

Distributed Compressed Sensing Algorithm for Hierarchical WSNs

N. Jiang, H. You, F. Jiang, L. Liu, Y. He

Nan Jiang*

College of Information Engineering, East China Jiaotong University
Nanchang, 330013, JiangXi, P.R.China
State Key Laboratory Breeding Base of Nuclear Resources and Environment,
East China Institute of Technology, Nanchang, 330013, JiangXi, P.R. China
*Corresponding author: jiangnan1018@gmail.com

Hui You, Lingfeng Liu

College of Information Engineering, East China Jiaotong University
Nanchang, 330013, JiangXi, P.R.China
youthui106@gmail.com, liulingfeng1983@gmail.com

Feng Jiang

School of Computer Science and technology, Harbin Institute of Technology
Harbin, 15001, HeiLongJiang, P.R.China
fjiang@hit.edu.cn

Yueshun He

College of Information Engineering, East China Institute of Technology
Nanchang, 330013, JiangXi, P.R.China
hys8418@163.com

Abstract: In the traditional theory of CS, each sensor node in Wireless Sensor Networks (WSNs) sends the information to sink node directly, and only considers the correlation between internal nodes information when recover, this would lead to the loss of the node information and also cause too much energy consumption. In this paper, combined with DEEC protocol, based on Simultaneous orthogonal matching pursuit (SOMP) algorithm, we propose a novel distributed compressed sensing algorithm for Hierarchical Wireless Sensor Networks (DCSH algorithm). This algorithm use the spatial correlation between the nodes and joint sparse model JSM-2, to compress and recover the sensor node information according to SOMP algorithm. Simulation results show that DCSH algorithm can not only obtain accurate reconstruction values of the node information and reduce the energy consumption greatly, but also can prolong the network lifetime.

Keywords: Distributed Compressed Sensing; DEEC algorithm; JMS-2 model; Cluster Architecture; Wireless Sensor Networks (WSNs).

1 Introduction

In the past few years, the capability of receiving and transmitting data in wireless sensor networks has been constantly enhanced, simultaneously, the amount of data, which need to be handled, is also growing very quickly. Traditional Nyquist sampling theory, needs the sampling rate of signal not less than 2 times the bandwidth of the signal; undoubtedly, this is higher requirement for signal processing capability and hardware equipments. Therefore, How to deal with bigger and faster data and find a new method is attracted more and more attentions. In 2004, Donoho and Candes et al proposed Compressed Sensing (CS) theory, it is a novel theory which make full use of sparse and compressibility of the signal [1] [2]. This theory shows that when the signal is sparse or compressible, it can realize the refactoring value of signal exactly

through collecting the projection values of small amount of signal. Different from traditional data processing method, sampling and compression of signal can conduct at the same time with a low rate, to the method of data processing, sampling process also completed the compression, so the amount of sampling data is greatly reduced, the Nyquist sampling is evenly spaced sampling, and compressed sampling is random sampling.

Generally, compressed sensing theory study how to use the intra-signal on compression. Considering distributed deployment and limited capability of sensor nodes in wireless sensor networks, it is necessary to use inter-signal with Distributed Compressed Sensing (DCS), D. Baron gave the basic concept and theory of Distributed Compressed Sensing [3], and then proved the upper and lower bounds of the number of measurements required for decoding [4]. In [5], the author presented another joint sparse model for the application scenarios such as the MIMO communication and speech signal, and designed the corresponding joint decoding algorithm. However, data fusion technology for wireless sensor networks based on the DCS theory is still at the starting stage. Therefore, it is necessary to study how DCS theory do the joint reconstruction through the observation vectors of each node in cooperation way, through define joint sparse of each node based on spatial correlation data.

Our contributions are showed as follows:

- Based on JSM-2 model, signals are related to each other, each signal has different coefficient value, while they are made up with the base vector of the same sparse coefficient set.
- By using SOMP algorithm, the decoding process can be simplified and corresponding measured values can also be reduced. In decoder side, the reconstruction efficiency will be improved.
- Cluster architecture is widely used in WSNs, such as classic LEACH, HEED, DEEC and so on. We try to bring cluster into CS theory. On one hand, it will save the transmission energy; on the other hand, sink nodes will receive more complete information in the case of limited sensor node processing capacity.

2 Simultaneous orthogonal matching pursuit(SOMP) algorithm

2.1 Distributed Source Coding [11]

According to the characteristics of the distributed sensor networks, a number of distributed coding algorithms are developed, these algorithms all involved the cooperation of sensor nodes, including predictive coding, distributed KLT and distributed wavelet transform, and also the three dimensional wavelet algorithm which uses inter-correlation between signal and in signal at the same time.

