Simulating the Need of Working Capital for Decision Making in Investments

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Abstract: Simulation is one of the main instruments within the financial techniques of modeling decisions in condition of risk. The paper compares a couple of simulation methods for Sales and their impact on the need of short term financing. For simulating the need of working capital, the original software implementation is based on the data analysis and statistical facilities of a common spreadsheet program. The case study aims at proving the utility of the software for furnishing results with three of the main known simulation methods and helping the decisional process.

Keywords: investment cycle, working capital, stochastic models, computer simulation, case study.

1 The premises of the operative financing in condition of risk

In the contemporary society that deals with a sum of unexpected events, the knowledge based management has to accept an uncontrollable component of the economic reality that needs corrective and, moreover, preventive actions. The managerial decisions taken in conditions of risk have to limit their effects to values complying with a tolerance set up in advance.

Managers have to be innovative and to find solutions that prevent the negative effects of the unexpected events. In the context of relaunching of the economy, in the new basic economic cycle, the main parameters have to be controlled in order to correctly assure the financial resources. As the usual forecasting methods are based on historical data, the decision maker takes in consideration the financial and time resources, the construction and validation of models for the behavior of the company in crisis conditions and the particular type of activities within the company. Forecasting the financial resources, that means a correct dimensioning of the working capital, is a pre-condition for fulfilling the company’s short or medium time strategies. [5]

Simulation of the company’s behavior and of the needed working capital integrates the inputs and outputs of the company’s budget. One of the main components of the budget are the Sales. Evaluating the historical data for the Sales, the cronograma can be divided in data belonging to the precedent cycle until the economic recession and data registered during the crisis. From a statistical point of view, in the post-crisis economical cycle, is recommended to consider only data before the cycle. In reality, such a simulation deals with errors due to neglecting the anti-crisis strategies and the already implemented corrective actions. A better approach is based on
the whole set of available historical data that includes also the present phase of relatively weak economical increase.

2 Theoretical aspects and proper simulation instruments

2.1 Financial calculation flow

Simulating the short term financing uses a set of relatively non-complicated arithmetical calculations. These are based on the formula for determining the cash conversion cycle, as deducing the average time for current debts’ payment from the sum of average transformation time of the stocks and debts in liquidity.

The need of operating working capital is calculated as the cash conversion cycle, measured in days, multiplied by the Sales - as resulting by the different simulation methods.

2.2 Simulation methods and instruments

For a good preview of the complex economical reality, the scenarios are built-up on repeated simulations that reflect possible values for monthly sales (x). As the literature presents many simulation methods, the decision maker has to choose the method for simulating the monthly sales that best fits his company [3].

The present paper deals with three modeling methods along with a user-friendly computer implementation, using built-in and user defined spreadsheet functions:

- simulation by using the Random generation number tool;
- simulation by using the inverse of the normal cumulative distribution for the specified mean and standard deviation;
- simulation by Monte Carlo method [6].

For the decision maker, the software implementation is almost fully automated, in the background being used the advanced tools included in the Data Analysis Tool Pack, a powerful add-in of MS-Excel.

The technique of Random Number Generation

The Random Number Generation is the most primitive simulation model. It consists of generating a set of random numbers based on the normal probability distribution of the simulated variable. The repartition function can be continue or discreet, depending on the type of the available historical data.

The Random number generator in MS Excel is a complex tool that allows the user to generate a set of values according to a normal probability distribution, a user defined histogram or a patterned distribution. [2]

The technique of using the inverse of the normal distribution

The technique of the reverse transformed considers for the simulated variable a probability function \( f(x) \) and a continue repartition function \( F(x) \). A random number \( r \in [0,1] \) is generated; the simulated variable takes the value that satisfies:

\[
F(x) = r
\]

that is \( x = F^{-1}(r) \),

\[
F(x) = r
\]
where $F^{-1}(r)$ is the inverse of the $F(x)$ repartition function of the considered variable.

The RAND() spreadsheet function is used for generating normally distributed, below unit positive numbers. These values are turned than into a set of simulated values by using the NORMINV (probability, average, standard deviation) function, where probability is the randomly generated $r$, average refers to the average of historical data and standard deviation is the measure of its variation.

