

New Failure Mode and Effects Analysis based on D Numbers Downscaling Method

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Abstract: Failure mode and effects analysis (FMEA) is extensively applied to process potential faults in systems, designs, and products. Nevertheless, traditional FMEA, classical risk priority number (RPN), acquired by multiplying the ratings of occurrence, detection, and severity, risk assessment, is not effective to process the uncertainty in FMEA. Many methods have been proposed to solve the issue but deficiencies exist, such as huge computing quality and the mutual exclusivity of propositions. In fact, because of the subjectivity of experts, the boundary of two adjacent evaluation ratings is fuzzy so that the propositions are not mutually exclusive. To address the issues, in this paper, a new method to evaluate risk in FMEA based on D numbers and evidential downscaling method, named as D numbers downscaling method, is proposed. In the proposed method, D numbers based on the data are constructed to process uncertain information and aggregate the assessments of risk factors, for they permit propositions to be not exclusive mutually. Evidential downscaling method decreases the number of ratings from 10 to 3, and the frame of discernment from 2^{10} to 2^3 , which greatly reduce the computational complexity. Besides, a numerical example is illustrated to validate the high efficiency and feasibility of the proposed method.

Keywords: Failure mode and effects analysis; Dempster-Shafer evidence theory; basic belief assignment; belief function; risk priority number; D numbers; evidential downscaling; aggregate assessment

1 Introduction

Failure mode and effects analysis (FMEA) was first developed to assess known and potential faults and prevent them from happening in the 1960s [5]. It is an efficient and powerful technology in engineering and management fields, including defining, identifying, and removing known and potential faults, errors, and risk from the system, process, design as well as service. [1, 51]. Besides, in other fields, such as risk management, healthcare management and engineer design, FMEA plays an important role [2, 8, 10, 11, 15, 24, 36]. Furthermore, FMEA not only identifies factors that induce fault but also classifies the likelihood and severity of such fault [11].

A traditional method in FMEA is risk priority number (RPN), which is acquired by multiplying the grades of occurrence, severity, and detection. Thus, how to aggregate the assessments of three risk factors is a key issue, especially when the evaluated information given by experts is uncertain. These years, a large number of approaches have been proposed to improve FMEA methodology [5, 11, 23, 26, 37, 43]. It's inevitable to deal with uncertainty in real world [20–22, 29, 44]. Due to the efficiency to handle linguistic information of fuzzy sets [13, 18], an intuitionistic fuzzy approach for FMEA is proposed in [15], which offers some advantages over earlier models as it accounts for degrees of uncertainty in relationships among various criteria or options, specifically when relations cannot be expressed in definite numbers. In addition, an integrating hesitant 2-tuple linguistic term sets and an extended QUALIFLEX approach is proposed by Liu H C et al. in [13]. Furthermore, as is known to all, D numbers [9] and grey theory are two popular methods to process the uncertain information. Therefore, based on the two theories, plenty of methods are presented [23].

Classical RPN is criticized due to its several shortcomings. For example, the importance weights of three risk factors are taken into consideration. Besides, classical RPN can do nothing about the uncertain information, etc. Aiming at these problems, a method which is effective to dispose the uncertainty of assessment is proposed by Chin et al. [4]. However, the algorithm is too complex computationally, the reason is that the numeric ratings of every risk factor are from 1 to 10 so that the number of frame of discernment is 2^{10} , which greatly increase the computational load [11]. In other to reduce the computational complexity, an evidential downscaling method is proposed in [11]. Nevertheless, the D-S combination theory it uses requests that propositions are exclusive mutually. As a matter of fact, traditional FMEA ratings are obtained by subjective judgment of the experts. Therefore, the boundary of two adjacent ratings is fuzzy, hence the propositions are not actually mutually exclusive. For purpose of solving this problem, in this paper, a new method to evaluate risk in FMEA based on D numbers and evidential downscaling method, named as D numbers downscaling method, is proposed. In the proposed method, on the one hand, the evidential downscaling is utilized to decrease the frame of discernment so that greatly reduces the computational complexity. On the other hand, according to the data, D number is constructed to processing uncertain information and aggregate the assessment of risk factors because D numbers permit propositions that are not exclusive mutually.

The rest of the paper is organized as follows. Key concepts and previous theories are briefly reviewed in Section 2. A new method to evaluate risk based on D numbers downscaling method is proposed in Section 3. A numerical example is illustrated to show the feasible of the proposed method in Section 4. A brief conclusion is drawn in Section 5.

2 Preliminaries

2.1 Risk priority number

The real systems are very complicated to be modelled [14,27,46]. Risk priority number (RPN) is a traditional and typical method to model and evaluate risk in FMEA of the complicated systems, which is acquired by multiplying the grades of occurrence assessment (O), severity assessment (S), and detection assessment (D). That is

$$RPN = OSD,$$

where O stands for the probability of occurrence of failure mode, S refers to the severity of failure mode and D refers to the probability of failure being detected. The three risk factors are evaluated by FMEA experts using a 1 to 10 numeric scale, as expressed in Tables 1 to 3 [11].

