

# Towards Pricing Mechanisms for Delay Tolerant Services

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**Abstract:** One of the applications of Delay Tolerant Networking (DTN) is rural networks. For this application researchers have argued benefits on lowering costs and overcoming challenging conditions under which, for instance, protocols such as TCP/IP cannot work because their underlying requisites are not satisfied. New responses are required in order to understand the true adoption opportunities of this technology. Constraints in service level agreements and viable alternative pricing schemes are some of the new issues that arise as a consequence of the particular operation mode. In this paper, we propose a novel model for pricing delay tolerant services, which adjusts prices to demand variability subject to constraints imposed by the DTN operation. With this model we also show how important parameters such as channel rental costs, cycle times of providers, and market sensitivities affect business opportunities of operators.

**Keywords:** Delay Tolerant Networks Pricing, Capacity Management, Economic Models.

## 1 Introduction

Networks deployed in low income isolated areas operate in challenging environments characterized by discontinuous power supply and long distance wireless link connections. Their architectures are adapted to reduce costs, but they are prone to fail due to severe bandwidth restrictions, an operation highly dependent on weather conditions, and low levels of redundancy on key elements. These facts lead to intermittent connectivity between end to end points, which breaks the main assumption for the proper operation of the TCP/IP protocol suite.

The research networking community introduced the concept of intermittent connected networks (ICN) to cope with this environment. According to [15], ICN can be defined as:

An infrastructure-less wireless network that supports the proper functionality of one or several wireless applications operating in stressful environments, where excessive delays and unguaranteed continuous existence of end-to-end path(s) between any arbitrary source-destination pair, result from highly repetitive link disruptions.

The core element is the uncertainty in connectivity partially solved by two mechanisms: store-and-forward and flooding. In the first mechanism, the network's nodes are capable of storing data during periods of disconnection and forwarding to other nodes closer to the destination when a link is established. In the second mechanism, flooding copes with the routing problem. As nodes cannot maintain origin-destination paths, the network is programmed to send many copies of the same data towards different nodes to increase the probability of reaching the final destination.

For isolated networks the first project introducing this concept was [19] with a two-fold objective: to reduce operational cost using offline data mule connections implemented by existing transportation methods such as buses, motorcycles, bicycles, etc., and to provide an infrastructure capable of overcoming interruptions. Then the idea was extended to use mechanic backhauls and the Delay Tolerant Network architecture (DTN) proposed by the Internet Engineering Task Force (IETF) to handle ICNs [4].

In recent years prototypes such as [5, 9] using DTN or similar architectures have appeared. Potential operators deploying networks in isolated regions would fulfil a mix of delay tolerant and real time services such as video conferencing for medical procedures. We envision that real time services are going to be operated using technologies resulting from projects deploying low-cost infrastructure for cellular networks, see [1, 21], or by wireless channels connected to infrastructure provided by states, i.e, the e-Mexico system [23], the National broadband plan in India [24], the broadband infraCo in South Africa [22], to name a few.

Although most of these proposals cover technical parts of the network, the economics of the services being fulfilled are not considered. Therefore, their sustainability is strongly compromised. In economic terms, providers establish service level agreements (SLAs) to offer alternatives regarding the quality of services and prices that customers are willing to pay. Right now, these agreements are based on measures including, but not limited to, packet loss ratio, connection latency and bandwidth. Once the agreement is established the provider charges services according to the rules defined in the SLAs. Nonetheless, potential operators in isolated areas cannot directly use these measurements; they are based on the assumption of end to end connectivity which is not guaranteed in the rural environment. So one of the problems faced by these operators is how to establish SLAs for a diverse set of customers in networks delivering delay tolerant and real time services.

In this paper we propose, as a first step, an integrated pricing and resource assignment policy for a delay tolerant service. This proposal is based on two service agreement elements and it optimizes the expected profits from the operator's point of view. Furthermore, using real data from a rural network we test the validity of this model to adjust to variability on traffic demand and study its behavior under different scenarios.

The remainder of this paper is organized as follows. Section II provides an overview of previous proposals on Internet service usage pricing. Section III introduces a novel pricing model tailored to a delay tolerant service. This model is based on traffic management and average time of delivery. Scenario test-bed settings and results are presented in section III. Finally, section IV summarizes and concludes the paper.

## 2 Related Work

From the introduction of the Internet, authors like [10] have argued the necessity of creating and using charging models to induce an optimal network resource usage. For Internet connectivity in low income regions, which can be isolated, an optimal resource assignment is even more important, howsoever, there is no room for light users to subsidize heavy users, nor enough capacity to deliver non-priority services without affecting high-priority services, the profit enablers for operators [2]. To the best of our knowledge this paper is the first to introduce charging methods for networks using the delay tolerant architecture. Despite its novelty, the proposal takes elements of general charging models for the Internet and, in particular, it recaptures conclusions to increase operator performance under time-variance consumption patterns, see [11, 16, 20] for an overview of alternative research approaches.