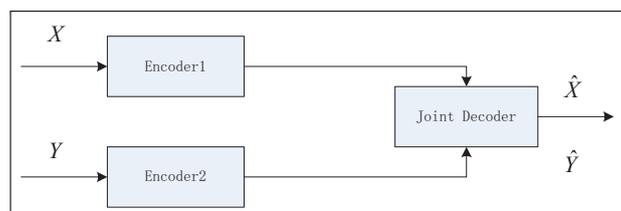


Figure 1: Distributed Source Coding.

Figure 1 is distributed source coding of two different source information. Processing of signal Y is completely independent, the transmission information is $H(Y)$, and processing of the

signal X need to use correlation with the signals Y . At the receiving side, the decoder $H(Y)$ can first restore the original signal Y ; as the edge information Y , use the correlation between the information X and Y that contained in $H(Y)$, and send the joint information $H(X|Y)$, complete the joint decoding of the source information X , here with the sum of the amount of the transmission information X and Y must be less than the joint entropy $H(X|Y)$, so it can ensure the compression effect of source information. In one word, for distributed source coding theory, the key issue is how to use the original related characteristics of the signals X and Y , and then to do the coding and joint decoding independently.

2.2 SOMP algorithm and JSM-2 model

JSM-2 [3]: The signals in this model have different coefficient value, while they are all made up with the base vector of the same sparse coefficient set. This model can be applied some important applications, such as MIMO telecommunication and audio signal array. The signals which are accorded with JSM-2 model may be sparsity in the fourier domain, for example, degenerative frequency component which is caused by different propagation paths.

In JSM-2 model, the expression of the original signal shown by the following formula:

$$X_j = \Psi \cdot \Theta_j, j \in 1, 2, \dots, J \quad (1)$$

Among them, the coefficient of each vector Θ_j , corresponding to a collection of indicators $I_j \subset 1, 2, \dots, N$, and I_j only has K elements, namely, that is $\|I_j\|_{l_0} = K$, the sparsity of each original signal X_j is K .

Paper [7] presented the greedy algorithm to recover the related signal, which is called Simultaneous Orthogonal Matching pursuit (SOMP) algorithm. The algorithm is very similar to the OMP algorithm, but there are some small changes. SOMP algorithm is based on the concept of distributed source coding. The basic idea of SOMP algorithm is: assuming that the sensor nodes in WSNs are all consistent with the JSM-2 model, namely each part of the original signal includes only part of the information and do the sparse representation on the same sparse basis, but the sparsity of each of the original signal sparse are not the same.

Suppose that there are B distributed sampling signals y_1, y_2, \dots, y_B , change the second step of the OMP algorithm to find the index, using the index to solve the following simple optimization problem:

$$\max_{\omega \in \Omega} \sum_1^B |\langle R_{k,t-1}, \varphi_j \rangle| \quad (2)$$

In which $R_{k,t-1}$ is the residual value of the first k distributed sample signal, the rest of the steps are the same with OMP algorithm. When $B = 1$, this process can be summarized as the standard OMP algorithm.

In conclusion, the SOMP algorithm is divided into the following steps:

- 1) Initialization: residual $r_0 = y$, the number of iterations $t = 1$, the index set $\Lambda_0 = \phi$;
- 2) Find the index λ_t , solve a simple optimization problem;
- 3) Update the indexes set $\Lambda_t = \Lambda_{t-1} \cup \lambda_t$, and update the selected column space $\theta_t = [\theta_{t-1}, \varphi_{j_t}]$. By convention, θ_t is an empty matrix;
- 4) By solving a least squares problem, get a new estimation of the signal;
- 5) Calculate the new approximate data and the new $a_t = \theta_t x_t$, $r_t = y - a_t$;
- 6) $t = t + 1$, if $t < m$, go back to step 2), otherwise, to step 7);

7) If $\|\theta_t\|_\infty > \gamma$, choose H_1 , otherwise choose H_0 . In which γ is nonzero threshold, $H_0 : \theta_s = 0$, $H_1 : \theta_s \neq 0$, θ_s is sparse coefficient.

3 DEEC clustering algorithm

DEEC [9] is a classical clustering algorithm, which can save the energy effectively and prolong the network lifetime. It can be able to construct optimized cluster structure for and data aggregation through the exchange of the message between the local nodes, and also balance network energy consumption effectively, then be better adapted to periodically collect data of sensor network applications. Simulation results show that DEEC can prolong the network lifetime of LEACH of about 45% under the smaller latency of network.