The larger RPN is, the more important degree it is supposed to be assigned, referring to the failure mode should be more priority to be corrected.

However, as a traditional method for FMEA, classical RPN has been controversial all the time for many reasons, and the most important ones are listed as follows:

- The three risk factors have the same importance in RPN, but in practical applications of FMEA, they might be treated with different weight.
- Other factors are not taken into consideration, such as economy factor.
- Different combination of risk factors might obtain the same RPN, nevertheless, the potential risk might be totally different.
- The mathematical formula for calculating RPN lacks the scientific basis. There is not any proof to clarify the reason why O, S and D should be multiplied to obtain RPN.
- In fact, the scores of the three factors are difficult to be determined accurately. Therefore, FMEA experts usually provide different types of assessment information, some of which are uncertain and incomplete data.

2.2 Dempster-Shafer evidence theory

Dempster-Shafer evidence theory (D-S evidence theory) constructs a basic probability assignment(BPA) in the frame of discernment. Through combining BPAs the imprecise and uncertain information can be fused [5]. With its rapid development, it is regarded as an important method that is extensively applied in many fields such as complex networks and systems [8, 10, 12, 19, 45, 50, 52], multisource information fusion [6, 7, 17, 28, 41, 42, 49], uncertainty modelling [1, 2, 16, 48, 53], pattern recognition [30, 31] and Imprecise payoff [34, 35].

Definition 1. Let $\Theta = \{H_1, H_2, \dots, H_N\}$ be a finite nonempty set, which is composed of N mutually exclusive and exhaustive elements. Denote $P(\Theta)$ as the power set composed of 2^N elements of Θ . The BBAs function is defined as a mapping of the power set $P(\Theta)$ to a number between 0 and 1, that is, $m : P(\Theta) \rightarrow [0, 1]$, and which satisfies the following conditions:

$$m(\emptyset) = 0; \tag{1}$$

$$\sum_{A \subseteq P(\Theta)} m(A) = 1. \tag{2}$$

The mass $m(A)$ represents how strongly the evidence supports A.

Definition 2. Two bodies of evidence A and B of Θ can be used to calculate the belief level for some new hypothesis C shown as follows: The measure of conflict K that is also called as the conflict coefficient between A and B is given as:

$$K = \sum_{A \cap B = \emptyset, \forall A, B \subseteq \Theta} m_i(A)m_{i'}(B); \tag{3}$$

and the mass function after combination is:

$$m(C) = \begin{cases} \frac{\sum_{A \cap B = C, \forall A, B \subseteq \Theta} m_i(A)m_{i'}(B)}{1 - K}, & \text{when } A \neq \emptyset \\ 0, & \text{when } A = \emptyset, \end{cases} \tag{4}$$

where K reflects the conflict between two bodies of evidence. Absolutely, $0 \leq K \leq 1$. $K = 0$ shows the absence of conflict between two bodies of evidence. $K = 1$ shows complete conflict between m_i and $m_{i'}$. When $K = 1$, the Dempster's rule of combination is no longer applicable.

2.3 The evidential downscaling method

An evidential downscaling method is proposed in [11], which is based on Euclidean distance from multi-scale to construct the BBAs. The new method, decreasing the frame of discernment from 2^{10} to 2^3 , is shown as follows:

Step 1. Calculate the group assessment into a crisp number with weighted average method.

Step 2. Construct the BBAs by Definition 2 with the Euclidean distances between group assessment and multi-scale ratings. When calculating the distance, an evidential downscaling method is presented to divide the rating into three scales: '10' for 'bad', '1' for 'good', and '5' for 'bad or good' because rating 10 expresses the most serious degree, rating 1 expresses that failure is almost impossible and rating 5 expresses the moderate degree, which decreases the number of frame of discernment from 2^{10} to 2^3 .

The BBAs can be constructed as follows:

$$m(\{B\}) = \frac{d(G)}{d(G) + d(B, G) + d(G)}, \quad (5)$$

$$m(\{G\}) = \frac{d(B)}{(d(G) + d(B, G) + d(G))}, \quad (6)$$

$$m(\{B, G\}) = 1 - m(\{B\}) - m(\{G\}). \quad (7)$$

Step 3. Combine the BBAs of risk factors by Dempster's combination rule.