The Monte Carlo technique

Monte Carlo method is similar to the statistical experiments as the characteristics of the probability distribution are calculated on the basis of multiple random experiments. The method is different as it is limited to a discreet probability for the simulated variable

$$A_{m,n} = \left( \begin{array}{c} x_1 \cdots x_i \cdots x_n \\ p_1 \cdots p_i \cdots p_n \end{array} \right)$$

(3)

that depends on a continuous probability function $f(x)$ and a continuous repartition function $F(x)$. However, it is also a normal distribution of positive below unit values for the simulated variable $x$.

The simulation method consists of the following sequential steps:

- building-up of a histogram that reflects the probability distribution of the variable, based on the historical data;
- simulating as many time as possible the probability of occurrence of each value for the variable according to the histogram;
- identifying the value of the variable according to the simulated cumulative distribution [6].

The simulated probably values $r$ are than transformed in values for the variable $x$ satisfying $x = F^{-1}(r)$, where $F^{-1}(r)$ is the inverse of the $F(x)$ repartition function of the considered variable.

If applying for monthly sales, based on the simulated probability, an integrated decision function is used:

$$IF(r < p_1; x_1; IF(r < p_2; x_2; \cdots; IF(r < p_{n-1}; x_{n-1}; x_n); \cdots))$$

(4)

The minimum number of iterations needed for obtaining relevant results with Monte Carlo method is given by:

$$n \geq \frac{z_{1-\frac{\alpha}{2}}^2 \cdot \sigma_{Sales}}{d}$$

(5)

where $\sigma_{Sales}$ is the standard deviation, $z_{1-\frac{\alpha}{2}}^2$ is the theoretic value for $\alpha$ confidence level and $d$ is the maximum admitted error for a chosen accuracy. [3]

2.3 Comparative sensitivity analysis

The comparative analysis underlines some aspects of the utility of simulation procedures in the decisional process and the sensitivity of the results, depending on the method chosen for building-up the sample of the simulated values.

In the context of simulating the monthly sales, on the one hand is important to calculate some basic indicators used in the decisional process - the forecasted need of operating working capital, the coefficient of variation and the confidence level for the forecast, and on the other hand, statistic tests are needed for comparing the results obtained with different techniques [1].
The need of working capital (NOWC) is the central indicator used for any analysis in condition of risk, being calculated by weighting the Sales by the probability of occurrence of each value:

\[
NOWC = \sum_{i=1}^{n} p_i \cdot Sales_i
\]  

(6)

For measuring the homogeneity of the simulated time series, the coefficient of variation is calculated:

\[
\% = \frac{\Delta}{NOWC} \cdot 100,
\]

(7)

where \( \Delta = \sqrt{\sum_{i=1}^{n} p_i (Sales_i - NOWC)^2 \cdot p_i} \)

(8)

is the standard deviation of the need of working capital calculated on the basis of the simulated probability distribution.

The values of the forecasted operating working capital cover a confidence interval

\[
NOWC - z_{1-\frac{\alpha}{2}} \cdot \sigma_{NOWC} < NOWC_{forecast} < NOWC + z_{1-\frac{\alpha}{2}} \cdot \sigma_{NOWC}
\]

(9)

where \( z_{1-\frac{\alpha}{2}} \) are the theoretical values for the Gauss-Laplace distribution [4].

From a practical point of view, important are the two last methods. The statistical tests aims at comparing for significant differences the values obtained by the technique of reverse transform and the Monte Carlo simulation. A \( z \)-test and a \( t \)-test are used.

