The Use of Relative Statistical Indicators, of Central Trend Indicators and Their Interpretation

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Abstract

Statistics is one of the economic evidence form in charge with the study of socioeconomic staple phenomenons, quantitatively speaking, in close connection with their qualitative side, in concrete conditions of space and time. The present article uses the importance of statistical indicators in the economic analysis of an economic entity, showing the calculation of theoretical and practical relative indicators, and the central trend indicators.

Keywords: statistical indicators, relative indicators, relative sizes, interpretation of statistical data, arithmetic mean, harmonic mean, geometric mean, mean square value, statistical interpretation, economic analysis.

1 Introduction

Relative indicators are expressed in relative quantities and are used to analyze the evolution of economic phenomena over time.

It is calculated as the ratio between the size it compares and the size taken as a basis for comparison.

This ratio is a coefficient, which can be multiplied by:

a. 100 and the result is called a percentage;
b. 1000 and the result is called a promil.

The types of relative sizes are [2]:

- Structure
- Of the dynamics
- Coordination
- Intensity
- Of the plan (provisions)

2 Types of Relative Sizes

2.1 Structure related sizes

It is calculated as the ratio between the size (in absolute amount) of a group and the size of the whole community [1].

Calculation formula:

- The size of a group
- The size of the community

They are also called specific weights or weights, i.e., how much a group represents in the community.

Obs: The sum of the group weights = 100%

Example:
Table 1. The structure of clients at a company with tourist activity in 2018 (conventional data)

<table>
<thead>
<tr>
<th>Categories of clients</th>
<th>Number of clients</th>
<th>Weight %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal clients</td>
<td>1.200</td>
<td>30</td>
</tr>
<tr>
<td>External clients</td>
<td>2.800</td>
<td>70</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>4.000</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Calculation of shares:

Internal clients no. = 1.200

\[ \frac{1.200}{4.000} \times 100 = 30\% \]

External clients no. = 2.800

\[ \frac{2.800}{4.000} \times 100 = 70\% \]

Interpretation: the number of external customers represents over 2/3 of the total number of customers (70%).

2.2 Dynamics related sizes

It is calculated as the ratio between the size of the indicator in the current period (1) and the size of the indicator in the base period [3].

Calculation formula:

\[ \frac{\text{The size of the indicator in the current period (1)}}{\text{The size of the indicator in the base period (0)}} \]

They can be expressed in percentages (%) and promils. The relative quantities of the dynamics express the evolution in time of the economic phenomena (increase or decrease) [4].

Example:

Table 2. The dynamics of the customer structure in 2018 compared to 2017 at a company with tourist activity

<table>
<thead>
<tr>
<th>Categories of clients</th>
<th>No. of clients year 2017 Base period (0)</th>
<th>No. of clients year 2018 Current period (1)</th>
<th>Weight (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal clients</td>
<td>1.200</td>
<td>1.760</td>
<td>30</td>
</tr>
<tr>
<td>External clients</td>
<td>2.800</td>
<td>2.640</td>
<td>70</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>4.000</strong></td>
<td><strong>4.400</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Calculation of customer shares in 2018:

Internal clients = \[ \frac{1.760}{4.400} \times 100 = 40\% \]

External clients = \[ \frac{2.640}{4.400} \times 100 = 60\% \]

Dynamics indicator:

Internal clients = \[ \frac{1.760}{1.200} \times 100 = 146.66\% \]

External clients = \[ \frac{2.640}{2.800} \times 100 = 94.28\% \]

Total clients = \[ \frac{4.400}{4.000} \times 100 = 110\% \]
Interpretation: The total number of tourists at the analyzed travel agency increased in 2018 compared to 2017 by 10% or 110% or 1.1 times. The situation by categories of customers is as follows: in the case of domestic tourists, the number of domestic customers increased in 2018 compared to 2017 by 46.66%; in the case of foreign tourists, the number of foreign customers decreased in 2018 compared to 2017 by 5.72%.

2.3 Coordination related sizes
They are calculated as the ratio between the values of two groups, phenomena (communities), different territorial units [5].