Step 4. Acquire the risk priority according to the aggregation of 'bad'. Let $m(\{B\})_{OSD}$, $m(\{G\})_{OSD}$ and $m(\{B, G\})_{OSD}$ be the aggregation of three risk factors for 'bad', 'good', and 'bad or good'. The final assessment of 'bad' and 'good' can be calculated as follows:

$$m(\{B\})'_{OSD} = m(\{B\})_{OSD} + \frac{m(\{B, G\})_{OSD}}{2}, \quad (8)$$

$$m(\{G\})'_{OSD} = m(\{G\})_{OSD} + \frac{m(\{B, G\})_{OSD}}{2}. \quad (9)$$

2.4 D numbers [9]

D numbers is an effective method to deal with uncertain information, which overcomes the shortcomings of Dempster-Shafer (D-S) theory. Nowadays, D number is extensively used in many fields such as dependence assessment, emergency management, and aggregating operator [33, 54, 55]. It can be defined as follows:

Definition 3. Let Ω be a finite nonempty set, D number is a mapping $\mathbf{D} : \Omega \rightarrow [0, 1]$, such that

$$\sum_{B \subseteq \Omega} D(B) \leq 1 \text{ and } D(\emptyset) = 0, \quad (10)$$

where \emptyset is an empty set and B is a subset of Ω . More importantly, different from the concept of frame of discernment in D-S theory, the elements of set Ω do not require to be mutually exclusive in D numbers. If $\sum_{B \subseteq \Omega} D(B) = 1$, the information is considered to be complete; otherwise, the information is regarded to be incomplete.

For a set $\Omega = \{b_1, b_2, \dots, b_i, \dots, b_n\}$, where $b_i \neq b_j$, if $i \neq j$. Then, a special D number can be expressed by

$$D(\{b_1\}) = v_1,$$

$$D(\{b_2\}) = v_2,$$

..... ,

$$D(\{b_i\}) = v_i,$$

..... ,

$$D(\{b_n\}) = v_n$$

or $D = \{(b_1, v_1), (b_2, v_2), \dots, (b_i, v_i), \dots, (b_n, v_n)\}$, where $v_i > 1$ and $\sum_{i=1}^n v_i \leq 1$. For the detailed information about D numbers combination rule, please refer [9]. A combination rule, a kind of add operation, is proposed to combine two D numbers.

3 The proposed method

As a traditional method for FMEA, classical RPN is acquired by multiplying the rating of three risk factors. Nevertheless, classical RPN is criticized for several disadvantages mentioned in 2.1. With the aim of overcoming these shortcomings, a large number of methods to evaluate risk for FMEA are proposed. However, existing methods either have a huge computing quantity, such as the Chinś method, or require the propositions to be mutually exclusive, like the method proposed by Du Y et al.. As a matter of fact, traditional ratings are divided by the subjective judgments of the experts which are based on the individual experience. Therefore, the boundary of two adjacent ratings is fuzzy, which means that two propositions in FMEA are not mutually exclusive. Hence, it is obviously unreasonable that Dempster-Shafer combination theory is used to aggregate assessment in the evidential downscaling method proposed in [11]. Aiming at these problems, a new method to evaluate risk in FMEA based on D numbers and evidential downscaling method is proposed. Based on the three risk factors as well, the proposed method, for one thing, decreases the number of the frame of discernment from 2^{10} to 2^3 by making use of the evidential downscaling method, which greatly reduce the computational complexity. For another, D numbers are utilized to manage the uncertain information. Because D numbers allow the propositions to not be exclusive, it is reasonable that D numbers are used to process uncertain information and aggregate the assessments. Furthermore, in consideration of the fuzzy information given by the experts on different failure modes, weighted averages are calculated to substitute them in the proposed method. Suppose there are N failure modes and M experts, the specific steps are shown as follows: The failures with higher values of ‘bad’ are assumed to be more important and should be given higher priorities.

- Step 1. Calculate the mathematical expectation of the score given by experts for evaluating each risk factor by Eq. (11).

Definition 4. Let $S_l^{i,j}$ be the mathematical expectation of the score S of failure mode i given by expert j in the assessment of risk factor l , it is calculated as follows:

$$S_l^{i,j} = \begin{cases} S, & \text{if } S \text{ is integer} \\ a \times c\% + b \times d\%, & \text{if } S \text{ is } (a : c\% , b : d\%) \text{ with } c\% + d\% = 100\% \\ a \times c\% + \frac{1}{9} \times (55 - a) \times (1 - c\%), & \text{if } S \text{ is } (a : c\%) \text{ but } 0 < c\% < 100\% \\ 5.5, & \text{if } S \text{ is missing} \end{cases} \quad (11)$$

where $i = 1, \dots, N, j = 1, \dots, M, l = O, S, D$.