Hande et al [8] introduce models to understand the relationship between the revenues of a monopolist Internet Service Provider(ISP) and the price sensitivity of users under a two part tariff. They conclude that in markets with high price sensitivity, normally found in low income regions, most of the revenues come from the usage based portion of the price. Even more, the paper suggests that the loss in revenue can be mitigated if the ISP implements charging based on consumption. Following these conclusions, the proposed model assumes that the ISP collects the usage part of the tariff and it has previous knowledge of the demand function based on the price sensitivity. This paper extends the formulation by including quality attributes on the demand function and costs. These extensions help to provide insights regarding ISP profitability and service quality experience.

Jian L. et al [12] introduce a price scheme to be implemented by a monopolist operator having a set of users, whose utilities not only depend on the traffic quantity, but on the specific access time. They show that the level of revenue extracted by the ISP depends on the information available. With perfect information, the ISP can extract the maximal revenue. When the ISP has only information about the traffic in different time-slots, a more realistic scenario, the loss in revenue compared with the maximal value is in general not bounded. Contrary to their proposal, we partially discard the additional utility received by users when access the network in a specific time-slot.

The present proposal utilizes a demand changing during the day and forecast prices. We expect that users being charged move to their preferred time-slot as they know pricing information, which somehow follows the results presented by [7]. For us, based on an actual deployment, the authors verify that repeating a game, in which users take into account net prices to decide either to wait or to use the network, helps not only to decrease congestion periods but to increase the network usage.

## 3 Proposed Pricing Model

Operators have two modes for connecting to the Internet. The first mode is a real time connection with cost and bandwidth capacity. Its cost is measured in value per unit of traffic volume and its bandwidth is measured in traffic volume per unit of time. The second mode is a mechanic backhaul connection with a by trip transit time (delivery time), cost, and capacity. Operators can divide traffic in both connections. Users receive in real time part of the content, for instance the web page's text, and, after the delivery time, the rest, images and videos. This way of operation is beneficial for users and providers. Users consume a reduced service instead of no service and providers increase the use of the network.

Information is relevant to users during a certain period of time. This is closely related to the value given by users to services, which is partially determined by whether the operator might

or not deliver the information within that period. Then, the delivery time becomes an intrinsic quality characteristic for delay tolerant services to be modelled as part of the demand.

In this proposal the idea of Grade of Service (GoS) was adapted to make quality operational for delay tolerant services. GoS is defined, see [6], as a set of traffic engineering variables used to measure the adequacy of a group of resources to fulfil a service under a specified condition. An important concept behind GoS is the average behavior, which is used to aggregate the performance of different resources into a unique measure. Following this idea, DTN operators are interested in measuring the adequacy of the resources (the two connection modes) for user's preferences on time of delivery. We propose that operators might use the weighted average time required to get answers to a user's request as a measure of the service's quality.

Our proposed model for pricing delay tolerant services includes price and average delivery time as factors determining aggregated demand. It creates an optimal policy for price and average delivery time searching for the maximum amount of profits. In the following subsections we present the main assumptions: the mathematical representation for the situation being modelled and the development of its solution.

### 3.1 Assumptions

#### Demand

In this paper demand constructions suggested by authors in [3] are used. They fixed the relationship between demand, price and time by the following linear function.

$$D(P, T) = a - \beta_P P - \beta_T T, \quad (1)$$

where  $a$  is the potential market size,  $\beta_P$  and  $\beta_T$  are the corresponding sensitivity of demand to price and to delivery time. The potential market size is generalized because demand fluctuates within the hours of days and between days. We propose  $a$  as a function of time, which is continuous and differentiable. The end form aggregated demand used in this proposal is:

$$D(t, P, T_{avg}) = a(t) - \beta_P P - \beta_T T_{avg} \quad (2)$$

#### Costs

The mechanic backhaul connection is charged by a third party. The vendor charges by the distance travelled and not by how much information is being travelled. From the operator's point of view this charge is a fixed cost by trip.

The real time connection is used for fulfil priority and non-priority services –delay tolerant services belong to the second category–, and it has a channel rental cost.

#### Constant capacity

The model assumes that connections cannot increase capacity dynamically as more demand is requested from users. The mechanic backhaul capacity is determined by the connection bandwidth and the total time that the vehicle is connected to the main gateway.

#### Constant consumption

Requests generated by users in the network are dissimilar in terms of data to be transferred. However, the operator needs to establish prices before observing the actual use of the network. The estimate of the average download and upload consumption is used to charge users. The

estimate is an input parameter for the model and it is included in the potential market size function  $a(t)$ .

