Multi-Objective Optimization for the m-PDPTW: Aggregation Method With Use of Genetic Algorithm and Lower Bounds

I. Harbaoui Dridi, R. Kammarti, M. Ksouri, P. Borne

Imen Harbaoui Dridi
LAGIS : Ecole Centrale de Lille,
Villeneuve d’Ascq, France
LACS : Ecole Nationale des Ingénieurs de Tunis,
Tunis - Belvédère, Tunisie
E-mail: imenharbaoui@gmail.com

Ryan Kammarti, Mekki Ksouri
LACS : Ecole Nationale des Ingénieurs de Tunis,
Tunis - Belvédère, Tunisie
E-mail: kammarti.ryan@planet.tn, Mekki.Ksouri@insat.rnu.tn

Pierre Borne
LAGIS : Ecole Centrale de Lille,
Villeneuve d’Ascq, France
E-mail: p borne@ec-lille.fr

Abstract: The PDPTW is an optimization vehicles routing problem which must meet requests for transport between suppliers and customers in purpose to satisfy precedence, capacity and time constraints. We present, in this paper, a genetic algorithm for multi-objective optimization of a multi pickup and delivery problem with time windows (m-PDPTW), based on aggregation method and lower bounds. We propose in this sense a brief literature review of the PDPTW, present our approach to give a satisfying solution to the m-PDPTW minimizing the compromise between total travel cost and total tardiness time.

Keywords: PDPTW, multi-objective, aggregation method, lower bounds.

1 Introduction

The vehicle routing planning and traffic management are considered a major logistical challenge in terms of supply, inter-plant transport or distribution transport. [1] Many studies have been directed mainly towards solving the vehicle routing problem (VRP). It’s an optimization vehicle routing problem to meet travel demands. Other researchers became interested on an important variant of VRP which is the PDPTW (Pickup and Delivery Problem with Time Windows) with capacity constraints on vehicles. The PDPTW is divided into two categories: 1-PDPTW (single-vehicle) and m-PDPTW multi-vehicle.

In the m-PDPTW problem which we are interested in, we consider a vehicles fleet $V_k$ of capacity $Q_k$ and a set of goods to transport providers to different destinations. The goal is to provide a set of customers under certain constraints concerning vehicles and their capacity, precedence between nodes, and this by minimizing the compromise between the total travel cost and total tardiness time. In this paper we present a literature review of the PDPTW followed by the proposed approach for the optimization of pick-up and delivery problem with time window, using the genetic algorithms, aggregation method and lower bounds.
2 Literature Review

2.1 Vehicle routing problem

The Vehicle Routing Problem (VRP) represents a multi-goal combinatorial optimization problem which has been the subject of many works and variations in the literature [2] [3]. The theory of the VRP is formulated as follows: given a depot \( D \) and a set of customers orders \( C = \{c_1, \ldots, c_n\} \), to build a package routing, for a finite number of vehicles, beginning and ending at the depot. In this routing, a customer must be served only once by a single vehicle and vehicle capacity transport for a routing should not be exceeded [4] [5]. The Meta heuristics were also applied to solve the vehicle routing problem. Among these methods, we can include ant colony algorithms, which were used by Montamenni, R et al for the resolution of DVRP [6], and by Sorin C. Negulescu et al to solve the Vehicle Route Allocation Problem (VRAP) [7]. Kaoru Hirota et al have presented a computational intelligence approach to VRSDP (Vehicle Routing, Scheduling, and Dispatching Problems). The objective of the VRSDP is to produce a delivery schedule for a group of vehicles, with respect to multiple users, so that while satisfying constraints delivery cost corresponding to users’ order is minimized [8]. Savelsbergh et al have shown that the VRP is a NP-hard problem [9].

