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An Uncertainty Measure for Interval-valued Evidences

W. Jiang, S. Wang

Wen Jiang*, Shiyu Wang
School of Electronics and Information,
Northwestern Polytechnical University
Xi'an, Shaanxi Province, 710072, China
*Corresponding author: jiangwen@nwpu.edu.cn
jiangwenpaper@hotmail.com, wangshiyu@mail.nwpu.edu.cn

Abstract: Interval-valued belief structure (IBS), as an extension of single-valued belief structures in Dempster-Shafer evidence theory, is gradually applied in many fields. An IBS assigns belief degrees to interval numbers rather than precise numbers, thereby it can handle more complex uncertain information. However, how to measure the uncertainty of an IBS is still an open issue. In this paper, a new method based on Deng entropy denoted as UIV is proposed to measure the uncertainty of the IBS. Moreover, it is proved that UIV meets some desirable axiomatic requirements. Numerical examples are shown in the paper to demonstrate the efficiency of UIV by comparing the proposed UIV with existing approaches.
Keywords: Dempster-Shafer theory, interval-valued belief structure, interval evidence, uncertainty measure, Deng entropy.

1 Introduction

Dempster-Shafer evidence theory, also known as D-S theory was proposed by Dempster [8] and extended by Shafer [45], it has received widespread attention and application in information processing [18,25,40,43,46,52]. As compared with classic probability theory, D-S theory allocates the belief to multi-subset proposition and does not require a priori information. Accordingly, D-S theory is used to process the uncertain information in many fields such as risk assessment [16,24,39,60], decision making [4,7,11,36,38,58], fault diagnosis [20,26,27,41,48,51], information fusion [2,9,12,19,35] and pattern classification [3,42,44,55].

Although the application of D-S theory has made considerable progress, there are still some common issues in urgent need to be solved. For instance, conflict processing should be taken into consideration when the obtained evidence is highly conflicting with each other [28,30,37,53], for we may get the count-intuitive results [29,59]. In view of this, many scholars have carried out extensive and profound research. Denœux [15] considered the evidence expressed by fuzzy-valued which acquire lots of application [57]. Moreover, the classic D-S theory demands precise belief degrees, yet it is not always available in some cases. For instance, in the decision making, the experts sometimes cannot provide an accurate assessment because of the lack of information. At this time, an interval-valued belief structure (IBS) [56] is more suitable for dealing with the uncertainty problem. About extending the D-S theory to IBS, many scholars have carried out some research such as Denœux [14] put forward a set of concepts about interval-valued belief structure and initially explored the combination and the uncertainty of it. Lee & Zhu [34] proposed the combination of two interval evidence. Wang [54] proposed the approach to combine and standardize the interval evidence in one step. However, it must be noted that there are still many unresolved issues about interval-valued belief structure.

One of the crucial issues is uncertainty measurement [10,50]. From the perspective of information theory, Klir elaborated the inner relationship between uncertainty and information [33].

Bronevich [5,6] discussed some of the issues and applications of the measurement of the uncertainty for imprecise probabilities. However, even how to measure the uncertainty of the mass function in D-S theory is still a considerable issue [21,23]. Dubois & Prade presented weighted Hartley entropy [17] to express the non-specificity of BPA. Klir & Wierman [32] explored five axiomatic requirements for the uncertainty measures including range, probabilistic consistency, set consistency, additivity and subadditivity, respectively. Abellán & Masegosa [1] have extended the axiomatic approach by appending new monotonicity requirement. Among existing uncertainty measures, aggregated uncertainty (AU) [22] and ambiguity measure (AM) [31] are two representative measures, yet they have their own shortcomings, such as low sensitivity and high computing complexity. Deng entropy [13] divided the belief for each focal element into all potential subsets. On the other hand, there is not many approaches about the uncertainty measure for interval-valued belief structure. Denoeux [14] proposed a rudiment to measure the uncertainty, yet it was immature and lacked the mathematical proof. Song [49] defined the axiomatic requirements for uncertainty measure and presented a new method IU to measure the uncertainty. But IU lost part of the information and may cause the counter-intuitive result because of the transformation from belief structures to probability distributions. Accordingly, how to effectively measure the uncertainty of interval-valued belief structure is still an open issue. In this paper, a new method based on Deng entropy to measure the uncertainty of the interval-valued belief structure and its axiomatic proof is presented as well. Several examples are shown to illustrated the rationality and effectiveness of the method.