Step 2. Calculate the Euclidean distances between group assessment and multi-scale ratings. Consistent with the evidential downscaling method mention in 2.3, then the new score $SG_l^{i,j}$ of each factor of each failure mode after downscaling can be obtained as follows:

$$d(B)_l^{i,j} = |S_l^{i,j} - 10|, \quad (12)$$

$$d(G)_l^{i,j} = |S_l^{i,j} - 1|, \quad (13)$$

$$d(B, G)_l^{i,j} = |S_l^{i,j} - 5|, \quad (14)$$

$$SG_l^{i,j} = \frac{d(B)_l^{i,j}}{(d(G)_l^{i,j} + d(B, G)_l^{i,j} + d(G)_l^{i,j})}, \quad (15)$$

where $i = 1, \dots, N, j = 1, \dots, M, l = O, S, D$.

Step 3. Construct D numbers on the basis of $SG_l^{i,j}$ in the light of Definition 5.

Definition 5. Let $D_{F_i}^{E_j}$ be the D number of failure mode i of experts j , which stands for the three assessments of experts j towards failure mode i , is modeled as follows:

$$D_{F_i}^{E_j} = \{(b_O^{ij}, v_O), (b_S^{ij}, v_S), (b_D^{ij}, v_D)\} \quad (16)$$

with

$$b_l^{i,j} = \lambda_j \times SG_l^{i,j} \quad (17)$$

where λ_j is the weight for expert j , and v_l ($l = O, S, D$) is the weight of the risk factor standing for the importance of it in FMEA. In addition, $i = 1, \dots, N, j = 1, \dots, M, l = O, S, D$.

Step 4. According to the processes of D_{F_i} numbers mentioned in [9] and Definition 6, for each failure mode, the combination of D numbers can be calculated and the $I(D)$ of each failure mode can be obtained as well. Most importantly, the failure modes with the lower values of $I(D)$ are assumed to be more important and should be given higher priorities.

Definition 6. Let D_{F_i} be the D number of failure mode i ($i = 1, \dots, N$) aggregate the assessment of M experts, thus, it can be calculate as follows:

$$D_{F_i} = D_{F_i}^{E_1} \oplus D_{F_i}^{E_2} \oplus \dots \oplus D_{F_i}^{E_j} \oplus \dots \oplus D_{F_i}^{E_M} \quad (18)$$

Figure 1 is the sub-flowsheet of the proposed method, that is, the procedure of downscaling and constructing D number. Figure 2 is the general flow-chart.

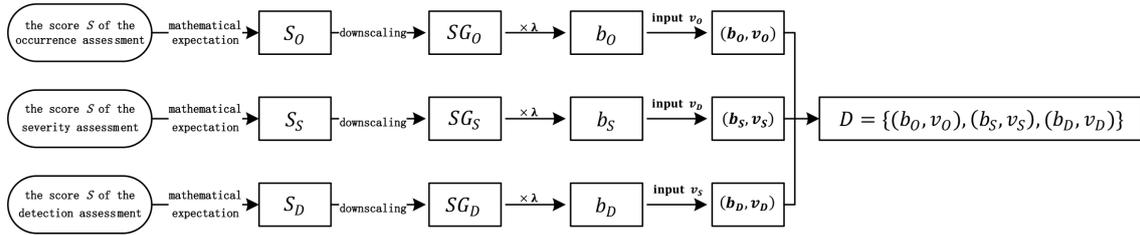


Figure 1: The procedure of downscaling and constructing D number

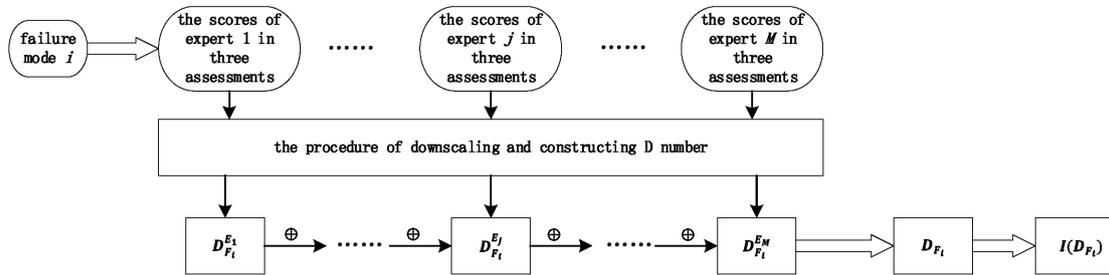


Figure 2: The general flow-chart of the proposed method

Table 1: Occurrence assessment by FMEA team members

Failure mode	Expert1	Expert2	Expert3	Expert4	Expert5
1	1	1		1	1
2	1 : 50%, 2 : 50%	1	1	1	1
3	2	2 : 90%	2	2	2
4	8	8	8 : 80%, 9 : 20%	8	8
5	6	6	6	6	6
6	2	13	2	2	2-3
7	2	2	2	9	2
8	1	1 : 75%, 2 : 25%	1	1	1
9	3	3	3	3	3
10	1 : 80%, 2 : 20%	1	1	1	1 - 2 : 85%, 3 : 15%
11	4	4	4	3 - 4 : 75%, 5 : 25%	4
12	9	9	9	9	9
13	8	8 : 80%	8	8	8
14	3	3	4	3	3
15	3	3	3	3	3 : 70%, 4 : 30%
16	1	1	1	1	1
17	3 - 5 : 90%, 6 : 10%	4	4	4	4
18	2	2	2 : 90%	2	2
19	7	7	7	7	7:80%
20	9	9	9	7 : 30%, 8 - 9 : 70%	9
21	9	8 - 9	9	9	9