### 3.2 Mathematical formulation

The operator needs a rule that defines traffic going to be served by the mechanic backhaul connection ( $F_1(t)$ ) or by the real time connection ( $F_2(t)$ ) and the price to charge. See figure 1. Each connection has a common set of parameters defined by the tuple  $(T, C)$  where  $T$  means time of delivery, and  $C$  means cost. The real time connection has an instant capacity  $K_2$ . The mechanic backhaul connection has a by trip capacity  $CAP_{mb}$ . The operator wants to maximize profits while maintaining the service mean delivery time ( $T_{avg}$ ). The operator cannot differentiate among customers willing to pay more for the time provision.

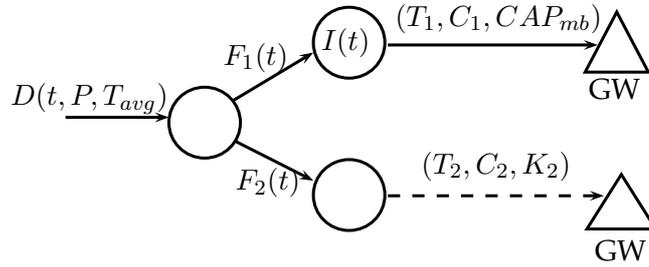


Figure 1: Elements in the general model

The general model could be stated as:

$$\begin{aligned} & \text{Max} \int_0^{T_1} P(t)(F_1(t) + F_2(t)) - C_1 F_1(t) - C_2 F_2(t) dt \\ & \text{Subject to} \\ & \quad I'(t) = -F_1(t) \tag{3a} \\ & \quad I(T_1) \geq 0 \tag{3b} \\ & \quad F_1(t) + F_2(t) = a(t) - \beta_T T_{avg} - \beta_P P(t) \tag{3c} \\ & \quad F_2(t) \leq K_2 \tag{3d} \\ & \quad (Z(t) - 1)F_1(t) \leq F_2(t) \tag{3e} \\ & \quad F_1(t) \geq 0 \tag{3f} \\ & \quad F_2(t) \geq 0 \tag{3g} \\ & \quad P(t) \geq 0 \tag{3h} \\ & \quad I(0) = CAP_{mb} \tag{3i} \end{aligned}$$

The first constraint indicates that the instantaneous change in the backlog of traffic going to be served by the mechanic backhaul connection is equal to the flow using that channel. The second constraint establishes that the aggregate capacity for services using the mechanic backhaul connection cannot be more than the capacity available. In particular, this condition must be true at the end of the cycle time.

The third constraint establishes that demand must go by either of the channels. It constitutes the relationship between channel use and price and service mean delivery time. Constraint four

indicates that the quantity of flow using the real time connection must be less than or equal to the capacity available.

Requests made at time  $t$  and using the mechanic backhaul must at least have a travel time equal to the vehicle cycle time  $T_1$ . Moreover, when the arrival time coincides with the vehicle being in transit, it has to wait the remaining cycle time and another cycle. Therefore, the traffic going by the mechanic backhaul as part of a request made at time  $t$  must wait  $2T_1 - t$ .

Then, the mean delivery time for services being handled at time  $t$  is given by:

$$\bar{L}(t) = \frac{(2T_1 - t)U_1(t) + T_2(t)U_2(t)}{U_1(t) + U_2(t)} \quad (4)$$

We can assume that  $T_2(t) = 0$  for all  $t$ . This measure according to the SLA maintained by the operator has to be less than or equal to  $T_{avg}$ . Defining the decreasing function  $Z(t) = 2T_1 - t/T_{avg}$  the expression is reduced to the following inequality which corresponds to constraint number five. The reader must note that it is valid only for the set  $\{t \leq T_1 : Z(t) \geq 1\}$ .

$$(Z(t) - 1)F_1(t) \leq F_2(t) \quad (5)$$

Finally, we define  $b(t) = a(t) - \beta_T T_{avg}$  as the function representing the potential market at time  $t$  after discounting the sensitivity of users for the average time.

### 3.3 Optimal Policy

In this subsection we develop the optimal policy for problem (3). First, the optimal policy for the formulation without the time average constraint is developed; then as a second step, the policy is extended to include the average time constraint.