Calculation formula:

\[
\frac{\text{The size of a group (communities), territorial unit to be compared (X}_A)}{\text{OR}} = \frac{\text{X}_B}{\text{X}_A}
\]

Example: Comparison of wheat production at two farms (data are conventional for 2018):
- farm A (1) the production was 30,000,000 lei
- farm B (2) the production was 24,000,000 lei

\[
\begin{align*}
\text{Wheat production at farm 1} &= 30,000,000 \text{ lei} \\
\text{Wheat production at farm 2} &= 24,000,000 \text{ lei}
\end{align*}
\]

a. Variant 1 = \(\frac{30,000,000}{24,000,000} \times 100 = 125\%\)

Result = 125%

Interpretation: Wheat production on farm 1 was 25% higher than wheat production on farm 2 in 2018.

b. Variant 2 = \(\frac{24,000,000}{30,000,000} \times 100 = 80\%\)

Result = 80%

Interpretation: Wheat production on farm 2 was 20% lower than wheat production on farm 1 in 2018.

2.4 Intensity related sizes
They are calculated as the ratio between the value of two related primary characteristics and express how much of the value of the compared characteristic returns per characteristic unit - basis of comparison.

Example:

Table 3. Average production per worker (W) at two companies in 2018 with the same object of activity (textile garments)

<table>
<thead>
<tr>
<th>Commercial company</th>
<th>Production value</th>
<th>No. of workers</th>
<th>Average production per worker</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>200,000</td>
<td>1,000</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>240,000</td>
<td>1,000</td>
<td>120</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{W}_B &= 120 \\
\text{Size related to intensity} &= \frac{120}{100} = 1.2
\end{align*}
\]
**Interpretation:** The average productivity per worker is 1.2 times higher in company B compared to company A, in 2018.

### 2.5 Provisions related sizes

It is calculated as relative quantities of:

- Planned volume in the current period

\[
\text{Plan tasks} = \frac{\text{Achieved volume in the base period}}{\text{Achieved volume in the current period}} \times 100
\]

This index can be:
- **Sub-unitary**, when a reduction is planned compared to the achievements of the previous period (expenses, costs);
- **Supra-unitary**, when an increase is planned compared to the plan load (production, sales)

\[
\text{Other plan achievements} = \frac{\text{Achieved volume in the current period}}{\text{Planned volume in the current period}}
\]

This index can be:
- **Sub-unitary**, when the achievements of the current period are lower than the provisions (reduction);
- **Supra-unitary**, when the achievements of the current period are higher than the provisions (increase).

**Example:**

**Table 4.** The volume of goods sales at the SC FRESH FRUCT SRL store in 2018

<table>
<thead>
<tr>
<th>Commercial company</th>
<th>Actual sales 2017</th>
<th>Provisions 2018</th>
<th>Actual sales 2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC FRESH FRUCT SRL</td>
<td>2,000,000</td>
<td>2,400,000</td>
<td>2,640,000</td>
</tr>
</tbody>
</table>

\[ \text{Provisions 2018} = 2,400,000 \]
\[ \frac{\text{Achievements 2018}}{\text{Provisions 2018}} \times 100 = 120\% \]

**Interpretation:** The company has forecast (scheduled) for 2018 an increase in sales volume by 20% (growth rate).

\[ \text{Achievements 2018} = 2,000,000 \]

\[ \frac{\text{Achievements 2018}}{\text{Provisions 2018}} \times 100 = 110\% \]

**Interpretation:** The company registered an increase in sales compared to the annual provisions by 10% in 2018. The central trend indicators used in the statistical analysis of mass socio-economic phenomena are studied using:

- **Average indices**
- **Position indices**

### 3 Average Sizes (Indices)

These average sizes are grouped by the nature of the characteristic into the following categories:

- Arithmetic mean
- Harmonic average
- Square mean
- Geometric average
- Chronological average
Each average is calculated as:
• Simple average: when the number of characteristic variants is equal to the number of observed units;
• Weighted average: when the same variant is found in several observation units.