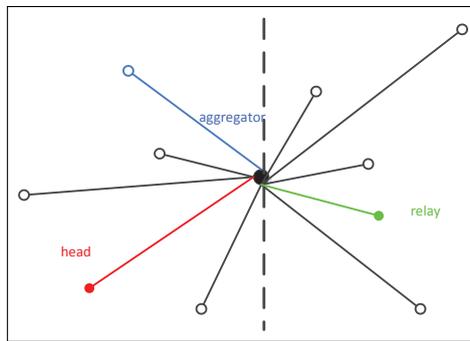


Figure 2: The structure of DEEC.

Clusters in the DEEC protocol contain four types of the nodes: the sensor, the head, the aggregator and the sender. Nodes in a network choose itself as the cluster head nodes with a certain probability value to form a cluster head node collection $Hhead_i$, become a cluster head node which is responsible for the set of clusters. The distance between each of the sensor nodes which are in the cluster i and the head nodes to meet the following conditions:

$$\min_{1 \leq k \leq |H|} dist(v_{ij}, head_k) \quad (3)$$

But the head is not responsible for data aggregation, fusion and sending information to the base station immediately, it plays a role of a "call" in the first place. In the process of the set of clusters, we determine the aggregator and the sender through the calculation. The former is used to gather the information which is sent by clusters and then make the data processing and fusion, the latter accept the information transmitted by the former and then send them to the base station. Of course, set clusters, gather data and send information to the base station, those three types of task may be borne by one or two nodes.

The process of the head selection of DEEC is similar to LEACH: each node generate a random number, if it less than the threshold, the node will be the cluster head. But the determination of the threshold here is different from LEACH, DEEC neither consider the effect of the current round nor consider whether the node within several rounds of cluster heads, for DEEC, cluster head set H only have the effect of dividing the network into several clusters. The threshold of DEEC is the probability of each node to become cluster head P_{head} , $P_{head} = K_{opt}/N$, in which K_{opt} is the optimal value of number of clusters in the network; N is the number of nodes in the network during initialization. The derivation of K_{opt} has a related discussion in the literature [8], here only assume that it is a system parameter during the initialization time of the network. When a node is selected as the cluster head, it broadcasts messages *INVITE* to

all other nodes, the nodes which receive the news choose the nearest cluster heads to *JOIN* and send a message to its cluster head *JOIN*(*ID*, *Position*). In this way, the entire network is divided into several clusters.

4 DCSH algorithm

In above algorithm, the ability of sensor nodes in WSNs is limited to only receive the information sent by a small amount of sensor nodes, and each sensor node has only its local information, therefore the joint decoding side cannot recover the full information, this will lead to the loss of the target source information when recover.

In this paper, based on our previous works [12] [13] [14], we propose an improvement scheme of DCS reconstruction algorithm (DCSH algorithm) which is based on SOMP. Its basic idea is: by using DEEC algorithm to divide sensor nodes into clusters, then select the cluster head, and the information of the nodes all gathered to the cluster head, then the information on the cluster head is the information within the entire cluster nodes. The sensor nodes send the information to the cluster head after data fusion and then use SOMP algorithm to recover the information. On one hand, the DCSH algorithm reduces the number of the nodes which transfer the information directly to the joint decoding end, avoids the loss of transmit information; on the other hand, by using the advantage of the DEEC protocol, our algorithm can achieve the goal of saving energy.

System model for DCSH algorithm is shown in Fig.3:

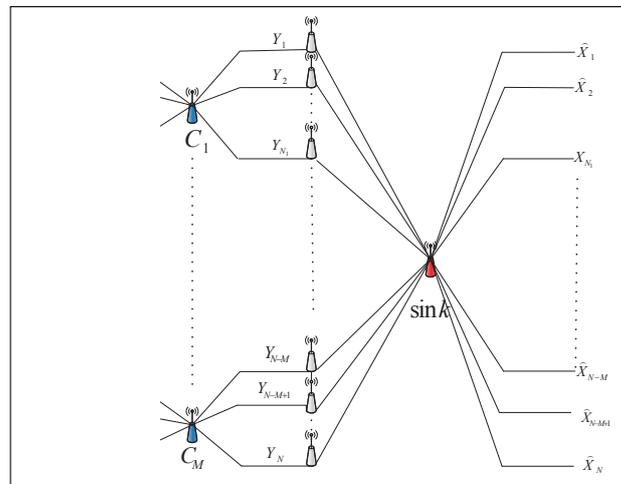


Figure 3: System Model for DCSH algorithm.