#### The optimal policy when providers can break the time average constraint

We introduce a new constraint in  $F_1(t)$  in order to make easier the optimal policy develop. This constraint limits the amount of flow using the mechanic backhaul connection at any time, which is written as  $F_1(t) \leq K_1$ . Multiplier functions  $\theta(t), \lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t), \lambda_5(t), \lambda_6(t)$  are respectively associated with (3a), (3c),  $F_1(t) \leq K_1$ , (3d), (3f), (3g), (3h) to calculate the Hamiltonian ( $H$ ) and Lagrangian ( $LH$ ), which are, see [14]:

$$f(t, I, F_1, F_2, P) = P(t)(F_1(t) + F_2(t)) - C_1 F_1(t) - C_2 F_2(t) \quad (6a)$$

$$g(t, F_1) = -F_1(t) \quad (6b)$$

$$h_1(t, P, F_1, F_2) = b(t) - \beta_P P(t) - F_1(t) - F_2(t) \quad (6c)$$

$$h_2(t, F_1) = K_1 - F_1(t) \quad (6d)$$

$$h_3(t, F_2) = K_2 - F_2(t) \quad (6e)$$

$$h_4(t, F_1) = F_1(t) \quad (6f)$$

$$h_5(t, F_2) = F_2(t) \quad (6g)$$

$$h_6(t, P) = P(t) \quad (6h)$$

$$H(t, I, F_1, F_2, P, \theta) = f(\cdot) + \theta(t)g(\cdot) \quad (6i)$$

$$LH(t, I, F_1, F_2, P, \theta, \boldsymbol{\lambda}) = f(\cdot) + \theta(t)g(\cdot) + \boldsymbol{\lambda} \mathbf{h} \quad (6j)$$

from which we obtain the necessary conditions for optimality:

$$LH_P = F_1(t) + F_2(t) - \lambda_1(t)\beta_P + \lambda_6(t) = 0, \quad (7a)$$

$$LH_{F_1} = P(t) - C_1 - \theta(t) - \lambda_1(t) - \lambda_2(t) + \lambda_4(t) = 0, \quad (7b)$$

$$LH_{F_2} = P(t) - C_2 - \lambda_1(t) - \lambda_3(t) + \lambda_5(t) = 0, \quad (7c)$$

$$LH_{\lambda_1} = b(t) - \beta_P P(t) - F_1(t) - F_2(t) = 0, \quad (7d)$$

$$\theta'(t) = -H_I = 0, \quad I'(t) = -F_1(t), \quad \theta(T_1) \geq 0, \theta(T_1)I(T_1) = 0, \quad (7e)$$

$$\lambda_4(t) \geq 0, \lambda_4(t)F_1(t) = 0, \quad \lambda_5(t) \geq 0, \lambda_5(t)F_2(t) = 0, \quad \lambda_6(t) \geq 0, \lambda_6(t)P(t) = 0, \quad (7f)$$

$$\lambda_2(t) \geq 0, \lambda_2(t)(K_1 - F_1(t)) = 0, \quad \lambda_3(t) \geq 0, \lambda_3(t)(K_2 - F_2(t)) = 0, \quad (7g)$$

From (7) and integrating  $\theta'(t) = 0$  we have  $\theta(t) = \theta_0$ . Without loss of generality we are going to restrict the optimal path to prices greater than zero, so  $\lambda_6(t) = 0$ . The reader must to observe:

1. Assuming  $C_1 < C_2$ , the unconstrained problem has as optimal price  $P_1^*(t) = b(t) + (C_1 + \theta_0)\beta_P/2\beta_P$ . It is important to note that the function  $f(t)$  is strictly concave in the parameter  $P$ ,  $f_{PP} = -2\beta_P < 0$ , and it is concave in the parameter  $F_1$ , Therefore, this value is a global optimum. Let  $\overline{D}_1(t) = b(t) - (C_1 + \theta_0)\beta_P/2$  the demand corresponding to  $P_1^*(t)$ .
2. Because the mechanic backhaul channel has lower cost ( $C_1 < C_2$ ), the optimal solution uses it as much as possible until the point  $\overline{D}_1(t)$ . Indeed, if  $K_1 > 0 \Rightarrow F_1(t) > 0$ .

Now, we calculate the optimal paths as functions of the maximal instantaneous capacity for the mechanic backhaul  $K_1$ . All cases are developed in the following paragraphs.

Case 1.  $K_1 > \overline{D}_1(t)$ . The constraint on the mechanic backhaul traffic is not binding. From observation two we have:  $F_2^*(t) = 0, \lambda_2^*(t) = 0, \lambda_3^*(t) = 0$ , and  $\lambda_4^*(t) = 0$ . Replacing these results on the set of equations 7, we can calculate the remaining optimal paths as:  $F_1^*(t) = b(t) - (C_1 + \theta_0)\beta_P/2, \lambda_1^*(t) = b(t) - (C_1 + \theta_0)\beta_P/2\beta_P$ , and  $\lambda_5^*(t) = C_2 - C_1 - \theta_0$ .

