Networked Predictive Fuzzy Control of Systems with Forward Channel Delays Based on a Linear Model Predictor

S.W. Tong, D.W. Qian, G.P. Liu

Shiwen Tong*
College of Automation
Beijing Union University
Beijing 100101, China
* Corresponding author: shiwen.tong@buu.edu.cn

Dianwei Qian
School of Control and Computer Engineering
North China Electric Power University
Beijing 102206, China
dianwei.qian@ncepu.edu.cn

Guo-Ping Liu
1.School of Engineering
University of South Wales
Portypridd CF37 1DL, UK
2.CTGT Center
Harbin Institute of Technology
Harbin 150001, China
guoping.liu@southwales.ac.uk

Abstract: This paper presents a novel networked control framework, using fuzzy logic control, for systems with network delays which are known to greatly weaken the control performance of the controlled system. To deal with the network delays, the predicted differences between the desired future set-points and the predicted outputs from a model predictor are utilized as the inputs of a fuzzy controller, thus a series of future control actions are generated. By selecting the appropriated control sequence in the plant side, the network delays are compensated. The simulative results demonstrate that the proposed method can obviously reduce the effect of network delays, and improve the system dynamic performance.

Keywords: networked control, model prediction, fuzzy control, delay compensation.

1 Introduction

The emergence of the network technology has changed the communication architecture of control systems from traditional point-to-point to current common bus. Sensors, actuators and controllers are connected through network, formed a feedback control system (namely, networked control system). This architecture has injected fresh blood to the classic and modern control theories and also arises higher challenges to the controller design at the same time. On the one hand, the introduction of the network to the control system brings many advantages such as low cost, easy maintenance and high reliability. On the other hand, the unavoidable time delay, data dropout and other complicated phenomenon existing in the network should be considered. In recent years, networked control theory and technology have become an important and hot research area. Scholars from different countries have made a lot of breakthroughs in the networked control [1]- [8].

Network delay has become one of the most concerned issues in the networked control system. Because the network delays can dramatically degrade the control performance of the systems
even makes the systems unstable. From recent published literatures, it can be seen that the
treatment of the network delays can be summarized in the following: The first one is to integrate
delay information into the controller design by designing a robust controller to decrease the effect
of time delay [3], [6]-[8]; The second one is to estimate delay information in the backward or
forward channel by using reason-rule-table, average-value or delay windows (DW) method [9,10];
The third one is to eliminate the network delays in the return path by using a cascade control
structure with P (proportion) control in the inner loop and fuzzy adaptive control in the outer
loop [4]; As we all known, one of the obvious characteristics of the networked control system is
that the communication networks can transmit a packet of data simultaneously. This feature
provides another solution to compensate for network delays in the forward channel [11,12]. Based
on it, Liu et al. [1, 2] proposed networked predictive control (NPC), using the strings of future
control actions, to compensate for the forward channel delays. In this paper, we try to design a
networked controller by using fuzzy control theories. Different from networked predictive control
method, we separate the model prediction from the controller design. The function of the model
predictor is just to produce future predicted outputs. A fuzzy controller is designed to generate
a series of future control sequence based on the errors between the desired future outputs and
the model predicted outputs. Then the strings of future control actions are packed and sent to
the plant side through the communication channel. Thus, the effect of delays in the forward
channel is lessened by using a delay compensator in the plant side.

Predictive control and fuzzy control are powerful tools. They have been used in the design
of the networked controller [1]-[5], [7, 8]. Some researchers have connected the prediction to
the fuzzy control and have proposed ‘predictive fuzzy control’ method [13, 14]. They use ‘future
error’ and ‘future error change’ as inputs of the fuzzy controller to produce the control actions
at the current time. To my knowledge, the combination of networked control system and the
predictive fuzzy control has not been reported except one paper written by us in 2007 [15]. We
call it ‘networked predictive fuzzy control’ with abbreviation NPFC. The core idea of the NPFC
is producing ‘future control actions’ by fuzzy controller design according to the ‘future error’ and
‘future error change’ from a model predictor. Then the network delays can be compensated by
choosing the ‘future control actions’.