For analyzing the impact of the simulation method, \( z \)-test is applied for the two sets of results for forecasting the working capital. As the means of the samples are positive, the univariat test is performed, with \( \alpha \) confidence level and the following null hypothesis:

\[
H_0 : NOWC_{reverse\_transformed} - NOWC_{Monte\_Carlo} = 0
\]

(10)

The \( z \) statistic uses the normal values \( z_{theoretic} = z_{1-\alpha} \)

(11)

and the alternative hypothesis is:

\[
H_a : NOWC_{reverse\_transformed} > NOWC_{Monte\_Carlo}
\]

(12)

For a normal distribution of the sample, the statistics of the test is:

\[
z_{calculated} = \frac{(NOWC_{random\_numbers} - NOWC_{monte\_carlo}) - 0}{\sqrt{\frac{\sigma^2_{reverse\_transformed} + \sigma^2_{Monte\_Carlo}}{n}}}
\]

(13)

having the mean : \( NFRE_{random\_numbers} - NFRE_{Monte\_Carlo} \)

(14)

and the spread of data about the mean is given by: \( \sigma^2_{reverse\_transformed} + \sigma^2_{Monte\_Carlo}[4] \).

The software instrument used for describing the probability distribution of the simulated values is \( FREQUENCY(data\_array; bin\_array) \) where \( data\_array \) is the array of previously simulated monthly sales and \( bin\_array \) refers to the intervals considered for counting the occurrence for each value of the simulated time series.

The \( z \)-test is applied using the appropriate statistical instrument included in the \( Data \ Analysis \ Tool \ Pack \).
3 Case study

Let’s consider "Crisis Ltd" a company that makes available data from its balance sheet for the last financial year and financial documents for the years 2002-2011.

3.1 The financial diagnosis

The items in Figure 1 are given by the balance sheet of the company for 2011. The monthly Sales of "Crisis Ltd" are presented in Figure 2 as data entry for the application. The chart in Figure 3 represents the Sales, along with a linear and a polynomial approximation.

<table>
<thead>
<tr>
<th>Item</th>
<th>Rotation rate</th>
<th>Inclusion rate</th>
<th>Item (days of sale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw material</td>
<td>40</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>Finished products</td>
<td>30</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td>Client debts</td>
<td>15</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Buildings</td>
<td>15</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Salaries</td>
<td>15</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Other current debts</td>
<td>15</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Working capital</td>
<td>10</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 1: Financial data from the balance sheet for 2011

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>3365</td>
<td>3620</td>
<td>3945</td>
<td>3559</td>
<td>4255</td>
<td>4385</td>
<td>4520</td>
<td>5059</td>
<td>5804</td>
<td>3200</td>
</tr>
<tr>
<td>February</td>
<td>3732</td>
<td>3832</td>
<td>3797</td>
<td>4152</td>
<td>3887</td>
<td>4157</td>
<td>4089</td>
<td>4809</td>
<td>4900</td>
<td>3550</td>
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<td>3603</td>
<td>4750</td>
<td>4200</td>
<td>3500</td>
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<tr>
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<td>3782</td>
<td>3612</td>
<td>4057</td>
<td>4882</td>
<td>4482</td>
<td>5012</td>
<td>4600</td>
<td>4200</td>
<td>4200</td>
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<tr>
<td>May</td>
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<td>4173</td>
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<td>4789</td>
<td>4213</td>
<td>4200</td>
<td>3800</td>
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<tr>
<td>June</td>
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<td>3829</td>
<td>4024</td>
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<td>4939</td>
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<td>4000</td>
<td>2800</td>
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</tr>
<tr>
<td>July</td>
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<td>4044</td>
<td>4079</td>
<td>4394</td>
<td>5104</td>
<td>3950</td>
<td>3200</td>
<td>4000</td>
</tr>
<tr>
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<td>3821</td>
<td>3805</td>
<td>3871</td>
<td>4061</td>
<td>4591</td>
<td>5171</td>
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<td>4500</td>
</tr>
<tr>
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<td>3739</td>
<td>4039</td>
<td>4254</td>
<td>4154</td>
<td>5019</td>
<td>5449</td>
<td>2540</td>
<td>3540</td>
<td>4500</td>
</tr>
<tr>
<td>November</td>
<td>3657</td>
<td>3687</td>
<td>3617</td>
<td>4287</td>
<td>4067</td>
<td>5007</td>
<td>5327</td>
<td>2200</td>
<td>3850</td>
<td>4800</td>
</tr>
</tbody>
</table>

Figure 2: Sales for 2002-2011

The descriptive statistics \[4\] in Figure 4 shows that the company is in recession since 2008 significant changes take place in the resources involved in the production and trade flows, in the need of operating working capital.