3.1 The arithmetic mean

It can only be calculated for quantitative variables. The arithmetic mean can be simple and weighted.

3.1.1 Simple arithmetic mean

It is calculated as the ratio between the sum of the values (levels) of the characteristics and their number.

\[ \bar{x} = \frac{\sum x_i}{n} \]

where:
- \( n \) = number of variants
- \( X_i \) = variants of the characteristics
- \( X \) = statistical characteristic

Example: A student obtained the following grades in the discipline "Statistics" in the school year 2017/2018: 4,5,6,7,8,9

The average student at the end of the school year is:

\[ \frac{4 + 5 + 6 + 7 + 8 + 9}{6} = 6,50 \]

3.1.2 Weighted arithmetic mean

\[ \bar{X} = \frac{\sum x_i n_i}{\sum n_i} \]

Where:
- \( n \) = frequencies
- \( i \) = number of characteristic variants

Example: The situation of students in a class according to their average in mathematics for a semester is as follows:

<table>
<thead>
<tr>
<th>Math average (x)</th>
<th>No. of students (n)</th>
<th>xn</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>42</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>48</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>30</strong></td>
<td><strong>205</strong></td>
</tr>
</tbody>
</table>

The general average of the students in this class in mathematics is calculated as follows:

\[ \frac{205}{30} = 6,83 \]

Example: Calculate the weighted arithmetic averages for a series of intervals: the distribution of students in a class of students at the end of a school year, according to the number of absences.
Table 6. Distribution of students in a class by number of absences

<table>
<thead>
<tr>
<th>Groups of students by number of absences</th>
<th>No. of students (f)</th>
<th>The center (middle) of the range (x)</th>
<th>x_f</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>18</td>
<td>5</td>
<td>90</td>
</tr>
<tr>
<td>11-20</td>
<td>4</td>
<td>15</td>
<td>60</td>
</tr>
<tr>
<td>21-30</td>
<td>3</td>
<td>25</td>
<td>75</td>
</tr>
<tr>
<td>31-40</td>
<td>3</td>
<td>35</td>
<td>105</td>
</tr>
<tr>
<td>41-50</td>
<td>2</td>
<td>45</td>
<td>90</td>
</tr>
<tr>
<td>TOTAL</td>
<td>30</td>
<td>-</td>
<td>420</td>
</tr>
</tbody>
</table>

Ma = Σxf / Σf = 420 / 30 = 14

Interpretation: The average number of absences per student is 14, it is between the first interval (with the highest number of students) and the second interval.

Example: Application for calculating the average for the alternative characteristic with answer variants: yes - no. At the end of a school year, in a class, the situation of students' promotion is as follows: out of 30 students, 24 are promoted and 6 are not promoted.

No. of promoted students
No. of promoted students (%) = ------------------------ = 24/30 = 80%
Total no. of students
No. of failed students
No. of failed students (%) = ------------------------ = 6/30 = 20%
Total no. of students

3.2 The harmonic mean

It is calculated as the inverse size of the arithmetic mean from the inverse values of the terms.

a) Simple: $\bar{X}_h = \frac{n}{\sum x^f}$, where
   n= number of series terms;
   x= characteristic variants.

b) Weighted: $\bar{X}_h = \frac{\Sigma f}{\sum x^f}$, where
   f= frequencies (weights)
   x= characteristic variants.

This average is used in different situations:
- Calculation of the average harvest per "hectare" on a farm;
- Calculating the average salary of a company, when the salary fund for different departments and the average salaries of employees are known.

Application:
Table 7. Calculation of the average salary at S.C. FRESH FRUCT S.R.L.