Case 2.  $K_1 \leq \overline{D}_1(t)$  and  $F_2^*(t) = 0$ . From these conditions and observation two we conclude  $\lambda_3^*(t) = 0$  and  $\lambda_4^*(t) = 0$ . These let us to find optimal values for the rest of the functions. Those values are:  $P_2^*(t) = b(t) - K_1/\beta_P, \lambda_1^*(t) = K_1/\beta_P, \lambda_2^*(t) = b(t) - 2K_1 - (C_1 + \theta_0)\beta_P/\beta_P, \lambda_5^*(t) = 2K_1 + C_2\beta_P - b(t)/\beta_P$ .

Case 3.  $K_1 \leq \overline{D}_1(t)$  and  $F_2^*(t) > 0$ . In this case  $P_3^*(t) = b(t) + C_2\beta_P/2\beta_P, F_1^*(t) = K_1, F_2^*(t) = b(t) - C_2\beta_P - 2K_1/2, \lambda_1^*(t) = b(t) - C_2\beta_P/2\beta_P, \lambda_2^*(t) = C_2 - C_1 - \theta_0, \lambda_3^*(t) = 0, \lambda_4^*(t) = 0, \lambda_5^*(t) = 0$ . Additionally, it is defined  $\overline{D}_2(t) = b(t) - C_2\beta_P/2$ .

Case 4.  $K_1 + K_2 \leq \overline{D}_2(t)$ . From observation one, the objective function concavity in  $P(t)$  for a given  $t$ , and the fact that it is increasing in  $F_1(t)$  and  $F_2(t)$  until the point  $\overline{D}_2(t)$ , it can be concluded as the optimal decision to assign  $K_1$  and  $K_2$  to  $F_1(t)$  and  $F_2(t)$  respectively. With these assignments we have  $\lambda_4^*(t) = 0, \lambda_5^*(t) = 0$ . Replacing this information on the set of equations, we have  $P_4^*(t) = b(t) - K_1 - K_2/\beta_P, \lambda_1^*(t) = K_1 + K_2/\beta_P, \lambda_2^*(t) = b(t) - 2(K_1 + K_2) - (C_1 + \theta_0)\beta_P/\beta_P, \lambda_3^*(t) = b(t) - 2(K_1 + K_2) - C_2\beta_P/\beta_P$ .

Case 5.  $K_1 = 0, F_2(t) > 0, K_2 \leq \overline{D}_2(t)$ . The objective function in  $t$  is increasing in  $F_2(t)$  as long as  $K_2 \leq \overline{D}_2(t)$ . Therefore,  $F_2^*(t) = K_2, \lambda_5^*(t) = 0$ . From  $K_1 = 0$ , we assign to the multiplier function  $\lambda_4^*(t) = 0$ . Replacing these results in the set of equations and resolving for the rest of functions, we have:  $P_5^*(t) = b(t) - K_2/\beta_P, \lambda_1^*(t) = K_2/\beta_P, \lambda_2^*(t) = b(t) - 2K_2 - (C_1 + \theta_0)\beta_P/\beta_P, \lambda_3^*(t) = b(t) - 2K_2 - C_2\beta_P/\beta_P$ .

Case 6.  $K_1 = 0, F_2(t) > 0, K_2 > \overline{D}_2(t)$ . From these conditions, then  $\lambda_3^*(t) = 0, \lambda_4^*(t) = 0$ , and  $\lambda_5^*(t) = 0$ . Additionally, replacing these conditions, the optimal paths are:  $P_6^*(t) = b(t) + C_2\beta_P/2\beta_P, \lambda_1^*(t) = b(t) - C_2\beta_P/2\beta_P, \lambda_2^*(t) = C_2 - C_1 - \theta_0$ .

The optimal  $\theta_0$  value is required for all cases, from sub-equation (3a) it is equal to: (where  $\chi$  represents the indicator function).

$$I(T_1) = \int_0^{T_1} -F_1(t)dt = \int_0^{T_1} -K_1\chi_{K_1 \leq \bar{D}_1} dt + \int_0^{T_1} -\bar{D}_1\chi_{K_1 > \bar{D}_1} dt \quad (8)$$

Let  $A_D = \int_0^{T_1} b(t)dt$  the total estimated market during the cycle time. From observation two, we conclude that operators want to send the maximal value by the mechanic backhaul channel. In case of enough mechanic backhaul capacity, i.e.,  $A_D - C_1\beta_P T_1/2 \leq CAP_{mb}$ , it must be assigned  $\theta_0 = 0$  and  $K_1 = \max\{b(t) - C_1\beta_P/2 : t \in [0, T_1]\}$ . When  $A_D - C_1\beta_P T_1/2 > CAP_{mb}$ , the constant value  $\theta_0$  can be obtained observing:

1. It can be verified that profits for cases 1 and 2 are greater or equal than profits for case 3, whenever  $0 \leq \theta_0 \leq C_2 - C_1$ .
2. The amount of flow not sent because not enough mechanic backhaul capacity is  $\theta_0\beta_P T_1/2$ . This issue is the result of using the optimal point  $b(t) - (C_1 + \theta_0)\beta_P/2$  instead of  $b(t) - C_1\beta_P/2$ .
3. From items 1 and 2, we conclude that the flow potentially lost is constrained in  $[0, (C_2 - C_1)\beta_P T_1/2]$ . If  $\frac{A_D - C_2\beta_P T_1}{2} \leq CAP_{mb} \leq \frac{A_D - C_1\beta_P T_1}{2}$ , then  $\theta_0 = \frac{A_D - 2CAP_{mb}}{\beta_P T_1} - C_1$  and  $K_1 = \max\{\frac{b(t) - C_1\beta_P}{2} : t \in [0, T_1]\}$ . If  $CAP_{mb} < \frac{A_D - C_2\beta_P T_1}{2}$ , then  $\theta_0 = C_2 - C_1$  and  $K_1 = \frac{CAP_{mb}}{T_1}$  must be assigned.

### The optimal policy for providers with the time average constraint

We use the Lagrangian  $LH$  defined in the previous section and include the constraint  $h_4 = F_2(t) - (Z(t) - 1)F_1(t)$ . Again, a continuous multiplier function  $\lambda_7(t)$  is associated with this constraint. The necessary optimality conditions for the modified Lagrangian is given by 7 and:

$$\lambda_7(t) \geq 0, \quad \lambda_7(t)(F_2(t) - (Z(t) - 1)F_1(t)) = 0 \quad (9)$$

Constraint (3e) is active when  $F_2(t) = (Z(t) - 1)F_1(t)$ ,  $Z(t) \geq 1$  and  $F_2(t)$  is constrained by  $K_2$ . Then combining constraints (3e) and (3d) we have  $K_2 \geq (Z(t) - 1)F_1(t)$ . From this inequality and  $K_1 \geq F_1(t)$ , we conclude that  $F_1(t)$  is constrained by  $\frac{K_2}{Z(t) - 1}$  as long as this value is less than  $K_1$ . Moreover, this value is the binding constraint for  $F_1(t)$  between the time 0 and time  $t^1$ , defined as:

$$t^1 = \max\left(0, 2T_1 - T_{avg} \left(\frac{K_2}{K_1} + 1\right)\right) \quad (10)$$

Now, we complete the optimal policy as a function of the instantaneous capacities  $K_1$  and  $K_2$ . If the constraint (3e) is active and  $t \in [0, t^1]$ , then the optimal policy is the result of the capacity  $K_2$  and the value  $\bar{D}_3(t) = \frac{b(t)Z(t) - \bar{C}_1\beta_P - C_2(Z(t) - 1)\beta_P}{2Z(t)}$  is attained, which is the maximum profit. See table 1, where  $\bar{C}_1 = C_1 + \theta_0$ . For  $t \in [t^1, t^2 = 2T_1 - T_{avg}]$  the optimal policy is governed by values  $K_1$  and  $\bar{D}_3(t)$ , see table 1. Finally, for  $t > t^2$  the optimal policy follows the results presented in the previous subsection.

Table 1: Optimal assignment policy for  $t \in [0, t^2]$ 

Case	$t$	$G(t)^*$	$P(t)^*$	$F_1(t)^*$	$F_2(t)^*$	$\bar{D}(t)$
$\frac{K_2 Z(t)}{Z(t)-1} \geq \bar{D}_3(t)$	$t \leq t^1$	$\frac{(Z(t)-1)(b(t)Z(t)-\beta_P(\bar{C}_1+C_2(Z(t)-1)))}{2Z(t)^2}$	$\frac{b(t)}{\beta_P} - \frac{G^* Z(t)}{\beta_P(Z(t)-1)}$	$\frac{G^*}{Z(t)-1}$	$G^*$	$\bar{D}_3(t)$
$\frac{K_2 Z(t)}{Z(t)-1} \leq \bar{D}_3(t)$	$t \leq t^1$	$K_2$	$\frac{b(t)}{\beta_P} - \frac{G^* Z(t)}{\beta_P(Z(t)-1)}$	$\frac{G^*}{Z(t)-1}$	$G^*$	$\frac{G^* Z(t)}{Z(t)-1}$
$Z(t)K_1 \leq \bar{D}_3(t)$	$t^1 \leq t \leq t^2$	$K_1$	$\frac{b(t)-Z(t)G^*}{\beta_P}$	$G^*$	$(Z(t)-1)G^*$	$Z(t)G^*$
$Z(t)K_1 > \bar{D}_3(t)$	$t^1 \leq t \leq t^2$	$\frac{(b(t)Z(t)-\bar{C}_1\beta_P-C_2(Z(t)-1)\beta_P)}{2Z(t)^2}$	$\frac{b(t)-Z(t)G^*}{\beta_P}$	$G^*$	$(Z(t)-1)G^*$	$\bar{D}_3(t)$

## 4 Results and discussion

### 4.1 Scenario and parameter settings

Overall performance evaluation is done using the software created for network rural planning in [17]. The real data presented in [13] and [18] are used as estimators for investment, maintenance costs, data traffic, and elasticities. The evolving potential market size  $a(t)$  and the sensitivity of demand to price  $\beta_P$  were calculated for business days and weekends. Data used for simulation is summarized in tables 2 and 3. Parameters taking values in a range are presented as three arguments: the initial, final, and increment values.