Suppose that there are N sensor nodes randomly distribute in the wireless sensor networks, meanwhile the sink node is in the center of the network area. There is a target source in the network, information on the target source will be sent to all nodes in the network, any sensor nodes can communicate with the sink node directly, and the signals all meet the joint sparse model JSM-2.

First, divide N nodes into M clusters, then elect the cluster heads C_1, C_2, \dots, C_M of M clusters respectively, and the number of the nodes in each cluster is N_1, N_2, \dots, N_M , transmit the cluster information to each cluster head and then use DCS algorithm to encode information on the cluster head, getting the measured value Y_1, Y_2, \dots, Y_M which is transmitted to the sink node for the joint decoding. At last, get the reconstruction value $\hat{X}_1, \hat{X}_2, \dots, \hat{X}_N$, thereby, recover the target source information accurately.

DCSH algorithm can be shown as follows:

- 1) Based on DEEC protocol, divide the sensor nodes into cluster, and choose the cluster head. The threshold P_{head} , $P_{head} = K_{opt}/N$ of DEEC is the probability value of each node to become cluster head, in which K_{opt} is the optimal value of number of clusters in the network; N is the number of nodes in the initialized network.
- 2) Determine the aggregator: when the head know location information of all nodes, aggregator can be determined through simple algorithm *FIND_AGGREGATOR* [10].
- 3) Determine the sender: using the algorithm *FIND_SENDER* [10].
- 4) Transmit the information of the sensor nodes to the cluster head, using their original measurement matrix to do the independently coding on the cluster head.
- 5) Use SOMP algorithms to joint reconstruction of the nodes information, thus recover the full information of the network.

5 The results of simulation and analysis

This paper uses Matlab as a tool for simulation, 100 sensor nodes are randomly distributed in deployment area , the center of the area is the cluster head, each of the node information conform to JSM-2 models.

First of all, we verify the performance of the SOMP algorithm. Get the reconstruction value \hat{X}_1 of the sensor node information X_1 by SOMP algorithm and OMP algorithm. The reconstruction error of is 5.1672×10^{-15} and 8.1596×10^{-15} . Figure 4 and Figure 5 show us the comparison between the two kinds of algorithms, and it apparently show that SOMP algorithm has the excellent effect of reconstruction. After compared with OMP algorithms, we will find SOMP algorithm has good performance and also the accuracy is higher.

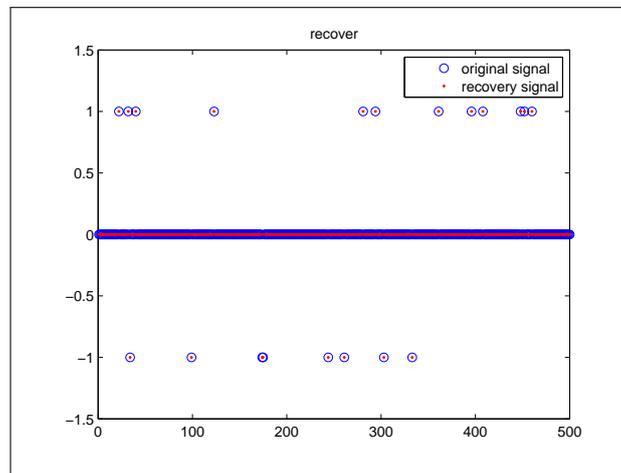


Figure 4: Comparison of SOMP algorithm.

Suppose that the LEACH protocol and DEEC protocol have the same number of the original data nodes, Figure 6 studies the performance under the conditions of same numbers of cluster nodes. In this figure, the two curves represent the protocol as LEACH and DEEC. Its specific results are shown in Figure 6, DEEC protocol has m less dead nodes When the algorithm run same time.