This paper is organized as follows: The architecture of networked predictive fuzzy control
(NPFC) is firstly presented. Secondly, a model predictor based on Diophantine equation is pro-
posed. Thirdly, the fuzzy controller using ‘future errors’ and ‘future error changes’ as inputs to
derive ‘future control actions’ is designed. Then the delay compensation mechanism is discussed
and the method is implemented in a servo control system. Finally, the conclusions are drawn in
Section 4.

2 Design of networked control systems

2.1 Structure of Networked Predictive Fuzzy Control Systems

The networked predictive fuzzy control system as shown in Fig. 1 mainly consists of three key
parts: the model predictor, the fuzzy controller and the delay compensator. The model predictor
is used to predict future outputs of the controlled system \( y(t|t), y(t + 1|t), \ldots, y(t + N - 1|t) \)
according to the delayed output \( y(t - 1) \) of the controlled system in the backward channel and
the control actions \( u(t - d - 1), u(t - d - 2), \ldots, u(t - d - n_b) \) in the past. The errors \( e(t|t), e(t + 1|t), \ldots, e(t + N - 1|t) \) between the desired future outputs \( r(t|t), r(t + 1|t), \ldots, r(t + N - 1|t) \)
and the predictive future outputs \( y(t|t), y(t + 1|t), \ldots, y(t + N - 1|t) \) of the controlled system
can be used to design a fuzzy controller to produce the future control sequences \( u(t|t), u(t + \ldots \)
1|t), · · · , u(t + N_u - 1|t). Then the future control sequences are packed and sent to the plant side through network. In the plant side, a delay compensator is used to compensate for the forward network delays by selecting appropriate control sequence.

![Figure 1: Structure of Networked Predictive Fuzzy Control System](image)

### 2.2 Model Predictor

The function of the model predictor is to produce a series of future outputs of the controlled object according to the delayed output in the backward channel and the control actions in the past. Models, no matter linear or nonlinear model, of such functions can all be the predictor. To simplified the issue and focus on the research work step by step, we only consider forward channel delays in this paper. This consideration is reasonable in some cases. For example, in a network that the transmission velocity in the backward channel is faster than the forward channel thus the delays in the backward channel can be neglected. Therefore a linear model predictor based on the Diophantine equation is proposed.

Consider a single-input and single-output process with the following form

\[ A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t) \]  

(1)

where,

\[ A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a} \]

\[ B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b} \]

Introducing a Diophantine equation to derive the model predictor.

\[ \Delta A(z^{-1}) E_i(z^{-1}) + z^{-i} F_i(z^{-1}) = 1 \]  

(2)

where, \( E_i(z^{-1}) \) is of order \( i - 1 \) and \( F_i(z^{-1}) \) is of order \( n_a \).

\[ E_i(z^{-1}) = 1 + \sum_{j=1}^{i-1} e_{i,j} z^{-j}, F_i(z^{-1}) = \sum_{j=0}^{n_a} f_{i,j} z^{-j}. \]
Define \( N \) as predictive horizon, \( N_m \) as model control horizon, from (1) and (2), the predicted value \( Y_p(t+1) \) of the controlled system can be obtained.

\[
Y_p(t+1) = P \Delta U(t-d) + Q \Delta U(t-d-1) + F y(t-1) \quad (3)
\]

where,

\[
P = \begin{bmatrix}
g_0 & 0 & \cdots & 0 \
g_1 & g_0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
g_{N-1} & \cdots & \cdots & g_0 \\
g_N & g_{N-2} & \cdots & g_{N-N_m}
\end{bmatrix}
\]

\[
Y_p(t+1) = [y_p(t), y_p(t+1), \ldots, y_p(t+N-1)]^T
\]

\[
\Delta U(t-d) = [\Delta u(t-d), \Delta u(t-d+1), \ldots, \Delta u(t-d+N_m-1)]^T
\]

\[
\Delta U(t-d-1) = [\Delta u(t-d-n_b), \Delta u(t-d-n_b+1), \ldots, \Delta u(t-d-1)]^T
\]

\[
F = [F_1(z^{-1}), F_2(z^{-1}), \ldots, F_N(z^{-1})]^T
\]

\[
Q = \begin{bmatrix}
g_{n_b} & \cdots & g_1 \\
\vdots & \ddots & \vdots \\
g_{n_b+N-1} & \cdots & g_N
\end{bmatrix}
\]