<table>
<thead>
<tr>
<th>Compartments</th>
<th>Average salary / employee ($\bar{S}$)</th>
<th>Salary fund / lei (FS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Sales</td>
<td>2.200</td>
<td>110,000</td>
</tr>
<tr>
<td>- Administrative</td>
<td>2.500</td>
<td>80,000</td>
</tr>
<tr>
<td>TOTAL S.C. FRESH FRUCT SRL</td>
<td>-</td>
<td>190,000 ($\Sigma FS$)</td>
</tr>
</tbody>
</table>

The number of employees is at the sales department 50, and at administrative 32. The average salary on the company in 2018 was 2,317.06 lei. Average salary per company is calculated:

$$\bar{S} = \frac{\Sigma FS}{\Sigma \frac{1}{FS}} = \frac{190,000}{\Sigma \frac{1}{FS}} = \frac{190,000}{50+32} = 2,317.06 \text{ lei}$$

The number of employees is calculated taking into account the average salary by compartments:

$$N_s = \frac{FS}{S} = \frac{1}{S} \cdot FS$$
In the "sales" section:

\[ N_{ang} = \frac{1}{2.200} \cdot 110.000 = \frac{1100}{11} = 100 \]

In the "administrative" section:

\[ N_{ang} = \frac{1}{2.500} \cdot 80.000 = \frac{8000}{125} = 64 \]

3.3 The square mean

It is calculated by extracting the square root from the arithmetic mean of the squared terms of the series.

It is used to calculate the variation of the characteristic (mean square deviation) or when it is indicated to give importance to higher levels in a series of statistics [6].

Calculation mode:

a) For simple series:

\[ \bar{X}_p = \sqrt{\frac{\sum X^2}{n}} \]

b) For the frequency series:

\[ \bar{X}_p = \sqrt{\frac{\sum X^2 f}{\sum f}} \]

Application: Calculation of the average number of products made by a worker, who works according to a company with production activity.

Table 8. Average number of products made by a worker

<table>
<thead>
<tr>
<th>Workers</th>
<th>No. of products (X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>TOTAL</td>
<td>60 ((\sum X))</td>
</tr>
</tbody>
</table>

If simple arithmetic mean is used,

\[ \bar{X}_a = \frac{\sum X}{\sum n} = \frac{60}{4} = 15 \]

where:

- n = no. of variants workers
- x = characteristic variants

If simple quadratic arithmetic mean is used,

\[ \bar{X}_p = \sqrt{\frac{\sum X^2}{\sum n}} = \sqrt{\frac{12^2 + 18^2 + 10^2 + 20^2}{4}} = \sqrt{\frac{968}{4}} = \sqrt{242} \approx 15.56 \]

So, the average number of products made by a worker calculated according to the arithmetic average is 15, and according to the square average is 15.56.

3.4 The geometric mean

It is based on the product relationship between the series terms and is used when the lower terms are given importance in the absolute value [7]. It is mainly used for the calculation of averages in the case of relative quantities of dynamics, respectively for establishing the average growth rate. The geometric mean is the value, which by replacing the terms of the series and the effect of their product, the resulting value is equal to the product of the terms, meaning:

\[ x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n = \Pi x, \]

where: \( \Pi x = the \ product \ of \ the \ series \ terms. \)

Geometric mean:

\[ \bar{X}_g = \sqrt[n]{\Pi x} \]

This average is used if there is a negative or zero term in the statistical series.
3.5 The chronologic mean

It is used in the case of the statistical series of dynamics (chronological) for the calculation of the average level.

\[
X_{cr} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n-1}
\]

where: \( x \) = characteristic variants;
\( n \) = number of terms.

Calculation mode:
- Make the sum between half the value of the first term and the last term, with the full value of the other terms of the series;
- Divide the total amount obtained by the number of series terms minus 1.

a) Simple:
It is used in the case of the dynamic series of moments with equal intervals, usually for the statistical calculation of the average stock (goods, materials)

Application:

Table 9. The situation of the stock of goods at a store for the second quarter of 2018

<table>
<thead>
<tr>
<th>Period</th>
<th>Goods stock (lei)</th>
<th>Average stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>April</td>
<td>10.000</td>
<td></td>
</tr>
<tr>
<td>May</td>
<td>10.400</td>
<td></td>
</tr>
<tr>
<td>June</td>
<td>11.600</td>
<td></td>
</tr>
<tr>
<td>July</td>
<td>10.800</td>
<td></td>
</tr>
</tbody>
</table>

The average stock of goods in each month of the second quarter was 7,133.33 lei.