Table 2: Severity assessment by FMEA team members

Failure mode	Expert1	Expert2	Expert3	Expert4	Expert5
1	7 : 20%, 8 : 80%	8	8	6-7 : 50%, 8-9 : 50%	8
2	8	8	8	8	8
3	7-9 : 90%	8	6-8	8	8
4	8	8	8	8	7-9 : 80%
5	8	7-9 : 90%	8	8	8
6	8	8	8	6-8	8
7	9 : 75%, 8 : 25%	9	9	9	9
8	4	4	4	4 : 50%, 5 : 50%	3-5 : 75%, 6-7 : 25%
9	2	2	2	2	2
10	2	2	1-2 : 60%, 3-4 : 60%	2	2
11	2	2-3	2	2	2
12	3	3	3 : 60%, 4 : 40%	3	3
13	2-3 : 80%, 3-4 : 20%	3	3	3	3
14	7	8	7	7	7
15	3	3	3	3	3
16		8	8	8	8
17	8	4	8	8	8
18	7	7	7	7	7
19	2	1-2 : 75%, 2-3 : 25%	2	2	2
20	8			8	8
21	3	3	3	3	3

4 Numerical example

Suppose that there are 5 experts evaluating risk in FMEA, who give their assessments on the three risk factors of 21 failure modes as shown in Table 1 to 3. The weights for the five experts are assumed to be 0.3, 0.3, 0.2, 0.1 and 0.1.

Take failure mode 1 of expert 1 for example. As shown in Table 1, three ratings the expert 1 gives are 1, (7:20%, 8:80%), and 3. Thus, according to Eq. (11), the mathematical expectation of score are 1, 7.8, and 3.

Then, the Euclidean distances between it and ‘bad’, ‘good’ and ‘bad or good’ are calculating as in Table 4.

Therefore, the new scores can be obtained as follows:

$$SG_O^{11} = \frac{d(B)_O^{11}}{d(G)_O^{11} + d(B, G)_O^{11} + d(G)_O^{11}} = \frac{9}{9 + 0 + 4} \approx 0.6923.$$

Table 3: Detection assessment by FMEA team members

Failure mode	Expert1	Expert2	Expert3	Expert4	Expert5
1	3	3	3	3	3 : 90%
2	3	3	3	3	3
3	4	4	4	4	3-4 : 80%, 5-6 : 20%
4	5	5	5	5	5
5	6	6	6 : 85%, 7 : 15%	6	6
6	1	1 : 85%, 2 : 15%	2		1
7	3	2	2	1-2 : 75%, 3-4 : 25%	2
8	3	3	3 : 80%, 4 : 20%	3	3
9	3	3	3-4 : 60%, 5 : 40%	3	3
10	4	4	4	4	4
11	3 : 70%, 5 : 30%	3	3	3	
12	7	7	7	7	7
13	6	6	6	5-7	6
14	4	4	4	4	4
15	4	4 : 95%	4	4	4
16	3	3	3	3	3
17	5	5	5	5	5
18	7	6-8	7	7	7
19	4	4	4	8-9 : 90%	4
20	4 : 60%	9	9	9	9
21	6	6	4-6	6	4 : 25%, 5-7 : 75%

Table 4: Euclidean distances between it and "bad", "good" and "bad or good"

Bad	Good	Bad or Good
$d(B)_O^{11} = 1 - 10 = 9,$	$d(G)_O^{11} = 1 - 1 = 0,$	$d(B, G)_O^{11} = 1 - 5 = 4;$
$d(B)_S^{11} = 7.8 - 10 = 2.2,$	$d(G)_S^{11} = 7.8 - 1 = 6.8,$	$d(B, G)_S^{11} = 7.8 - 5 = 2.8;$
$d(B)_D^{11} = 3 - 10 = 7,$	$d(G)_D^{11} = 3 - 1 = 2,$	$d(B, G)_D^{11} = 3 - 5 = 2;$