With the proposed pricing model, only one delay tolerant service can be handled. Authors in [13] report two kinds of traffic that potentially could belong to this category: Http and e-mail, with e-mail being more delay tolerant than Http. Because the same elasticity value is reported for both services, we can consolidate their traffic and manage them as one service. Extending the model to more than one service and real time technology requires additional control variables and modifying the inventory constraint. This extension is left for future research.

Table 2: Network general parameters

Parameter	Value
Channel(Kbps)	32
Annual investment rate(%)	2,14,6
Monthly fixed cost (Usd)	330
Cost by channel (Usd)	90,170,10
Mechanic Backhaul cost (Usd)	10,70,15
Initial working Hour	6
Final working Hour	19
Cycle time ( $T_1$ )	7
Time average ( $T_{avg}$ )	7
Investment periods	60

Table 3: Service parameters

Parameter	Delay Tolerant	Real Time
Elasticity	1.45	1.337
Demand contribution (%)	75	25
$\beta_T$	0.1,1,0.01	N/A
Base price ( $Usd/unit$ )	0.016	0.016,0.05,0.016

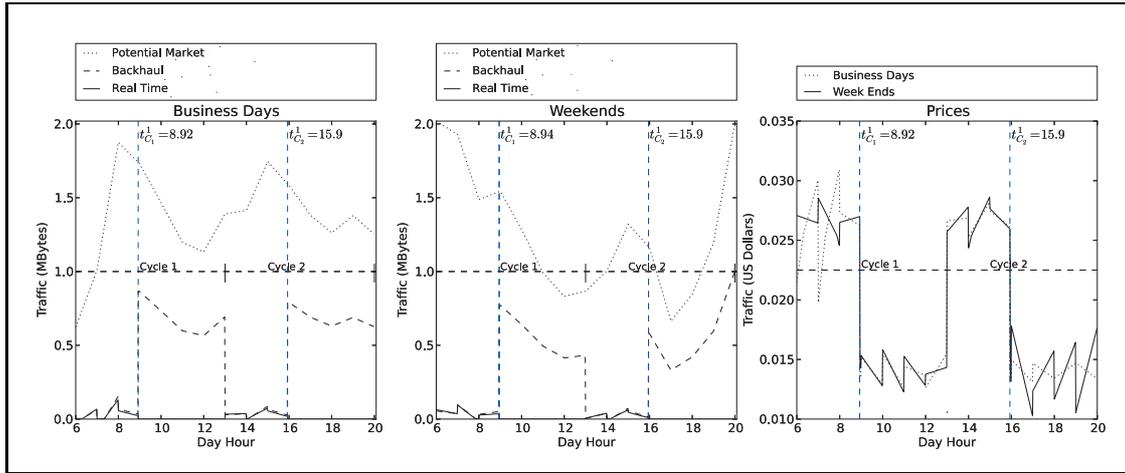


Figure 2: Traffic and price behavior

## 4.2 Results

The evolution of traffic by connection and price is presented in Figure 2. The two figures on the left show the potential market size and the optimal use of the real and mechanic backhaul connections. The two cycles are delimited and for each of them the time interval  $t \in [0, t^1]$  in which constraint 3e is active and determined by  $K_2$ , and when it is determined by  $K_1$ ,  $t \in [t^1, t^2]$ . Traffic use is strongly affected by the average time constraint regardless of business days or weekends. As long as the policy has to use the real time connection, it increases prices to cover link cost (see right graph). Hence, the optimal demand decreases limiting the network usage. Once the time constraint is more relaxed, the mechanic backhaul traffic increases as well as optimal demand. In fact, for the hypothetical scenario of  $T_1 \leq t^2$  the maximum mechanic backhaul connection use is determined by the potential market.

The last sub-figure in 2 corresponds to the optimal price. As it can be seen the figure presents jumps which are the result of changing  $\beta_P$  every hour within the days. Explicitly, we find the size of jumps to be proportional to demand changes between hours. From our tests we inference that maintaining a constant  $\beta_P$  eliminates jumps in prices, but under some scenarios decreases profits for operators. These jumps introduce comprehension complexities for users and operators, so we argue that maintaining a constant sensitivity to price for weekdays and weekends is the best decision for the whole system.