The remainder of this paper is organized as follows. Section 2 starts with a brief presentation of D-S evidence theory and some other indispensable related concepts. In Section 3, we present a new method to measure the uncertainty of the interval-valued belief structure. Some numerical examples are given to demonstrate the validity of our new method in Section 4. Conclusions are summarized in Section 5.

2 Preliminaries

2.1 Dempster-Shafer evidence theory

Dempster-Shafer evidence theory, as introduced by Demster [8] and expanded later by Shafer [45], has been widely used in dealing with uncertainty. Some basic concepts in D-S theory are introduced as follows.

Let Θ be a finite set of worlds, which is called a frame of discernment (FOD). Θ consists of some propositions, which are mutually exclusive and exhaustive, and indicated by

$$\Theta = \{\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_N\}. \tag{1}$$

Let 2^Θ be the power set of Θ, namely

$$2^\Theta = \{\varnothing, \theta_1, \theta_2, \dots, \theta_N, \{\theta_1 \cup \theta_2\}, \dots, \{\theta_1 \cup \theta_2 \cup \cdots \cup \theta_i\}, \dots, \Theta\}. \tag{2}$$

For a FOD Θ, a mass function is a mapping $m : 2^\Theta \to [0,1]$, it is also called the basic probability assignment (BPA) or the belief structure. BPA must satisfy the following condition

$$\begin{cases} \sum\limits_{A \in 2^\Theta} m(A) = 1, \\ m(\varnothing) = 0. \end{cases} \tag{3}$$

For a BPA, its belief function $Bel : 2^\Theta \to [0,1]$ is defined as

$$Bel(A) = \sum_{B \subseteq A} m(B), \tag{4}$$

the plausibility function $Pl : 2^\Theta \to [0,1]$ is defined as

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B\cap A\neq\varnothing} m(B). \tag{5}$$

Assume there are two BPAs m_1 and m_2 with the same FOD, it can be combined by Dempster's combination rule.

$$m(A) = \frac{1}{1-k}\sum_{B\cap C=A} m_1(B)m_2(C), \tag{6}$$

where

$$k = \sum_{B\cap C=\varnothing} m_1(B)m_2(C). \tag{7}$$

k is between [0,1], which is called the coefficient of conflict. When $k = 1$, Dempster's combination rule will be invalid.

2.2 Interval-valued belief structure

Uncertainty is sometimes no longer described by a unique belief structure, but by a convex set of belief structures verifying certain constraints. A set of concepts of interval-valued belief structure (IBS) is given as follows [14].

Let Θ be the frame of discernment, $F_1, F_2, \ldots, F_N$ be N subsets of Θ and $[a_i, b_i]$ be N intervals with $0 \leqslant a_i \leqslant b_i \leqslant 1$, $(i = 1, 2, \ldots, N)$. An interval-valued belief structure (IBS) m is a belief structure on Θ such that

$$a_i \leqslant m(F_i) \leqslant b_i, \tag{8}$$

where

$$0 \leqslant a_i \leqslant b_i \leqslant 1,\ i = 1, 2, \ldots, N, \tag{9}$$

$$\sum_{i=1}^{N} a_i \leqslant 1 \text{ and } \sum_{i=1}^{N} b_i \geqslant 1, \tag{10}$$

$$m(A) = 0 \quad \forall A \notin \{F_1, F_2, \ldots, F_N\}. \tag{11}$$

Obviously, m are non-empty imposes certain constraints on the a_i and b_i. If the singleton m is an IBS with $a_i = b_i = m(F_i)$ for $\forall F_i$, m degenerates to a precise belief structure (BS). An IBS means the interval associated to each subset of Θ is [0,1]. It may be interpreted as reflecting "second-order" ignorance, that is, ignorance of what the state of belief of an agent may be.

Let m be an interval-valued belief structure, namely $a_i \leqslant m(F_i) \leqslant b_i$ for $i = 1, 2, \ldots, N$. If $\forall k \in \{1, 2, \ldots, N\}$, a_i and b_i satisfy

$$\sum_{i=1}^{N} a_i + (b_k - a_k) \leqslant 1, \tag{12}$$

$$\sum_{i=1}^{N} b_i - (b_k - a_k) \geqslant 1. \tag{13}$$

Then, m is called a normalized interval-valued belief structure (NIBS) [54].

