \\ &\hat{\Phi} = \begin{bmatrix} \Phi_{11}+(A-I)^TU_1(A-I) & \Phi_{12}+(A-I)^TU_1B & 0 & 0 \\ * & \Phi_{22}+B^TU_1B & \Phi_{23} & 0 \\ * & * & \Phi_{33} & \Phi_{34} \\ * & * & * & \Phi_{44}\end{bmatrix} \\ &\xi(k) = \begin{bmatrix} x^T(k) & x^T\left(k-h(k)\right) & x^T(k-\alpha) & x^T(k-h_M)\end{bmatrix}^T\end{aligned} \tag{20}$$

Obviously, $\Delta V(k) < 0$ for $k \in \Omega_1$ if $\hat{\Phi} < 0$. Using the Schur complement, it is easy to see that $\Delta V(k) < 0$ holds if $\Phi < 0$ and $h(k) \in [0, \alpha]$.

Case 2. Similarly, for $k \in \Omega_2$, i.e. $\alpha + 1 \le h(k) \le h_M$, using the Schur complement, it is easy to see that $\Delta V(k) < 0$ holds if $\Psi < 0$.

From the above discussions, we can see that for all $k \in Z^+$ if (4)-(5) hold, $\Delta V(k) < 0$, which completes the proof. □

Remark 3. Theorem 2 presents a stability result which depends on the maximum delay bound h_M. The conditions in Theorem 2 are expressed in terms of LMIs, and therefore, they can be easily checked by using standard numerical software.

Remark 4. The delay interval $[k - h_M,\ k - 1]$ in the ID-D LFK is divided into two unequal subintervals: $[k - h_M,\ k - \alpha - 1]$ and $[k - \alpha,\ k - 1]$, where $0 < \alpha < h_M$ is a tuning parameter.

Consequently, the different weighing matrices in the Lyapunov functional are used in various subintervals and the information of delayed state $x(k-\alpha)$ can be taken into full consideration. Further, using the subintervals and Lemma 1, the upper bounds of some terms in $\Delta V_3(k)$ are more accurately estimated than by using the previous methods since the upper bound h_M of delay $h(k)$ on the interval $0 \le h(k) \le h_M$ is substituted with two less conservative upper bounds α and h_M on the subintervals $0 \le h(k) \le \alpha$ and $\alpha < h(k) \le h_M$, respectively. So the decomposition method presented in Theorem 2 can reduce the value of MAUB.

An algorithm for seeking a corresponding values of α $(0 < \alpha < h_M)$ subject to (4)-(5), such that the MAUB of h_M has maximal value, can easily be obtained.

Algorithm 5. *Step 1. Let* $h = 0$ *and* $\alpha = 0$.

Step 2. $h = h + 1$.

Step 3. $\alpha = \alpha + 1$.

Step 4. If inequalities (4)-(5) is feasible, then $\alpha_m = \alpha$, $\alpha = 0$ *and go to step 2; otherwise, go to step 5.*

Step 5. If $\alpha = h - 1$, *go to step 6; otherwise, go to step 3.*

Step 6. The maximal delay is $h_M = h - 1$ *and the minimal value of tuning parameterαis* α_m.

3 Numerical examples

In this section, two examples are presented. The obtained results have been compared with several existing criteria in the literature.

Example 1. Consider system (1) with the time-varying delay $h(k)$ satisfying (2) and

$$A = \begin{bmatrix} 0.8 & 0 \\ 0 & \lambda \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \quad \lambda \in \{0.91,\ 0.97\}$$

Case 1 ($\lambda = 0.91$). This system was considered in [13] and [22]. Table 1 lists the MAUB of delay obtained from Theorem 2 of this paper. For comparison, results from [13, 22] are also listed in the table. It is clear that Theorem 2 leads to better results than those in [13, 22].

Table 1. Comparison of MAUB of delay based on different existing methods for $\lambda = 0.91$

Method	h_M
[13]	41
[22, Corollary 1]	42
Theorem 2 in this paper	46 for $\alpha = 19, \cdots, 30$

Case 2 ($\lambda = 0.97$). For comparison, the results from [2, 6, 7, 9] and this paper are listed in Table 2. It is clear that Theorem 2 gives much better results than the existing delay-dependent criteria.

Example 2. Consider system (1) with the time-varying delay $h(k)$ satisfying (2) and

Case 1. [1, 14, 15]

$$A = \begin{bmatrix} 0.6 & 0 \\ 0.35 & 0.7 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}$$

Case 2. [8, 9, 15, 24]

$$A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}$$

Table 2. Comparison of MAUB of delay based on different existing methods for $\lambda = 0.97$	
Method	h_M
[9, Theorem 1]	4
[7, Theorem 3]	8
[6, Lemma 2]	8
[2, Theorem 1]	10
Theorem 2 in this paper	17 for $\alpha = 9, 10, 11$

For the above systems, the LMIs conditions for delay-independent stability [16]

$$P = P^T > 0, \quad Q = Q^T > 0, \quad \begin{bmatrix} -P+Q & 0 & A^TP \\ * & -Q & B^TP \\ * & * & -P \end{bmatrix} < 0$$

are feasible, from which we deduce that both systems are stable for $0 \le h(k) < \infty$. Using the existing delay-dependent criteria, it can be obtained only the finite value of MAUB, which will guarantee the stability of the given systems. Tables 3 (Case 1) and 4 (Case 2) list the intervals of time delay for different methods. Based on Theorem 2 in this paper, large numerical values of MAUB are obtained ($h_M \to \infty$). Hence, using of Theorem 2 leads to better results than those in [1, 8, 9, 14, 15, 25].

Table 3. Interval of time delay for Case 1	
Method	Interval
[1, Th. 3.1]	$2 \le h(k) \le 10$
[15, Theorem 1], [15, Theorem 2]	$2 \le h(k) \le 13$
[14, Theorem 3.2]	$0 \le h(k) \le 12$
[2, Theorem 1]	$2 \le h(k) \le 15$
Theorem 2 in this paper	$0 \le h(k) \le 10 \cdot 10^{21}$

Table 4. Interval of time delay for Case 2	
Method	Interval stability
[9, Theorem 1]	$0 \le h(k) \le 6$
[15, Theorem 2]	$0 \le h(k) \le 10$
[8, Theorem 1]	$0 \le h(k) \le 12$
[24, Theorem 5]	$2 \le h(k) \le 19$
[24, Theorem 7]	$2 \le h(k) \le 20$
Theorem 2 in this paper	$0 \le h(k) \le 9.61 \cdot 10^8$

4 Conclusion

In this paper, the problem of obtaining delay dependent stability conditions for a class of systems with interval time-varying delay is discussed. A new interval delay-dependent Lyapunov–Krasovskii functional is constructed by splitting the delay interval into two unequal intervals by tuning parameter α. The free-weighting matrices and model transformation are not used in order to derive delay-dependent criteria. Numerical examples show that the results proposed in this paper are much less conservative while comparing the maximum allowable upper bound of delay with the existing results in the literature.

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