Let \( G_i(z^{-1}) = B(z^{-1})E_i(z^{-1}) = g_0 + g_1 z^{-1} + \cdots + g_{n_b+i-1} z^{-(n_b+i-1)} \), we can construct matrix \( P \) and \( Q \) by solving the Diophantine equation.

It should be noted that the item \( \Delta U(t-d) = [\Delta u(t-d), \Delta u(t-d+1), \ldots, \Delta u(t-d+N_m-1)]^T \) in equation (3) can not be obtained at the current time \( t \). We assume \( \Delta u(t-d+i) = 0, i = 0,1, \ldots, N_m-1 \). This assumption is reasonable. On the one hand, the dynamic response of the future time can be reflected by using the past control actions and the previous process outputs. On the other hand, the item \( y(t-1) \) is corresponding to the correction of the predicted model. That is \( y(t-1) = y_m(t-1) + e(t-1) \), where \( y_m(t-1) \) is the model output and \( e(t-1) \) is the error between the process output and the model output.

### 2.3 Fuzzy Controller

The successful implementation of networked predictive fuzzy control relies on an assurance that the fuzzy controller must calculate all the pre-defined control actions in every sample time. So the fuzzy control algorithm should be simple and suitable to the real-time control. A real-time simplified fuzzy controller [16] is used to generate the future control actions. Unlike the ordinary fuzzy control, which uses error and error change at the present time to produce the current control action, the networked predictive fuzzy controller uses 'future error' and 'future error change' to derive 'future control actions'. The ordinary fuzzy control can be regarded as a special condition of the networked fuzzy control when the predictive step is equal to zero. This networked predictive fuzzy controller has two-input one-output. One input is the error \( e \) between the desired future output and the predicted output. The other one is the change of the error \( ec \). The output of the fuzzy controller is the change of the future control action \( \Delta u \). The membership functions of \( e \) and \( ec \) are adopted as triangular forms and the membership function of the output \( \Delta u \) is adopted discrete form as Fig. 2 shown.

The 'Simplification' of the fuzzy controller relays on the inference process. For the typical two-input-one-output fuzzy controller, only four control rules are excited at each cycle time with
the form 'If $e$ is $L_e$ and $ec$ is $L_{ec}$, then $\Delta u$ is $L_{\Delta u}$', where $L$ is the linguistic variables PB, PM, PS, ZE, NS, NM, NB. Due to the characteristic of the triangular membership function, $e$ is at most belong to two membership functions $\mu_{i}^{e}$ and $\mu_{i}^{e+1}$, $ec$ is at most belong to two membership functions $\mu_{j}^{ec}$ and $\mu_{j}^{ec+1}$, thus $\Delta u$ has $2 \times 2$ combinations, that is four control rules:

- If $e$ is $L_e^{(i)}$ and $ec$ is $L_{ec}^{(j)}$ then $\Delta u$ is $L_{\Delta u}^{(i,j)}$
- If $e$ is $L_e^{(i)}$ and $ec$ is $L_{ec}^{(j+1)}$ then $\Delta u$ is $L_{\Delta u}^{(i,j+1)}$
- If $e$ is $L_e^{(i+1)}$ and $ec$ is $L_{ec}^{(j)}$ then $\Delta u$ is $L_{\Delta u}^{(i+1,j)}$
- If $e$ is $L_e^{(i+1)}$ and $ec$ is $L_{ec}^{(j+1)}$ then $\Delta u$ is $L_{\Delta u}^{(i+1,j+1)}$