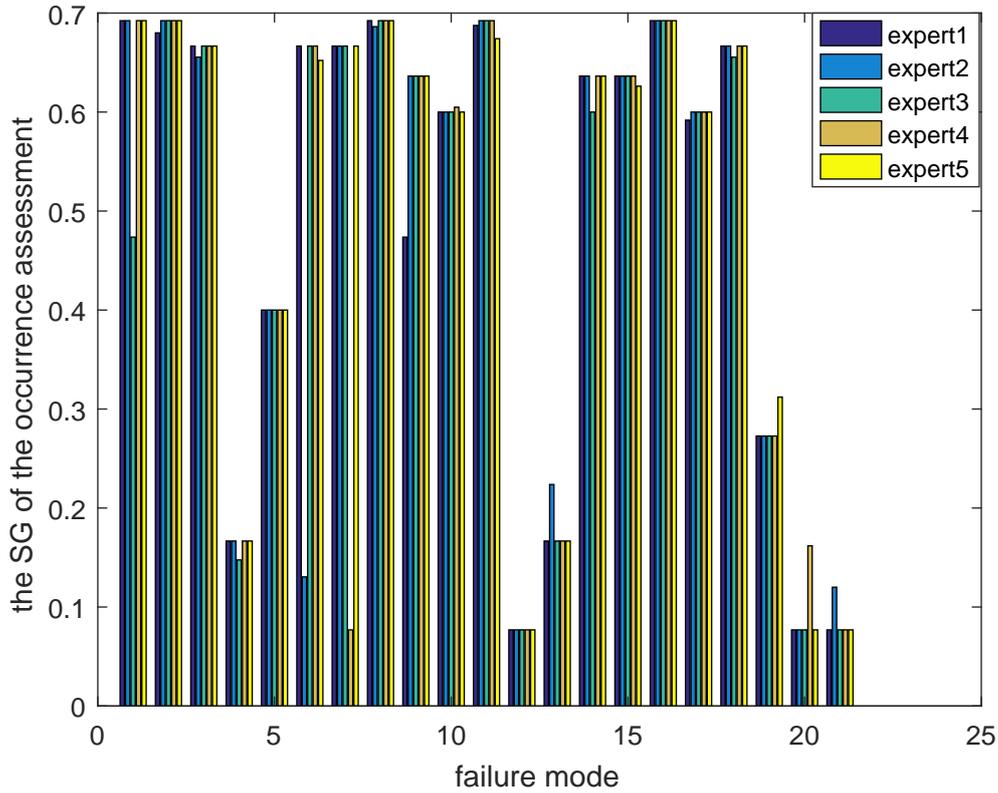


Figure 3: The new score of Occurrence assessment

$$SG_S^{11} = \frac{d(B)_S^{11}}{d(G)_S^{11} + d(B, G)_S^{11} + d(G)_S^{11}} = \frac{2.2}{2.2 + 6.8 + 2.8} \approx 0.1864.$$

$$SG_D^{11} = \frac{d(B)_D^{11}}{d(G)_D^{11} + d(B, G)_D^{11} + d(G)_D^{11}} = \frac{7}{7 + 2 + 2} \approx 0.6364.$$

The data of the other three assessments is treated in the same way, among which the result of occurrence assessment with the new scores are shown in Figure 3. As shown in Figure 3, the higher initial rating tends to the lower new score while the lower rating tends to the higher one.

Then, the first element b can be calculated by multiply the weight of the expert 1 and the new score:

$$b_O^{11} = \lambda_1 \times SG_O^{11} = 0.3 \times 0.6923 = 0.2077.$$

Table 5: The representation of D numbers for failure mode 1 (F_1)

F_1	D numbers
Expert 1	$D_{F_1}^{E_1} = \{(0.2077, \frac{1}{3}), (0.0559, \frac{1}{3}), (0.1909, \frac{1}{3})\}$
Expert 2	$D_{F_1}^{E_2} = \{(0.2077, \frac{1}{3}), (0.0500, \frac{1}{3}), (0.1909, \frac{1}{3})\}$
Expert 3	$D_{F_1}^{E_3} = \{(0.0947, \frac{1}{3}), (0.0333, \frac{1}{3}), (0.1273, \frac{1}{3})\}$
Expert 4	$D_{F_1}^{E_4} = \{(0.0692, \frac{1}{3}), (0.0217, \frac{1}{3}), (0.0636, \frac{1}{3})\}$
Expert 5	$D_{F_1}^{E_5} = \{(0.0692, \frac{1}{3}), (0.0167, \frac{1}{3}), (0.0627, \frac{1}{3})\}$

$$b_S^{1,1} = \lambda_1 \times SG_S^{1,1} = 0.3 \times 0.6923 = 0.0559.$$

$$b_D^{1,1} = \lambda_1 \times SG_D^{1,1} = 0.3 \times 0.6923 = 0.1909.$$

With the same process, the b of other D numbers can be obtained. Next, the data is treated using the D numbers. It needs to be emphasized that the weight of each rich factor is treated as the second parameter v . It is worth mentioning that the judgments for the relative importance of each criterion are various from different expert.