For a non-normalized interval-valued belief structure m, which violates Eq. (10), it can be normalized by following equations.

$$\hat{a}_i = \frac{a_i}{a_i + \sum_{j=1, j\neq i}^{N} b_j},\ i = 1, 2, \ldots, N, \tag{14}$$

$$\hat{b}_i = \frac{b_i}{b_i + \sum_{j=1,j\neq i}^{N} a_j},\ i = 1, 2, \ldots, N. \tag{15}$$

On the other side, if m has already satisfied Eq. (10), but not Eqs. (12) and (13), it can be normalized by following two equations.

$$\hat{a}_i = \max\left\{a_i, 1 - \sum_{j=1,j\neq i}^{N} b_j\right\},\ i = 1, 2, \ldots, N, \tag{16}$$

$$\hat{b}_i = \min\left\{b_i, 1 - \sum_{j=1,j\neq i}^{N} a_j\right\},\ i = 1, 2, \ldots, N. \tag{17}$$

The concepts of belief function and plausibility function may easily be generalized to an interval-valued belief structure. Since these quantities are linear combinations of belief masses constrained in closed intervals, their ranges are both closed intervals.

Let m be a normalized interval-valued belief structure on Θ. For $\forall A \in \Theta$, its belief function and plausibility function are defined respectively as

$$Bel(A) = \left[\min \sum_{F_i \subseteq A} m(F_i), \max \sum_{F_i \subseteq A} m(F_i)\right], \tag{18}$$

$$Pl(A) = \left[\min \sum_{F_i \cap A \neq \varnothing} m(F_i), \max \sum_{F_i \cap A \neq \varnothing} m(F_i)\right], \tag{19}$$

where

$$\min \sum_{F_i \subseteq A} m(F_i) = \max\left[\sum_{F_i \subseteq A} a_i, \left(1 - \sum_{F_i \not\subseteq A} b_i\right)\right], \tag{20}$$

$$\max \sum_{F_i \subseteq A} m(F_i) = \min\left[\sum_{F_i \subseteq A} b_i, \left(1 - \sum_{F_i \not\subseteq A} a_i\right)\right], \tag{21}$$

$$\min \sum_{F_i \cap A \neq \varnothing} m(F_i) = \max\left[\sum_{F_i \cap A \neq \varnothing} a_i, \left(1 - \sum_{F_i \cap A = \varnothing} b_i\right)\right], \tag{22}$$

$$\max \sum_{F_i \cap A \neq \varnothing} m(F_i) = \min\left[\sum_{F_i \cap A \neq \varnothing} b_i, \left(1 - \sum_{F_i \cap A = \varnothing} a_i\right)\right]. \tag{23}$$

2.3 Deng entropy

Since Shannon entropy [47] was proposed to quantify the expected value of the information volume contained in a message, it has became a significant approach to measure the uncertainty. However, for a mass function in D-S theory, Shannon entropy cannot calculate its uncertainty because the mass function includes multiple subset elements. To measure the uncertainty of the mass function, Deng [13] proposed Deng entropy as follows

$$E_d(m) = -\sum_{A \subseteq \Theta} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1}, \tag{24}$$

where m is a BPA defined on the frame of discernment Θ, A is the focal element of m, and $|A|$ is the cardinality of A.

Deng entropy is analogous with the classical Shannon entropy, but the belief for each focal element A is divided by $(2^{|A|}-1)$ which indicates the potential supports in A.

3 Proposed uncertainty measure for interval-valued belief structures

In an interval-valued belief structure, the belief degree for each subset is not a precise value but an interval. So contrasted with single-valued belief structures, an interval-valued belief structure is more vague and more uncertain, since an IBS has the "second-order" ignorance. Thus, how to measure the uncertainty of the IBS is an essential issue. In this paper, A new method to measure the uncertainty of IBS is proposed.