From Fig. 2(c) shown, output $\Delta u$ is adopted discrete form membership function. It is assumed that output domain has been divided into $c_m(k), m = 1, 2, \cdots, n$. Do minimum and maximum operator, calculate

\[
\tilde{\mu}^{(m)}_{L_{\Delta u}^{(i,j)}} = \wedge(\mu_{L_e^{(i)}}, \mu_{L_{ec}^{(j)}}, \mu_{L_{\Delta u}^{(i,j)}}^{(m)})
\]
\[
\tilde{\mu}^{(m)}_{L_{\Delta u}^{(i,j+1)}} = \wedge(\mu_{L_e^{(i)}}, \mu_{L_{ec}^{(j+1)}}, \mu_{L_{\Delta u}^{(i,j+1)}}^{(m)})
\]
\[
\tilde{\mu}^{(m)}_{L_{\Delta u}^{(i+1,j)}} = \wedge(\mu_{L_e^{(i+1)}}, \mu_{L_{ec}^{(j)}}, \mu_{L_{\Delta u}^{(i+1,j)}}^{(m)})
\]
\[
\tilde{\mu}^{(m)}_{L_{\Delta u}^{(i+1,j+1)}} = \wedge(\mu_{L_e^{(i+1)}}, \mu_{L_{ec}^{(j+1)}}, \mu_{L_{\Delta u}^{(i+1,j+1)}}^{(m)})
\]

and

\[
\tilde{\mu}^{(m)}_{L_{\Delta u}} = \vee(\tilde{\mu}^{(m)}_{L_{\Delta u}^{(i,j)}}, \tilde{\mu}^{(m)}_{L_{\Delta u}^{(i,j+1)}}, \tilde{\mu}^{(m)}_{L_{\Delta u}^{(i+1,j)}}, \tilde{\mu}^{(m)}_{L_{\Delta u}^{(i+1,j+1)}})
\]

where $m = 1, 2, \cdots, n$

It is not easy to directly get the inference rules of the future; however, the typical dynamic of the second order system can be obtained ahead of time. Fig. 3 presents the phase plane of
Figure 3: Phase plane and fuzzy control rules analysis of typical second order linear system.

The typical second order linear system with the x-axis standing for variable $e$ and y-axis standing for variable $ec$. From this figure, it can be seen that the points in x-axis and y-axis are crucial. If these points are controlled very well, the control performance is guaranteed. So the main inference rules are deduced as shown in Table 1.

Table 1: Main control rules of the fuzzy controller in NPFC

<table>
<thead>
<tr>
<th>No.</th>
<th>Control rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>if $e = PB$ and $ec = ZE$ then $\Delta u = PB$</td>
</tr>
<tr>
<td>2</td>
<td>if $e = ZE$ and $ec = NB$ then $\Delta u = NB$</td>
</tr>
<tr>
<td>3</td>
<td>if $e = NB$ and $ec = ZE$ then $\Delta u = NB$</td>
</tr>
<tr>
<td>4</td>
<td>if $e = ZE$ and $ec = PB$ then $\Delta u = PB$</td>
</tr>
<tr>
<td>5</td>
<td>if $e = PS$ and $ec = ZE$ then $\Delta u = PS$</td>
</tr>
<tr>
<td>6</td>
<td>if $e = ZE$ and $ec = NM$ then $\Delta u = NM$</td>
</tr>
<tr>
<td>7</td>
<td>if $e = NS$ and $ec = ZE$ then $\Delta u = NS$</td>
</tr>
<tr>
<td>8</td>
<td>if $e = ZE$ and $ec = PM$ then $\Delta u = PM$</td>
</tr>
<tr>
<td>9</td>
<td>if $e = ZE$ and $ec = NS$ then $\Delta u = NS$</td>
</tr>
<tr>
<td>10</td>
<td>if $e = ZE$ and $ec = PS$ then $\Delta u = PS$</td>
</tr>
<tr>
<td>11</td>
<td>if $e = ZE$ and $ec = ZE$ then $\Delta u = ZE$</td>
</tr>
</tbody>
</table>