In order to compare with other methods which do not take it into consideration and prove the feasibility of the proposed method as well, the importance of three factors are assumed to be same. That is, v is supposed to be identically equal to $\frac{1}{3}$.

Therefore, the assessment result being disposed through above process are expressed in the forms of D numbers. According to the data of expert 1, a D number $D_{F_1}^{E_1}$ is constructed, where $D_{F_1}^{E_1} = \{(0.2077, \frac{1}{3}), (0.0559, \frac{1}{3}), (0.1909, \frac{1}{3})\}$. Similarity to $D_{F_1}^{E_1}$, According to the data of expert 2, expert 3, expert 4 and expert 5, four D numbers $D_{F_1}^{E_2}$, $D_{F_1}^{E_3}$, $D_{F_1}^{E_4}$ and $D_{F_1}^{E_5}$ are generated. Table 4 shows these D numbers. In the same method, for failure mode F_i , and expert E_j , each D number $D_{F_i}^{E_j}$ can be constructed, too.

For each failure mode, all experts' data represented by D numbers are combined according to the combination rule of D numbers mentioned in [9]. Take the failure mode 1 (F_1) for example, the integrated assessment of three experts is the aggregation of $D_{F_1}^{E_1}$, $D_{F_1}^{E_2}$, $D_{F_1}^{E_3}$, $D_{F_1}^{E_4}$ and $D_{F_1}^{E_5}$,

$$D_{F_1} = D_{F_1}^{E_1} \oplus D_{F_1}^{E_2} \oplus D_{F_1}^{E_3} \oplus D_{F_1}^{E_4} \oplus D_{F_1}^{E_5}.$$

In accordance with the processes of D_{F_i} numbers mentioned in [9], D_{F_1} is calculated. Therefore, the $I(D_{F_1})$ of D_{F_1} can be obtained. Using the same method, D_{F_i} and $I(D_{F_i})$ are acquired, which are shown in Table 5. In the meantime, according to $I(D_{F_i})(i = 1, 2, \dots, 21)$, the ranking of 21 failure modes are obtained, where the failure modes with the lower values of $I(D)$ are given higher priorities.

In Table 6, the risk priority rankings of the evidential downscaling method are shown. Compared with the ranking of the evidential downscaling method, the proposed method's is similar. As seen from Table 6, failure mode 3, failure mode 9, failure mode 12, failure mode 13, failure mode 15, failure mode 18, failure mode 20 and failure mode 21 have the same risk priority ranking in both methods. Besides, it is indicated that the five of highest risk priority rankings are