2.2 The PDPTW: Pickup and Delivery Problem with Time Windows

The PDPTW is a variant of VRPTW where in addition to the existence of time constraints, this problem implies a set of customers and a set of suppliers geographically located. Every routing must also satisfy the precedence constraints to ensure that a customer should not be visited before his supplier. [10] A dynamic approach for resolving the 1-PDP without and with time windows was developed by Psaraftis, H.N considering objective function as a minimization weighting of the total travel time and the non-customer satisfaction [11]. Jih, W et al have developed an approach based on the hybrid genetic algorithms to solve the 1-PDPTW, aiming to minimize combination of the total cost and total waiting time. [12] Another genetic algorithm was developed by Velasco, N et al to solve the 1-PDP bi-objective in which the total travel time must be minimized while satisfying in priority the most urgent requests. In this literature, the method proposed to resolve this problem is based on a No dominated Sorting Algorithm (NSGA-II). [13] Kammarti, R et al treat the 1-PDPTW, minimizing the compromise between the total travel distance, total waiting time and total tardiness time, using an evolutionary algorithm with special genetic operators, taboo search to provide a set of viable solutions. [14] [15]. This work has been extended, by proposing a new approach based on the use of lower bounds and Pareto dominance method, to minimize the compromise between the total travel distance, total waiting time and total tardiness time. [16] About the m-PDPTW, Sol, M et al have proposed a branch and price algorithm to solve the m-PDPTW, minimizing the vehicles number required to satisfy all travel demands and the total travel distance. [17] Quan, L et al have presented a construction heuristic based on the integration principle with the objective function, minimizing the total cost, including the vehicles fixed costs and travel expenses that are proportional to the travel distance. [18] A new metaheuristic based on a tabu algorithm, was developed by Li, H et al to solve the m-PDPTW. [19] Li, H et al have developed a "Squeaky wheel" method to solve the m-PDPTW with a local search. [20] A genetic algorithm was developed by Harbaoui Dridi I et al treating the m-PDPTW to minimize the total travel distance and the total transport cost [21]. This work has been extended, by proposing a new approach based on the use of Pareto dominance method to give a set of satisfying solutions to the m-PDPTW minimizing total travel cost, total tardiness time and the vehicles number. [22] [23]
3 Mathematical Formulation

Our problem is characterized by the following parameters:

- \( N \): Set of customers, supplier and depot vertices,
- \( N' \): Set of customers and supplier vertices,
- \( N^+ \): Set of supplier vertices,
- \( N^- \): Set of customers vertices,
- \( n \): Size of the initial population,
- \( K \): Vehicle number,
- \( d_{ij} \): Euclidian distance between the vertex \( i \) and the vertex \( j \). If \( d_{ij} = \infty \) then the road between \( i \) and \( j \) doesn’t exist,
- \( t_{ijk} \): Time used by the vehicle \( k \) to travel from the vertex \( i \) to the vertex \( j \),
- \([e_i, l_i] \): Time window of the vertex \( i \),
- \( s_i \): Stopping time at the vertex \( i \),
- \( q_i \): Goods quantity of the vertex \( i \) request. If \( q_i > 0 \), the vertex \( i \) is a supplier; if \( q_i < 0 \), the vertex \( i \) is a customer and if \( q_i = 0 \) then the vertex was served.
- \( Q_k \): Capacity of vehicle \( k \),
- \( i = 0...N \): Predecessor vertex index,
- \( j = 0...N \): Successor vertex index,
- \( k: 1...K \): Vehicle index,
- \( X_{ijk} = \{ \begin{array}{ll} 1 & \text{if the vehicle travel from the vertex } i \text{ to the vertex } j \\
0 & \text{else} \end{array} \)
- \( A_i \): Arrival time of the vehicle to the vertex \( i \),
- \( D_i \): Departure time of the vehicle from the vertex \( i \),
- \( y_{ik} \): The goods quantity in the vehicle \( k \) visiting the vertex \( i \),
- \( C_k \): Travel cost associated with vehicle \( k \),
- A vertex is served only once,
- A vertex is served only once,
- The capacity constraint must be respected,
- The depot is the starting and finishing vertex for the vehicle,
- The vehicle stops at every vertex for a period of time to allow the request processing,
- If the vehicle arrives at a vertex \( i \) before its time windows beginning date it waits.
The function to minimize is given as follows:

\[
\text{Minimize } f = \left( \lambda_1 c_1 \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} C_k d_{ijk} X_{ijk} + \lambda_2 c_2 \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} \max(0, D_i - l_i) X_{ijk} \right)
\]

(1)

Where \( \lambda_i \) and \( c_i \) are weights and scaling coefficients.