Considering the impact of uncertain elements on Sales’ evolution, the model explains the main tendency (82.20%) while the random factors are responsible for 16.43% of the Sales chronograma. The seasonality represents only 1.37% in the sales evolution. Applying an F-test / ANOVA on the monthly means (Figure 5) with a null hypothesis of equal means, leads to \[F_{calculated} = 0.749 < F_{0.95;11} = 1.887\]. The hypothesis of equal means is accepted and confirms the weak influence of the seasonality.

The modeling of the sales will be based either on 360 values, representing monthly average sales or on scenarios built on the probability distribution of sales.

The size of the sample is justified by the minimum number of iterations needed for obtaining relevant results with Monte Carlo method. According to (5), for \[\sigma_{Salesri} = 1036.72\], \[\alpha = 5\%\], \[z_{0.95}=1.96\], 2.6% tolerance, \[d=0.026 \times 2343.71 = 110.10\], the relevant sample has \[n \geq 341\] values.
Figure 3: Monthly Sales (chart)

Figure 4: Descriptive statistics for Sales (historic data)

Figure 5: ANOVA on monthly Sales
3.2 Results of the simulation with different techniques

The simulation spreadsheet is built-up on the presented theoretical basis, applying the probability distribution specific to each method. In order to emphasize the automatic calculation for distributions and the simulated values for the need of working capital, the same spreadsheet is presented in three different views.

- Figure 6: The distribution based on the Random number generation
- Figure 7: The distribution based on the normal cumulative distribution

3.3 Comparative analysis of the results

An intermediary step for implementing the Monte Carlo distribution is based on the probability distribution of the historical data, presented in the right-upper corner of Figure 6.

A first visual comparison for the three distributions is presented in the histogram in Figure 9 and the chart in Figure 10.
The coefficient of variation for the three methods is rather similar and high, proving a low homogeneity of the time series simulated by each method. The explanation is given by including in the simulation models the period of crisis, when the sales decreased. The confidence level for the three considered models is also similar.

As the results obtained by using the Random number generation and the inverse of the normal cumulative distribution are almost identical, further comparison will take in consideration only the last two methods: the inverse of the normal cumulative distribution and the Monte Carlo distribution.

As the maximum values for the need of working capital differs according to the model, it’s obvious that the probability of fulfilling an optimistic scenario differs too.

The historic data is characterized by the means presented in Figure 11.

Applying z-Test: Two Sample for Means to the sample consisting of 360 values for each method gives the results presented in Figure 12.

The variance shows the spread of statistic data to the mean, being calculated from the
Figure 10: Comparative chart for the NOWC calculated with the different methods

Figure 11: Average of historic data

Figure 12: z-Test for comparing the two methods
historical data. Obviously, it presents the same value for both samples: \( \sigma = 10880.40 \text{ u.m.} \).

The z-test reveals a significant difference between the two samples, as \( |z_{\text{calculated}}| = 2186.038 > z_{\text{theoretic}} = 1.645 \). This can be explained by the fact that results based on the inverse of the normal cumulative distribution are closer to historical data than results based on Monte Carlo simulation. Moreover, it confirms the theory of central limit and recommends the use of continuous repartition functions rather than the discreet repartition.

### 4 Conclusions and further work

The three considered simulation methods generate well balanced results for the need of working capital, bearing with similar coefficients of variation. As regarding the means, a significant difference is registered between the mean of the values simulated with the inverse of the cumulative normal distribution and Monte Carlo method.

In the simulation based on the inverse of the cumulative normal distribution, for converting the randomly generated numbers, a discreet function is recommended, such as the Poisson repartition function [7].

The spreadsheet can be further developed by fully automating the z-test in order to avoid any intervention of the decision maker in the calculating process. However, the present software implementation proves the utility of spreadsheet programs in decision making and offers a relevant set of data for the need of working capital that can improve management in investments.

### Bibliography