Definition 1. Let m be a normalized interval-valued belief structure on the frame of discernment $\Theta=\{F_1,F_2,\ldots,F_N\}$, and it satisfies $a_i \leqslant m(F_i) \leqslant b_i$, which means the accurate belief $m(F_i)\in [a_i,b_i]$. Then the uncertainty measure of the IBS m is as follows

$$UIV(m)=\sum_{i=1}^{2^N}[\min_{m(F_i)\in[a_i,b_i]}\widetilde{E_d(F_i)},\max_{m(F_i)\in[a_i,b_i]}\widetilde{E_d(F_i)}], \tag{25}$$

where

$$\widetilde{E_d(F_i)}=-m(F_i)\log_2\frac{m(F_i)}{2^{|F_i|-1}}, \tag{26}$$

and $|F_i|$ is the cardinality of F_i.

The new measurement method we proposed is based on Deng entropy, not Shannon entropy, so our method is more suitable to handle the proposition of multi-subsets. For Deng entropy, the belief of the focal element $m(F_i)$ is divided by the number of potential subsets $2^{|F_i|}-1$ that demonstrates the non-specificity of the evidence. The more single elements are contained in focal elements, it is obvious that the greater the uncertainty. The term $-m(F_i)\log_2 m(F_i)$ is analogous to Shannon entropy and is the measure of discord of the evidence. Thereby, it is also appropriate to quantify the uncertainty of interval-valued belief structure. Obviously, UIV is an interval number. Its value embodies the belief distribution of different proposition in IBS, and its interval length reflects the ambiguity generated by the belief expressed in intervals.

Song [49] proposed the axiomatic requirements for a measure of uncertainty for a normalized interval-valued belief structure m.

Theorem 2. *Let U be a measure of uncertainty for a normalized interval-valued belief structure m on the FOD $\Theta=\{\theta_1,\theta_2,\ldots,\theta_N\}$, then U must content the following condition.*

1. *Whenever the NIBS defines a precise probability distribution, U degenerates to Shannon entropy.*
2. *When the NIBS assigned to all subsets of Θ are completely unknown, its uncertainty is maximum. Thus, U reaches its maximum value.*
3. *If the NIBS assigns to a certain singleton of Θ is 1, the uncertainty of it is 0. Therefore, U gets its minimum value 0.*

It will be shown that our new method satisfies the above-mentioned axiomatic requirement.

Proof:

1. If the NIBS m defines a precise probability distribution on $\Theta = \{F_1, F_2, \ldots, F_N\}$,

$$\begin{aligned} UIV(m) &= \sum_{i=1}^{2^N} [\widetilde{E_d(F_i)}, \widetilde{E_d(F_i)}] \\ &= \sum_{i=1}^{N} -m(F_i) \log_2 \frac{m(F_i)}{2^{|1|} - 1} \\ &= -\sum_{i=1}^{N} m(F_i) \log_2 m(F_i). \end{aligned}$$

From the above equation, we can see that when m defines a precise belief structure on Θ, UIV degenerates to Deng entropy. Moreover, when m defines a precise probability distribution, UIV degenerates to Shannon entropy.

2. When the NIBS assigned to all subsets of Θ are completely unknown, that is for $\forall F_i \in 2^{\Theta}$, $[a_i, b_i] = [0, 1]$. It is apparent that

$$UIV(m) = \sum_{i=1}^{2^N} [\min_{m(F_i)\in[0,1]} \widetilde{E_d(F_i)}, \max_{m(F_i)\in[0,1]} \widetilde{E_d(F_i)}$$

where

$$\widetilde{E_d(F_i)} = -m(F_i) \log_2 \frac{m(F_i)}{2^{|F_i|-1}},$$

and it can be seen as a function of F_i, now the independent variable F_i is $\in [0, 1]$. Therefore, the minimum value of $\widetilde{E_d(F_i)}$ is 0 and the maximum value may be mutative with the change of $|F_i|$ yet it can always get its maximum value for any F_i, that is

$$\max_{m(F_i)\in[0,1]} \widetilde{E_d(F_i)} = \max \widetilde{E_d(F_i)}$$

So,

$$UIV(m) = [0, \sum_{i=1}^{2^N} \max \widetilde{E_d(F_i)}].$$

In this case, the value and the interval length of UIV are both the maximum value, which indicates that m is totally uncertain, that is, its uncertainty is maximum.