Thus, the incremental control actions

$$\Delta u(t + \eta) = \frac{\sum_{m=1}^{n} \tilde{\mu}_{\Delta u}^{(m)} * c_{m}}{\sum_{m=1}^{n} \tilde{\mu}_{\Delta u}^{(m)}}$$

and the predicted control actions

$$u(t + \eta) = u(t + \eta - 1) + \Delta u(t + \eta), \eta \text{ is integer and } 0 \leq \eta \leq N_u - 1$$

can be given, where $N_u$ is the control horizon. If $e$ is the error and $ec$ is the error change at present time, then $\Delta u$ is the incremental control action at present time. If $e$ and $ec$ are the values of the future time, then the future incremental control actions can be derived.
As paper [17]-[19] presents, variable domain implemented in the fuzzy control can greatly improve the control accuracy, which has successfully been applied to the control of a quadruple inverted pendulum [20] and the wing rock phenomenon [21]. This is achieved by the domain contraction and expansion. Domain contraction is equivalent to the increase of the control rules. Although the real-time algorithm has considered only 11 rules, together with the variable domain strategy, the fuzzy controller can acquire satisfactory control performance. The varies of domain can be achieved by multiplying flex factors $\alpha(e)$, $\beta(ec)$ and $\gamma(e, ec)$ of domains $E$, $EC$ and $\Delta U$, respectively. There are many different forms of flex factors. In this paper, we adopt the exponential form of functions:

$$
\alpha(e) = \left[ \frac{|e|}{E} \right]^{\tau_1}, 0 < \tau_1 < 1
$$

$$
\beta(ec) = \left[ \frac{|ec|}{EC} \right]^{\tau_2}, 0 < \tau_2 < 1
$$

$$
\gamma(e, ec) = \left[ \left( \left[ \frac{|e|}{E} \right]^{\tau_1} \left[ \frac{|ec|}{EC} \right]^{\tau_2} \right)^{\tau_3} \right], 0 < \tau_1, \tau_2, \tau_3 < 1
$$

To summarized, the networked predictive fuzzy controller has eleven parameters to be designed. Four parameters are related to the model predictor. They are the order of the predictive model $n_a$ and $n_b$, the predictive horizon $N$, and the model control horizon $N_u$. Seven parameters are belong to the networked fuzzy controller. They are the control horizon $N_u$, the scaling gains $K_e$, $K_{ec}$, $K_{\Delta u}$ of error $e$, error change $ec$ and incremental control action $\Delta u$, and the variable domain parameters $\tau_1$, $\tau_2$ and $\tau_3$. The program steps for the networked predictive fuzzy control plus variable domain strategy are summarized below:

**Step 1:** Use equation (3), calculate the future outputs $y_p(t|t)$, $y_p(t+1|t)$, $\cdots$, $y_p(t+N-1|t)$ of the controlled system according to the delayed output of the feedback channel and the previous control actions.

**Step 2:** Calculate the differences between the desired future outputs $r(t|t), r(t+1|t), \cdots, r(t+N-1|t)$ and the model predicted values $y_p(t|t), y_p(t+1|t), \cdots, y_p(t+N-1|t)$ to get $e(t|t), e(t+1|t), \cdots, e(t+N-1|t)$ and $ec(t|t), ec(t+1|t), \cdots, ec(t+N-1|t)$.

**Step 3:** Adjust input and output domain using equation (8) in terms of $e(t+\eta)$ and $ec(t+\eta)$.

**Step 4:** Calculate membership functions of input $e(t+\eta)$ and $ec(t+\eta)$ and output $\Delta u(t+\eta)$.

**Step 5:** Use minimum-maximum inference method [see equation (4) and (5)].

**Step 6:** Calculate the predicted control actions $u(t+\eta)$ using equation (6) and (7).

**Step 7:** Let $\eta = 0$ to $N_u - 1$, repeat step 3-6.

**Step 8:** Send the control actions $\Delta u(t), \Delta u(t+1), \cdots, \Delta u(t+N_u-1)$ with a packet to the plant side.