Subject to:

\[
\sum_{i=1}^{N} \sum_{k=1}^{K} x_{ijk} = 1, j = 2, \ldots, N
\]

(2)

\[
\sum_{j=1}^{N} \sum_{k=1}^{K} x_{ijk} = 1, i = 2, \ldots, N
\]

(3)

\[
\sum_{i \in N} X_{i0k} = 1, \forall k \in K
\]

(4)

\[
\sum_{j \in N} X_{0jk} = 1, \forall k \in K
\]

(5)

\[
\sum_{i \in N} X_{iku} - \sum_{j \in N} X_{ujk} = 0, \forall k \in K, \forall u \in N
\]

(6)

\[
X_{ijk} = 1 \Rightarrow y_{jk} = y_{ik} + q, \forall i, j \in N; \forall k \in K
\]

(7)

\[
y_{0k} = 0, \forall k \in K
\]

(8)

\[
Q_k \geq y_{ik} \geq 0, \forall i \in N; \forall k \in K
\]

(9)

\[
D_w \leq D_v, \forall w \in N^+; \forall v \in N^-
\]

(10)

\[
D_0 = 0
\]

(11)

\[
X_{ijk} = 1 \Rightarrow D_i + t_{ijk} \leq D_j \forall i, j \in N; \forall k \in K
\]

(12)

The constraint (2) and (3) ensure that each vertex is visited only once by a single vehicle. The constraint (4) and (5) ensure that the vehicle routing is beginning and finishing in the depot. The constraint (6) ensures the routing continuity by a vehicle. (7), (8) and (9) are the capacity constraints. The precedence constraints are guaranteed by (10), (11) and (12).

4 Genetic Algorithm For Optimization Of The m-PDPTW

4.1 Generation of the initial populations

In our case, we generate two types of populations. A first population noted \( P_{\text{node}} \) (Figure 1), which represents all nodes to visit with all vehicles, according to the permutation list coding. The second population noted \( P_{\text{vehicle}} \) (Figure 2) indicates nodes number visited by each vehicle.
4.2 Correction procedures

Before beginning the construction of the population $P_{node/vehicle}$, we proceed to the correction procedures of precedence and capacity between nodes. We consider the following couples customer / supplier: (1,5), (2,8), (9,7), (10,3) and (4,6), noting that $Q_{k,max} = 60$ and $q = 20$, we present, respectively, in figures 3 and 4 the principle of correction precedence and capacity.

4.3 Computation procedure

Taking into account the population $P_{node}$, correction procedures and $P_{vehicle}$ we illustrate in Figure 5 an example of an individual from the population $P_{node/vehicle}$. Knowing that it is necessary to verify that a couple is visited by only one vehicle. [24]

with: $N' = 10$ and $K = 2$

Figure 6 represents the process to determine the population $P_{node/vehicle}$.

The principles of different genetic operations such as crossover and mutation operator are detailed in our work [21].
4.4 Multi-criteria evaluation

A multi-objective problem is defined as an optimization vector problem, which seeks to optimize several components of a vector function cost.

Pareto dominance method

A multi-criteria problem $P$ is composed of $n$ variables, $m$ inequality constraints, $p$ equality constraints and $k$ criteria that can be formulated as follows:

$$
P \Rightarrow \begin{cases} 
\min f(x) = [f_1, f_2, f_3, \ldots, f_k(x)] \\
g_i(x) \leq 0 \quad i=0 \ldots m \\
g_j(x) = 0 \quad j=0 \ldots p
\end{cases} \quad (13)$$

However, it is necessary to find solutions representing a possible compromise between the criteria. The Pareto optimality concept introduced by the economist V. Pareto in the twentieth century is frequently used [25]. V. Pareto formulated the following concept: in a multi-criteria problem, there is equilibrium so that we can not improve one criterion without deteriorating at least one other. This equilibrium has been called Pareto optimal A solution is noted Pareto optimal if it is dominated by any other point in solutions space. These points are noted non dominated solutions.

A point $X \in E$ dominates $Y \in E$ if:

$$\begin{cases} 
\forall i, f_i(x) \leq f_i(y) \\
\text{and } \exists j, \text{such as } f_j(x) < f_j(y)
\end{cases} \quad (14)$$

Figure 7 shows an example where we seek generations of the initial populations to minimize $f_1$ and $f_2$. The points 1, 3 and 5 are not dominated. On the contrary point 2 is dominated by point 3, and point 4 is dominated by point 5.