Using mixed technologies, the behavior of prices and demand might indicate a cost of real time channels too high for isolated areas. As this cost decreases, the model starts to decrease prices and the demand is stimulated for both channels. So we can suggest that mixed providers could be profitable in areas where WIMAX technologies could be installed ( rental costs less that 90 USD dollars for 32 kbps month).

Second, operators desire to know how traffic and income are affected by different parameters used in the model. The impact of average time, cost of the real time connection, and market's time sensitivity are presented in Figure 3. If the operator only considers the time average parameter, his optimal decision would be to increase it as much as possible. However, this decision is correct as long as users do not have great sensitivity to time, low  $\beta_T$  values. If demand is closely related with time, high  $\beta_T$  values, the operator must establish an equilibrium between income gained as more traffic uses the mechanic backhaul connection and the corresponding demand lost.

The real time connection cost is a critical parameter. When the time average constraint is

active, it determines the optimal demand  $\bar{D}_3$ , price, and quantity send by the two connection types. In the simulated test-bed, the most extreme case is assumed in which the operator has to use a VSAT connection. Surprisingly, these results indicate that deployment regions closer to connected Internet areas can benefit from mechanic backhaul connections when services require high volume traffic and users are to a certain degree insensible to delay.

There exists a correlation between real time costs and the traffic pattern used. It can be seen in figure 3 that a decrease in channel rental costs for business days produces more income than the same increase in weekends. The demand lost during the interval  $[0, t^1]$  explains this outcome, which is the consequence of the variation in the potential market size throughout time. Therefore, we can conclude the need for using continuous models for adapting policies to demand variations.

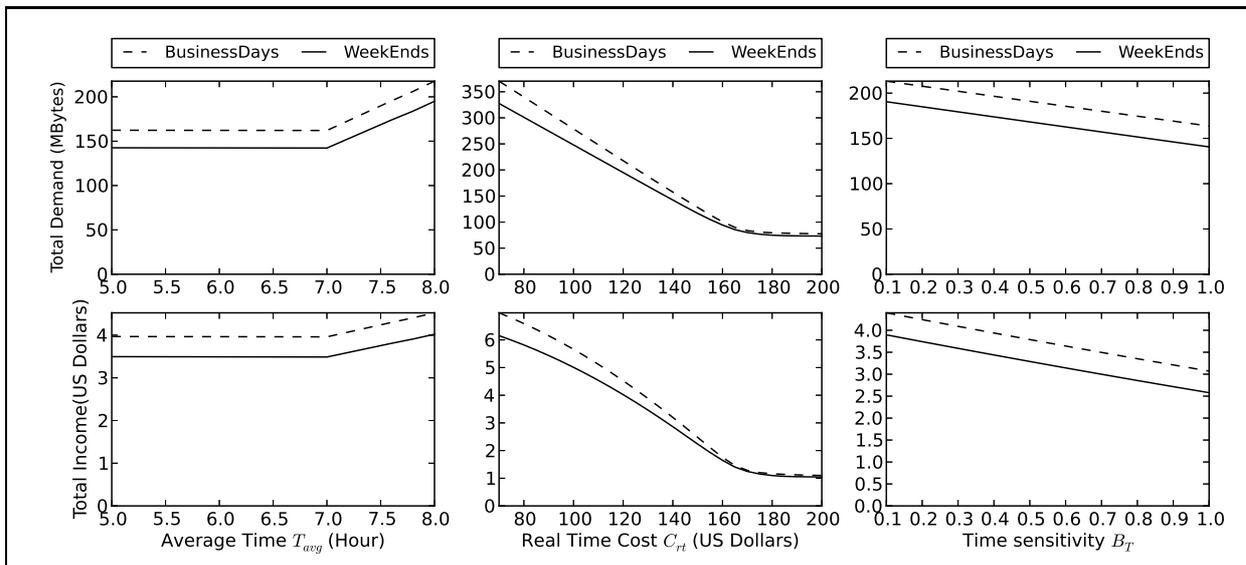


Figure 3: Change in aggregated results due to parameter shifts

## 5 Conclusions

We propose a model with price and delay time as demand predictors in which operators compromise themselves to deliver requests to maximize their profits. The results presented suggest that continuous models, like the one proposed, are pertinent to establish prices for delay tolerant services as long as they can handle variability of demand. Contrary to our first assumption, we observed that networks markets located close to Internet connected points can benefit from these services. Although real time channel lease cost continues to be a critical factor determining the demand offered to the market, results suggest that mechanic backhaul connections somehow mitigate the situation. In this paper we do not research, to name a few, the consequences of monopolistic operators' behavior, the extent to which sustainability of network deployments is reached, and the presence of economies of scale. All these research directions remain open.

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