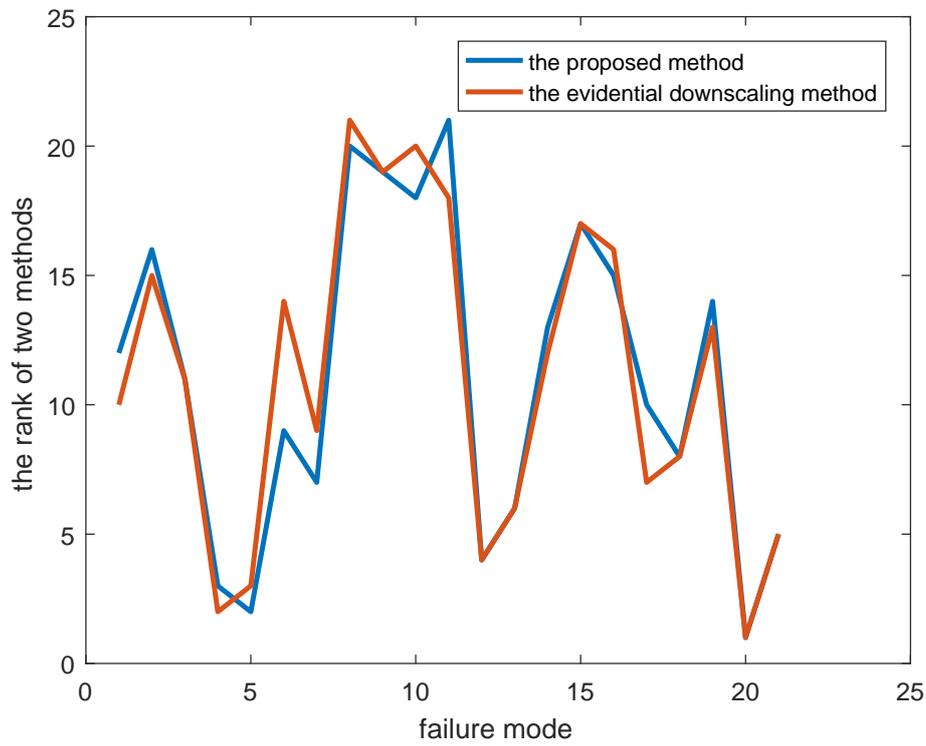


Figure 4: The comparison of risk priority ranking by two method

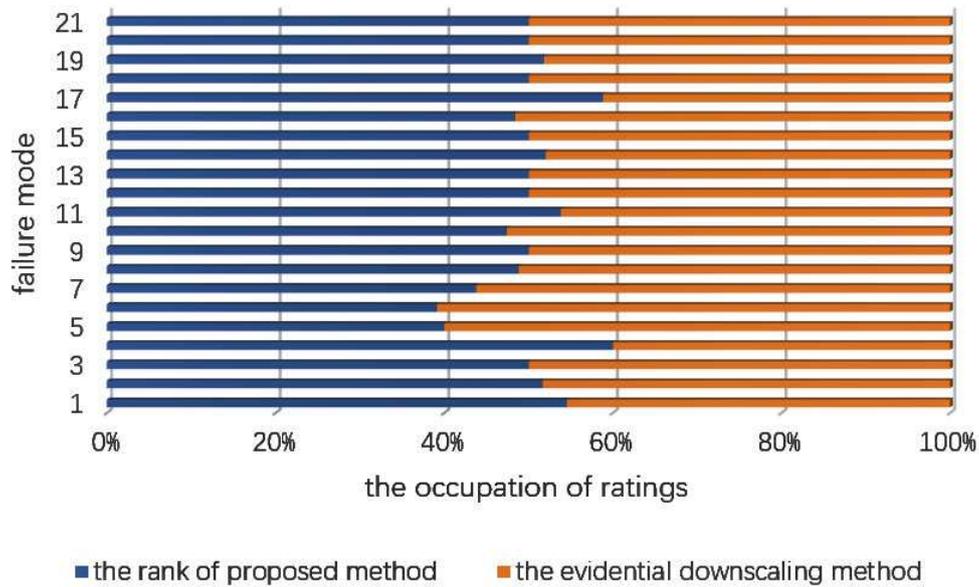


Figure 5: The occupation of ratings between two method

failure mode 20, 4, 5, 12 and 21, which means that these 5 faults are most likely to occur in both two methods. Moreover, in both two methods, the five of the lowest priorities are failure mode 8, 9, 10, 11 and 15, indicating that these 5 failures are supposed to be.

Figure 4 shows the comparison of risk priority rankings by the proposed method and the

Table 6: The comparison of risk priority ranking by two methods

Failure mode	Result	The rank of proposed method	The evidential downscaling method
1	0.06634	12	10
2	0.07464	16	15
3	0.06597	11	11
4	0.04524	3	2
5	0.04210	2	3
6	0.06309	9	14
7	0.05789	7	9
8	0.08779	20	21
9	0.08743	19	19
10	0.08616	18	20
11	0.08824	21	18
12	0.04596	4	4
13	0.05568	6	6
14	0.06670	13	12
15	0.08518	17	17
16	0.06896	15	16
17	0.0633	10	7
18	0.06064	8	8
19	0.06781	14	13
20	0.02350	1	1
21	0.05345	5	5

evidential downscaling method, in which the X-axis shows the rankings and Y-axis shows the failure modes. As is shown in Figure 4, it indicates that the rank result generated by proposed method has similar trend with the evidential downscaling method, which proves the proposed method is valid. However, in consideration that it is out of reality to request the propositions to be exclusive mutually, the proposed method is more reasonable. In 5, the occupation of ratings between two methods for each failure mode is shown. From Figure 5, the edge of the area around with different color is almost near the midline of the graph, which proves that the proposed method similarly has a good effect for risk assessment.

5 Conclusion

The conventional FMEA method has been criticized for its deficiencies especially in the evaluation of risks of failure mods and computation of classical RPN. A large number of new methods for FMEA have been proposed. Nevertheless, some deficiencies still exist in these methods, such as huge computing quality. Although the evidential downscaling method presented by Du Y et al. is capable to greatly reduce the amount of calculation, it is not reasonable that the propositions are requested to be exclusive mutually. Therefore, a new method to evaluate risk in FMEA based on D numbers downscaling method is proposed with the purpose of solving this problem.

Obviously, the proposed method can handle the uncertain information well in FMEA. Compared with the traditional RPN, the proposed method not only can dispose the scores of three risk factors even if they are uncertain, but also takes into consideration the relative importance weights of risk factors. Therefore, the result of the proposed method is more reliable. In comparison to the existing method, the proposed has more succinct calculation formulas, which has a smaller amount of calculation so that it is more worthy of promotion. Furthermore, because of

the use of D numbers, the propositions are not required to be exclusive mutually, which proves the proposed method is more reasonable than the evidential downscaling method. In conclusion, the proposed method is a feasible and efficient method to risk assessment in failure mode and effects analysis. In addition, although the proposed method takes the weights of three factors into consideration, it does not make sure the method to obtain the weights, which would be the further exploration.

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