3. If the NIBS assigns to a certain singleton of Θ is 1, there is no harm in supposing that for singleton F_k, $m(F_k) = 1$, and the belief degree of all the rest subsets is 0. Then

$$\begin{aligned} UIV(m) &= \sum_{i=1}^{2^N} [\min_{m(F_i)\in[a_i,b_i]} \widetilde{E_d(F_i)}, \max_{m(F_i)\in[a_i,b_i]} \widetilde{E_d(F_i)}] \\ &= [\min_{m(F_i)\in[1,1]} \widetilde{E_d(F_k)}, \max_{m(F_i)\in[1,1]} \widetilde{E_d(F_k)}] + \sum_{\substack{i=1 \\ i\neq k}}^{2^N} [\min_{m(F_i)\in[0,0]} \widetilde{E_d(F_i)}, \max_{m(F_i)\in[0,0]} \widetilde{E_d(F_i)}] \\ &= -1 \times \log_2 \frac{1}{2^1 - 1} - \sum_{\substack{i=1 \\ i\neq k}}^{2^N} (0 \times \log_2 \frac{0}{2^{|F_i|} - 1}) = 0 \end{aligned}$$

In fact, the UIV at this time is not 0, but [0,0]. This result thoroughly explains the m under this circumstance is totally definite, and it is also in line with intuition.

Table 1: NIBSs in Example 3

	$\{F_1\}$	$\{F_2\}$	$\{F_3\}$	$\{F_1, F_3\}$
m_1	[0.2,0.3]	[0.1,0.35]	[0.4,0.6]	[0,0]
m_2	[0.2,0.3]	[0.1,0.35]	[0.35,0.7]	[0,0]
m_3	[0.2,0.3]	[0.1,0.35]	[0,0]	[0.4,0.6]
m_4	[0.2,0.3]	[0.1,0.35]	[0.2,0.3]	[0.2,0.3]

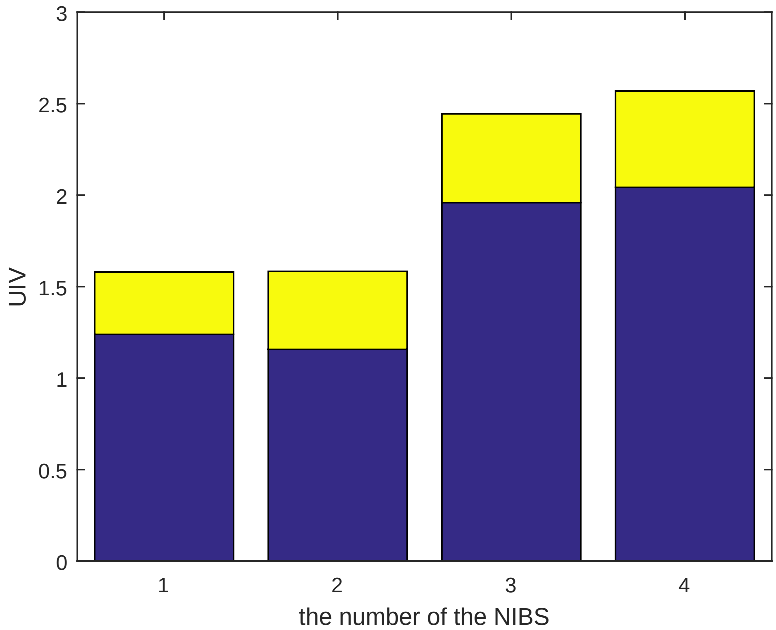

Figure 1: The UIV of each NIBS in Example 3

□

4 Numerical examples

In this section, several examples are given to demonstrate the effectiveness of UIV.

Example 3. *Assume a frame of discernment* $\Theta = \{F_1, F_2, F_3\}$, *and consider four NIBSs defined as shown in Table 1.*

We can calculate the UIV of the NIBSs as follows

$$UIV(m_1) = [1.239, 1.580] \quad UIV(m_2) = [1.157, 1.583]$$
$$UIV(m_3) = [1.959, 2.444] \quad UIV(m_4) = [2.042, 2.569]$$

and they are also graphically shown in Fig. 1. The yellow portion represents the endpoint of the interval of the UIV. The range of $UIV(m_2)$ is larger than $UIV(m_1)$ from the figure, since $m_2(F_3)$ is more uncertain than $m_1(F_3)$. However, the value of $UIV(m_2)$ is close to $UIV(m_1)$ because the belief distribution in m_1 and m_2 are about the same. Considering $UIV(m_3)$ and $UIV(m_1)$, it is obvious that both the length and the value of $UIV(m_3)$ are bigger since the