Known that the initial energy of the nodes is $50J$, the energy of the nodes decrease with the number of the election round, Figure 7 shows the relationship between the rounds and active

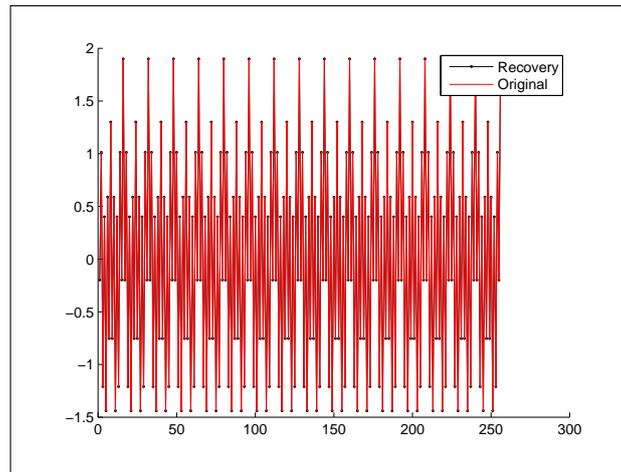


Figure 5: Comparison of OMP algorithm.

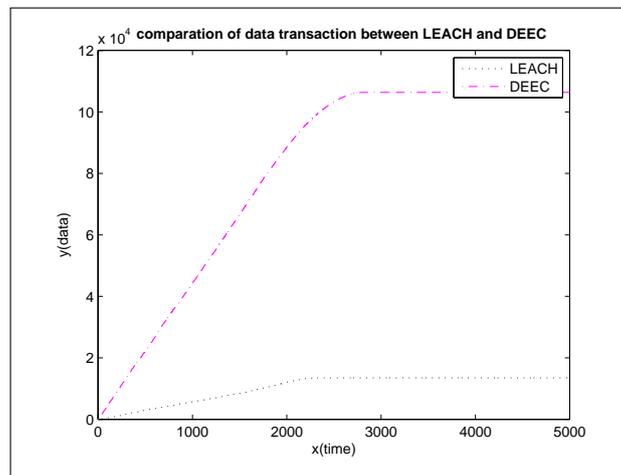


Figure 6: The comparison of the lifecycle.

nodes of DEEC algorithm. We can conclude that, in the same case, networks lifetime of DEEC is longer than traditional LEACH protocol, to a great extent, improve the network cycle.

For the further study of these two reconstruction algorithms, once again, we compare the average absolute error performance of the SOMP and OMP algorithm. Figure 8 demonstrates the relationship between the absolute error performances of the two algorithms, it can be seen that, in the case of the same number of measured values, the absolute error performance of SOMP algorithm is lower than the absolute error performance of OMP algorithm. To sum up, the comparison and analysis of the performance of SOMP algorithm and OMP algorithm in Figure 8 give a conclusion that in the same conditions, SOMP algorithm can ensure the accuracy rating of the reconstruction while reducing the number of measured values. In this way we can save the energy of the network and improve the efficiency of the signal reconstruction.

6 Conclusions

This paper proposes a distributed compressed sensing algorithm for hierarchical Wireless Sensor Networks (DCSH algorithm). This algorithm utilize DEEC algorithm to get the accurate information by recovering the information of sensor nodes in the network. The sensor nodes

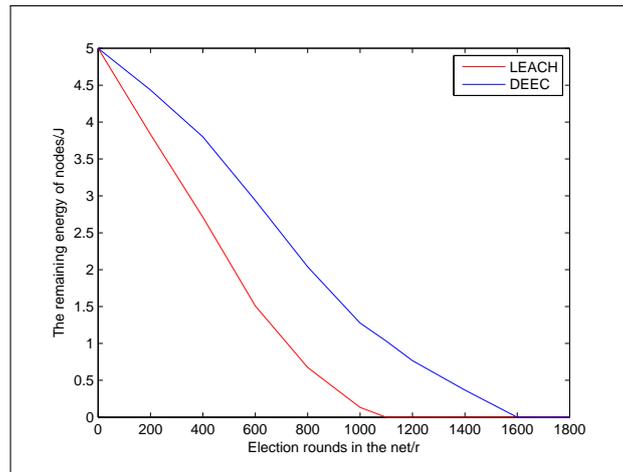


Figure 7: The comparison of the nodes and the wheel.

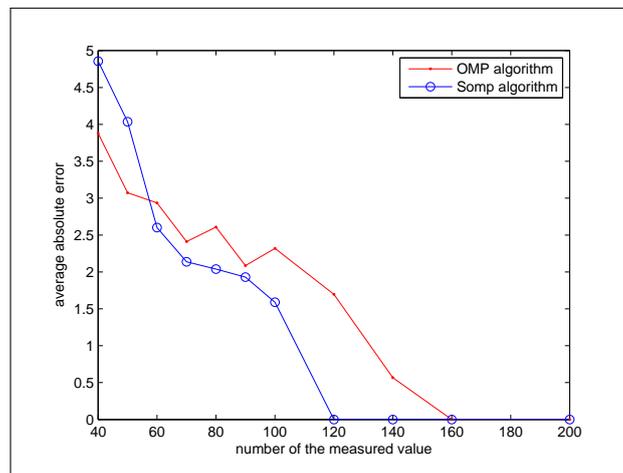


Figure 8: The change of two kinds of algorithm.