Average level of stock of goods = \( \frac{10.000 + 10.400 + 11.600 + 10.800}{3} = \frac{5.000 + 5.200 + 5.800 + 5.400}{3} = \) \( \frac{21.400}{3} = 7.133.33 \) lei

b) Weighted in the case of dynamic series with unequal intervals.

\[
X_{cr} = \frac{x_1 + x_2 + x_3 + \ldots + x_n + \sum_{i=1}^{n-1} x_i \cdot f_i}{f_1 + f_2 + \ldots + f_n}
\]

Application:

Table 10. The situation of raw material stocks at a company with production activity in 2018

<table>
<thead>
<tr>
<th>Period</th>
<th>Good stock (lei) (x)</th>
<th>Time intervals (in months) are:</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>16.000 (x_1)</td>
<td>( f_1 = 2 )</td>
</tr>
<tr>
<td>March</td>
<td>13.000 (x_2)</td>
<td>( f_2 = 4 )</td>
</tr>
<tr>
<td>July</td>
<td>10.000 (x_3)</td>
<td>( f_3 = 2 )</td>
</tr>
<tr>
<td>September</td>
<td>15.000 (x_4)</td>
<td>( f_4 = 3 )</td>
</tr>
<tr>
<td>December</td>
<td>12.000 (x_5)</td>
<td></td>
</tr>
</tbody>
</table>

Average annual stock = \( \frac{16.000 + 13.000 + 15.000 + 12.000}{2 + 2 + 2 + 2 + 3} = \frac{15.000 + 12.000}{2 + 2 + 2 + 3} = \frac{29.500 + 46.000 + 25.000 + 40.500}{11} = \frac{141.000}{11} = 12.818.18 \) lei

The average annual stock of raw materials for 2018 was 12,818.18 lei.

4 Position Sizes (Indicators)

They are used for distribution series.
4.1 The median

Represents the central value of a statistical series ordered ascending or descending, which divides the terms of the series into equal parts: half of the terms are smaller than the median and half are larger than the median.

If the series has an odd number of terms, the median represents the value of the central term, which is calculated by the relation \( \frac{n+1}{2} \), where \( n \) is the serial term number.

Example:
Statistical series 3,7,8,9,4,2,5, \( n = 7 \)
Place of the median = \( \frac{7 + 1}{2} = \frac{8}{2} = 4 \)
The median is 9, i.e., the fourth term.

If the series has an even number of terms, the value of the median is calculated as the arithmetic mean of the two central terms.

Example:
Statistical series in ascending order: 6,8,9,12,15,16
The median = \( \frac{9+12}{2} = \frac{21}{2} = 10.5 \)
The series has an odd number of terms, the series is ordered ascending: 5,5,6,7,7; the series has 5 terms, therefore the formula is used: \( \frac{n+1}{2} \)
The place of the median is \( \frac{5+1}{2} = 3 \)
The median is 6, occupying the 3rd place (middle).

Application:
The series has an even number of terms (4).
The series is ordered in ascending order: 10,12,18,20.
Place of median between term 2 and 3:
The median = \( \frac{12+18}{2} = \frac{30}{2} = 15 \)

4.2 The module

Represents the characteristic level that has the highest frequency. Calculated only for frequency distribution statistical series. In table no. 1, the maximum frequency is 7, so the modulus, i.e., the corresponding value is 5 and 6 (averages of 5 and 6). In table no. 4, the highest frequency is 20, so the module corresponds to worker no. 4, which made this number of products (20).

5 Conclusions

The importance of the statistical calculation lies not only in the calculation of the arithmetic ratio of these indicators, but in the examples and their interpretation, from which the top-level management (managers) can come out to conclusions and arguments in order to correct the business evolution. The theoretical and practical presentation of these statistical indicators is addressed to practicing accountants, economists, students enrolled in an economic profile, departments of economic analysis within companies, as well as those interested in the introduction of the calculation methods and the interpretation of statistical indicators.

REFERENCES