**Step 9:** Select the control action $u(t|t − k)$ and add to the controlled process.

**Step 10:** In the next sample time, repeat step 1-9.

### 2.4 Network Delays Compensation

It is assumed that the network communication delay in the forward channel is not greater than the length of the predicted control horizon. To make use of the ‘packet transmission’ characteristic of the network, a string of future control actions which contain $u(t), u(t+1), \cdots, u(t+N_u-1)$ at sample time $t$ are sent to the plant side at the same time. Then the control value from the latest control sequence available on the plant side is chosen as a control input of the plant to compensate for the forward channel delay. For example, if the latest control sequence...
on the plant side is
\[
\begin{bmatrix}
    u(t - k|t - k) \\
    u(t - k + 1|t - k) \\
    \vdots \\
    u(t - k + Nu - 1|t - k)
\end{bmatrix}
\]

Then the output selected control signal will be
\[u(t) = u(t|t - k)\]

Figure 4: The step responses of NPC and NPFC with 1-step forward channel delay.

3 Simulations

Consider the servo control system as reference [2] shown. The system model with discrete form is as follows:
\[
G(z^{-1}) = \frac{-0.00886z^{-1} + 1.268227z^{-2}}{1 - 1.66168z^{-1} + 0.6631z^{-2}} \tag{9}
\]
where sample time is 0.04 second.

Suppose there is one step delay in the forward channel, following Section 2.2 and Section 2.3, we design a linear model predictor and a real-time fuzzy controller. The parameters of the model predictor are: \(n_a = 2, n_b = 2, N = 12, N_m = 10\). The parameters of the networked fuzzy controller are: \(N_u = 10, K_e = 0.00125, K_{ec} = 0.02, K_{\Delta u} = 0.6, \tau_1 = 0.1, \tau_2 = 0.1\) and \(\tau_3 = 0.01\). The NPC parameters are set to \(N = 12, N_u = 10, \rho = 1500\). Fig. 4 shows the control performance of NPFC and NPC. The dot line is the set-point. The solid line stands for the NPFC method. The dash line stands for NPC method. The dash-dot line stands for the NFPC method without delay compensation. From the figure, it can be seen that the NPFC can be regulated better than the NPC in control performance with rapid dynamic and small overshoot. The delay compensation mechanism is very effective.

Suppose the case that six step delays exist in the forward channel. The NPFC controller parameters are adjusted as: \(K_e = 0.0004, K_{ec} = 0.008, K_{\Delta u} = 0.08\), and the NPC parameters
are set to $N = 25$, $N_u = 10$, $\rho = 100000$. To testify the control performance of the networked predictive fuzzy control method, the results of the NPC and the NPFC are presented in Fig. 5. Through model prediction, fuzzy controller design and delay compensation, the NPFC presents very obviously better performance than NPC method. The rising time of NPFC is about 1.1 seconds while 1.5 seconds for NPC method. Moreover, NPC has 3.75% overshoot while NPFC has nearly no overshoot. When NPFC method not considers delay compensation, static errors can be seen in Fig. 5. On the contrary, the dynamic response reaches steady state after 1.4 seconds when the delay compensator is acting.

![Networked Predictive Fuzzy Control](image)

Figure 5: The step responses of NPC and NPFC with 6-step forward channel delay.

4 Conclusions

This paper proposes a network predictive fuzzy controller structure. By solving the Diophantine equation, the linear model predictor generates a series of predictive outputs of the controlled system according to the outputs and the control sequence in the past time. The errors between the desired future outputs and the predictive outputs from the linear model predictor and the error change are used to design a real-time fuzzy controller. So a series of future control sequence is produced in the controller side. By selecting the appropriate control sequence in the plant side, the delays in the forward channel are compensated. Because NPFC has more parameters which can be regulated, the control performance can be adjusted better than NPC method.

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