choose the cluster head by DEEC protocol. We take the residual energy of nodes into consideration while in the process of choosing the cluster head, prevent some nodes energy exhausted prematurely, this can not only extend the network lifetime effectively, but also reduce the number of the nodes which transmit data directly to the gathering node. Then based on the spatial correlation of the cluster nodes and joint sparse model (JSM-2), by using distributed compressed sensing reconstruction algorithm(SOMP), recover the original node information from a small amount of information accurately. The simulation results show that this algorithm has better performance, which can improve the network lifetime, and can be better adapted to periodically collect data of the WSNs application.

Acknowledgments

This work is supported by National Natural Science Foundation of China under Grant No. 61063037 and No. 51364001, Open Program of State Key Laboratory Breeding Base of Nuclear Resources and Environment under Grant No.NRE1102, Key Projects in the Science and Technology Pillar Program of Jiangxi Province of China under Grant No. 20111BBG70031-2 and 20133BBE50033, Educational Commission of Jiangxi Province of China under Grant No.

GJJ13335 and GJJ13354, and Foundation for Young Scientists of Jiangxi Province of China under Grant No. 20133BCB23016.

Bibliography

- [1] Donoho D L.(2006); Compressed sensing, *IEEE Transactions on Information Theory*, 2006, ISSN 0018-9448, 52(4): 1289-1306.
- [2] Candés E.(2006); Compressive sampling, *In: Proceedings of International Congress of Mathematicians*, ISBN 978-3-03719-022-7, 1433-1452.
- [3] D.Baron et al (2005); Distributed compressive sensing, *Technical Report* ,pre-print.
- [4] D.Baron et al (2005); An Information Theoretic Approach to Distributed Compressed Sensing, *In: Conference on Communication, Control, and Computing* , ISBN: 9781604234916.
- [5] M.F.Duarte et al (2005); Distributed Compressed Sensing of Jointly Sparse Signals, *In: Proceeding of the 39th Asilomar Conference on Signals, Systems and Computation* , ISSN 1058-6393, 1537-1541.
- [6] J Tropp; A. Gilbert; M Strauss(2006); Algorithms for simultaneous sparse approximation, Part I: Greedy pursuit, *Journal of Signal Processing*, ISSN 0165-1684, 86: 572-588.
- [7] W Dai; O Milenkovic(2009); Subspace pursuit for compressive sensing signal reconstruction. *IEEE Transactions on Information Theory*, ISSN 0018-9448, 55(5): 2230-2249.
- [8] L.Zhao; L.Q.Lian(2005); Distributed and Energy Efficient Self-organization for On-off Wireless Sensor Networks, *International Journal of Wireless Information Networks*, ISSN 1068-9605, 12(1) : 211-215.
- [9] L.Qing; Q.Zhu; M.Wang(2006); Design of a distributed energy-efficient clustering algorithm for heterogeneous wireless sensor networks, *ELSEVIER, Computer Communications*,ISSN 0140-3664, 29(12): 2230-2237.
- [10] Y.X.Liu et al(2010); Regularized Adaptive Matching Pursuit Algorithm for Signal Reconstruction Based on Compressive Sensing, *Journal of electronics and information*, ISSN 1009-5896, 32(11): 2713-2717.
- [11] D. Slepain; J. K. Wolf(1973); Noiseless coding of correlated information sources. *IEEE Transaction on Information Theory*, ISSN 0018-9448, 19(9): 471-480.
- [12] Nan Jiang(2014). WDEM: Weighted Dynamics and Evolution Models for Energy-Constrained Wireless Sensor Networks, *Physica A: Statistical Mechanics and its Applications*, ISSN 0378-4371, 404: 323-331.
- [13] Nan Jiang; Sixin Jin; Yan Guo; Yueshun He(2013); Localization of Wireless Sensor Network Based on Genetic Algorithm, *International Journal of Computers Communications & Control*, ISSN 1841-9844, 8(6): 825-837.
- [14] Nan Jiang; Rigui Zhou; Qiulin Ding(2009); Dynamics of Wireless Sensor Networks, *International Journal of Distributed Sensor Networks*, ISSN 1550-1329, 5(6): 693-707.