Table 2: UIV in Example 4

Cases	UIV
A={1}	[2.080,3.803]
A={1,2}	[3.216,4.886]
A={1,2,3}	[3.949,5.864]
A={1,2,...,4}	[4.609,6.743]
A={1,2,...,5}	[5.238,7.581]
A={1,2,...,6}	[5.851,8.400]
A={1,2,...,7}	[6.458,9.209]
A={1,2,...,8}	[7.062,10.013]
A={1,2,...,9}	[7.663,10.816]

multi-element can take along more uncertainty than single element even though in the same interval. It is worth noting that compared with $UIV(m_3)$, $UIV(m_4)$ is close but slightly larger. Although a great deal of belief are assigned on the multi-element in m_3 and it conveys illegibility, the allocation form which distributes the belief to more subsets is more excursive and this result is we take for granted.

Example 4. *Suppose that we have a frame of discernment* $\Theta = \{1, 2, \dots, 10\}$. *A NIBS* m *is shown as follows.*

$$m(2,3) = [0.1, 0.25],\ m(A) = [0.6, 0.8],\ m(\Theta) = [0.1, 0.2]$$

where A *is a varying subset of* Θ. *A starts at* $A = \{1\}$, *increases one more element every time and ending with* $A = \{1, 2, \dots, 9\}$. *The* UIV *of* m *are shown in Table 2 and Fig. 2. The yellow portion represents the endpoint of the interval of the* UIV.

From Fig. 2, the result shows that UIV increases monotonically with the number of elements in A. This is rational because the more elements contained in a subset, the more uncertain it is. From the example it can be seen that UIV is capable of reflecting such a feature.

In the first two examples, some superior properties are demonstrated. Then an example from Song [49] are used to illustrate our proposed UIV and contrast it with Song's uncertainty measure IU. The formula of Song's measurement are shown as follows.

Definition 5. Let m be a normalized interval-valued belief structure on the FOD $\Theta = \{F_1, F_2, \dots, F_N\}$, and it satisfies $a_i \leqslant m(F_i) \leqslant b_i$. Then IU of the IBS m is as follows

$$IU(m) = \sum_{i=1}^{N} \left(-\frac{a_i + b_i}{2} \log_2 \frac{a_i + b_i}{2} + \frac{b_i - a_i}{2}\right) \tag{27}$$

Example 6. *The example Song used in the paper is shown in Table 3, and to make a comparison with Song's method, the consequents of IU and our new method* UIV *are both demonstrated in Table 4.*

For the NIBSs from m_1 to m_5, we can see their belief intervals are completely consistent, merely the corresponding subsets are disparate. The uncertainty degree IU proposed by Song, are so similar that it is difficult to measure the uncertainty accurately. Moreover, the belief assignment of m_1 and m_5 are entirely different, yet their IU are almost identical. For UIV, m_5 with more belief assigned to multiple elements has a higher uncertainty, m_2 and m_3 take

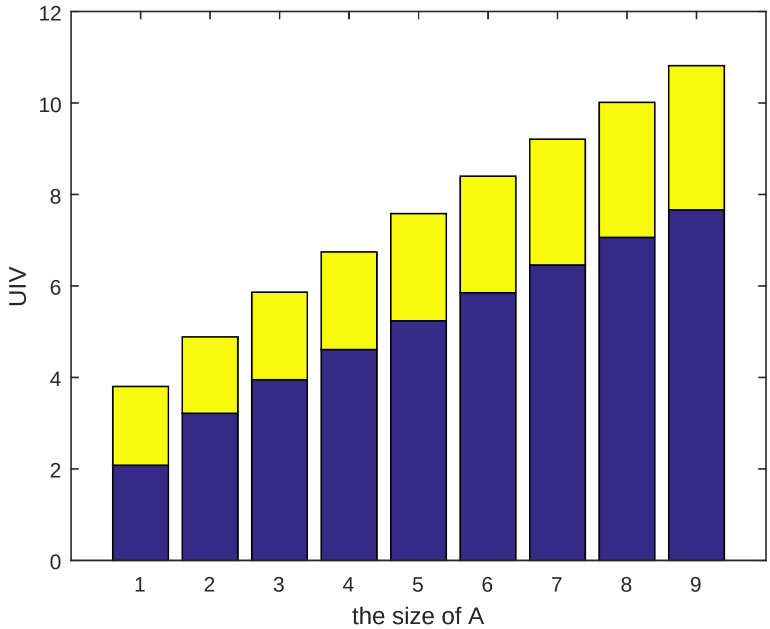

Figure 2: UIV in Example 4

Table 3: NIBSs in Song's example ($\Theta = \{F_1, F_2, F_3\}$)