Aggregation method

In the resolution of MOP (Multi objectives Problem), several traditional methods are transforming the MOP into a single objective problem. Among these methods we find the aggregation method. This is one of the first methods used to generate Pareto optimal solutions. It is to transform the problem (MOP) in a problem ($PMO_\lambda$) which combines the different cost functions of the problem into a single objective function $F$ generally linear [26]:

$$F(x) = \sum_{i=1}^{n} c_i \lambda_i f_i(x) \quad (15)$$

Where $\lambda_i$ and $c_i$ are weights and scaling coefficients, according to the application, that the different objectives are not necessarily commensurable. The constants $c_i$ are usually initialized to $\frac{1}{f_i(x^*)}$ where $f_i(x^*)$ is the optimal solution associated to the objective function $f_i$ considered.
Figure 6: Computation procedure
Multi-objective optimization and computing of lower bounds

The computing of lower bounds has been studied in literature for several scheduling problems which we quote: the problems with a machine [27], with parallel machines [28] and [29], the hybrid flow-shop problems [30] and the flexible job shop problems [31]. These proposed methods for computing of the lower bounds are generally based on the relaxation of constraints (preemption of tasks, constraints related resources ...) to minimize one or more criteria for optimal scheduling. Based on these methods and seen that we don’t have information on the optimal solutions associated with different cost functions $f_i$ for our problem we should compute the minimum value to determine the scaling constants $c_i$. For this objective, we use the relaxation of various constraints. To find a minimum value associated with the criterion of total travel cost $f_1 = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} C_k d_{ij} X_{ijk}$, we have treated this problem to the travelling salesman problem when we try to minimize $\sum_{i \in N} \sum_{j \in N} d_{ij} X_{ij}$. We subsequently determine the routing crossed by a single vehicle, minimizing the total travel distance by incorporating the constraints and capacity precedence. What gives us $d_{\text{min}, \text{p.c.}}$. 

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Figure 7: Dominance example

Figure 8: Pareto Solutions + Optimal solution by aggregation
\[ d_{\text{min,p.c}} = \min(\sum_{i \in N} \sum_{j \in N} d_{ij} X_{ij}) \] (16)

By setting \( K \), the number of vehicles used, we get:

\[ d_{\text{min,k,p.c}} = \frac{d_{\text{min,p.c}}}{K} \] (17)

Consequently, we acquire a value \( f_{1b} \) that represents a minimum value of the total travel cost.

\[ f_{1b} = \sum_{k \in K} C_k d_{\text{min,k,p.c}} \] (18)

To determine the minimum value of the total tardiness time, we fix \( K \) the vehicles number and find out the population \( P_{\text{node/vehicle}} \). Thus, we calculate the total tardiness time for each individual and determine the minimum.

\[ f_{2b} = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} \max(0, D_{ij} - l_i) X_{ijk} \] (19)

Knowing that for the criterion of tardiness, a better lower bound is zero. We will have therefore:

\[ \begin{cases} c_2 = \frac{1}{f_{2b}} \\ s.c. f_{2b} \neq 0 \end{cases} \] (20)

### 4.5 Computational results

To test our approach, we use benchmark problem instances generated by Li and Lim [19] from Solomon’s ones [32]. Corresponding to Solomon’s classification of C1, C2, R1, R2, RC1 and RC2, their data sets were also generated in six classes: LC1, LC2, LR1, LR2, LRC1 and LRC2. The LC problems are clustered whereas in the LR problems, providers and customers are randomly generated. Therefore in the LRC problems the providers and the customers are partially clustered and partially randomly distributed. While LC1, LR1 and LRC1 problems have a short scheduling horizon, LC2, LR2 and LRC2 have longer scheduling one. [33] In our work, we consider a vehicle number \( k \) ranging between 1 and 25. Table 1 shows the results of our simulation using the parameters of the problem LRC1. Of course, for every given solution, we note the corresponding routing, crossed by each vehicle.

\( N_{sol} \): represents the number of non dominated solutions.
\( N_k \): represents the vehicles number used.

We observe that our approach generates a multiple number of solutions that give flexibility of choice for the decision maker and that by using two different methods to determine the vehicles number used, minimizing the compromise between the total travel cost and the total tardiness time. We also observe that we obtain a total tardiness equal to zero with a tolerable cost.

### 5 Conclusion

In this paper, we have presented our approach to solve the m-PDPTW, based on Pareto dominance method, with use of genetic algorithm and lower bounds. Our purpose was in a first part a brief literature review on the VRP, 1-PDPTW and m-PDPTW. The mathematical formulation of our problem is detailed in second part. Then, we have detailed the use aggregation
method and lower bounds to determine a set of solutions, minimizing our objective functions. Simulation was presented in a last part by using benchmark’s data.

## Bibliography