	$\{F_1\}$	$\{F_2\}$	$\{F_3\}$	$\{F_1, F_2\}$	$\{F_1, F_3\}$	$\{F_2, F_3\}$	$\{F_1, F_2, F_3\}$
m_1	[0.2,0.4]	[0.1,0.3]	[0.3,0.6]	[0,0.1]	[0,0]	[0,0]	[0,0]
m_2	[0.2,0.4]	[0,0]	[0,0]	[0,0.1]	[0.3,0.6]	[0,0]	[0.1,0.3]
m_3	[0,0]	[0.1,0.3]	[0,0]	[0,0.1]	[0.2,0.4]	[0.3,0.6]	[0,0]
m_4	[0,0]	[0,0]	[0.3,0.6]	[0,0.1]	[0.3,0.6]	[0.1,0.3]	[0.2,0.4]
m_5	[0,0]	[0,0]	[0,0]	[0,0.1]	[0.3,0.6]	[0.1,0.3]	[0.2,0.4]
m_6	[0,1]	[0,1]	[0,1]	[0,0]	[0,0]	[0,0]	[0,0]
m_7	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,1]

Table 4: IU and UIV of the NIBSs

	IU	UIV
m_1	1.930	[1.239,2.073]
m_2	1.609	[2.074,3.778]
m_3	1.889	[2.110,3.568]
m_4	1.575	[1.714,3.181]
m_5	1.939	[2.513,4.532]
m_6	3	[0,1.592]
m_7	1.793	2.807

Table 5: NIBSs and their IU and UIV in Example 7

	$\{F_1\}$	$\{F_2\}$	$\{F_1, F_2\}$	IU	UIV
m_1	[0.1,0.25]	[0.3,0.4]	[0.4,0.5]	1.163	[2.016,2.323]
m_2	[0.2,0.3]	[0.4,0.45]	[0.2,0.4]	1.163	[1.764,2.213]

second place, as well as m_1 is the most precise of these NIBSs. Furthermore, UIV is suitable for measurement for the reason that the difference in calculated values is significant and thus has a degree of discrimination.

Another detail of concern is m_6 and m_7. The uncertainty of m_7 is low, while the maximum uncertainty degree occurs on m_6. The cause of this consequence as Song said in [49], "This is caused by the transformation from belief structures to Bayesian belief structures, which will cause information loss." $UIV(m_6)$ is comparatively small because m_6 only distribute the belief to singleton. In addition, m_7 actually is not a normalized interval-valued belief structure. It turns into a NIBS $m_7(\{F_1, F_2, F_3\}) = 1$ by Eqs. (16) and (17). After standardization, $UIV(m_7)$ is a precise number and its uncertainty can be effectively measured.

Example 7. *Let a frame of discernment be $\Theta = \{F_1, F_2\}$. Two NIBSs, their IU and UIV are shown in Table 5.*

We can calculate that both two Bayesian belief structures of m_1 and m_2 are $m(a) = [0.3, 0.5]$, $m(b) = [0.5, 0.65]$, and IU is not competent to measure the uncertainty in this situation. Because for two unrelated NIBSs with significant differences in the degree of uncertainty, their IU are equivalent. Through the above analysis, it is found that UIV is more reasonable to measure the uncertainty of the interval-valued belief structures.

5 Conclusion

D-S theory has been widely used in information processing and information fusion. In many applications, we can only obtain an interval-valued belief structure instead of a basic probability assignment defined on single values, due to lack of information and some other reasons. It is indispensable to measure the uncertainty of the IBS, there is still an open issue.

The main contribution of this paper is a new method based on Deng entropy, UIV is proposed to measure the uncertainty of an IBS. It is proved that UIV meets some axiomatic properties. Numerical examples are illustrated to show the effectiveness of UIV and discuss its characteristic. Moreover, it is found that UIV is more reasonable and sensitive in comparison with existing methods.

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