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Feedback Linearization with Fuzzy Compensation for Uncertain Nonlinear Systems

M.C. Tanaka, J.M.M. Fernandes, W.M. Bessa

Marcelo C. Tanaka, Josiane M.M. Fernandes,
Wallace M. Bessa
Departamento de Engenharia Mecânica,
Universidade Federal do Rio Grande do Norte
Campus Universitário Lagoa Nova, Natal,
RN 59072-970, Brazil
marcelotanaka.eng.mec@gmail.com,
josiane.eng.mec@gmail.com, wmbessa@ct.ufrn.br

Abstract:
This paper presents a nonlinear controller for uncertain single-input–single-output (SISO) nonlinear systems. The adopted approach is based on the feedback linearization strategy and enhanced by a fuzzy inference algorithm to cope with modeling inaccuracies and external disturbances that can arise. The boundedness and convergence properties of the tracking error vector are analytically proven. An application of the proposed control scheme to a second-order nonlinear system is also presented. The obtained numerical results demonstrate the improved control system performance.
Keywords: feedback linearization, fuzzy logic, nonlinear control, Van der Pol oscillator.

1 Introduction

Due to its simplicity, feedback linearization scheme is commonly applied in industrial control systems, specially in the field of industrial robotics. The main idea behind this control method is the development of a control law that allows the transformation of the original dynamical system into an equivalent but simpler one [11]. Although feedback linearization represents a very simple approach, an important handicap is the requirement of a perfectly known dynamical system, in order to ensure the exponential convergence of the tracking error.

On this basis, much effort has been made to combine feedback linearization with intelligent algorithms in order to improve the trajectory tracking of uncertain nonlinear systems. The most common strategies are based on artificial neural networks [2,4,9,10,13] or fuzzy logic [1,3,5,6]. A drawback of these approaches is that both neural networks or fuzzy logic are used to model the entire plant, which means that a large computational effort is normally required to characterize system dynamics.

Considering that the designer of the control system usually has at least some knownledge of the plant to be controlled, a nonlinear controller is proposed in this paper to compensate for the uncertainties of single-input-single-output (SISO) nonlinear systems. The adopted approach is based on the feedback linearization method, but enhanced by a fuzzy inference system to cope with modeling imprecisions and external disturbances that can arise. This approach requires a reduced number of fuzzy sets and rules and consequently simplifies the design process. The boundedness and convergence properties of the closed-loop signals are analytically proven and numerical simulations are carried out in order to demonstrate the improved performance of the proposed control scheme.

2 Feedback Linearization

Consider a class of n^{th}-order nonlinear systems:

$$x^{(n)} = f(\mathbf{x}, t) + b(\mathbf{x}, t)u + d \tag{1}$$

where u is the control input, the scalar variable x is the output of interest, $x^{(n)}$ is the n-th time derivative of x, $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]$ is the system state vector, $f, b : \mathbb{R}^n \to \mathbb{R}$ are both nonlinear functions and d is assumed to represent all uncertainties and unmodeled dynamics regarding system dynamics, as well as any external disturbance that can arise.

In respect of the disturbance-like term d, the following assumption will be made:

Assumption 1. *The disturbance d is unknown but continuous and bounded, i. e. $|d| \leq \delta$.*

Let us now define an appropriate control law based on conventional feedback linearization scheme that ensures the tracking of a desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$, *i. e.* the controller should assure that $\tilde{\mathbf{x}} \to 0$ as $t \to \infty$, where $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x}, \dot{\tilde{x}}, \ldots, \tilde{x}^{(n-1)}]$ is the related tracking error.

On this basis, assuming that the state vector $\mathbf{x}$ is available to be measured and system dynamics is perfectly known, *i. e.* there is no modeling imprecision nor external disturbance ($d = 0$) and the functions f and b are well known, with $|b(\mathbf{x}, t)| > 0$, the following control law:

$$u = b^{-1}(-f + x_d^{(n)} - k_0\tilde{x} - k_1\dot{\tilde{x}} - \cdots - k_{n-1}\tilde{x}^{(n-1)}) \tag{2}$$

guarantees that $\mathbf{x} \to \mathbf{x}_d$ as $t \to \infty$, if the coefficients k_i $(i = 0, 2, \ldots, n-1)$ make the polynomial $p^n + k_{n-1}p^{n-1} + \cdots + k_0$ a Hurwitz polynomial [11].

The convergence of the closed-loop system could be easily established by substituting the control law (2) in the nonlinear system (1). The resulting dynamical system could be rewritten by means of the tracking error:

$$\tilde{x}^{(n)} + k_{n-1}\tilde{x}^{(n-1)} + \ldots + k_1\dot{\tilde{x}} + k_0\tilde{x} = 0 \tag{3}$$

where the related characteristic polynomial is Hurwitz.

However, since in real-world applications the nonlinear system (1) is often not perfectly known, the control law (2) based on conventional feedback linearization is not sufficient to ensure the exponential convergence of the tracking error to zero.

Thus, we propose the adoption of fuzzy inference system within the control law, in order to compensate for d and to enhance the feedback linearization controller.

3 Fuzzy Inference System

Because of the possibility to express human experience in an algorithmic manner, fuzzy logic has been largely employed in the last decades to both control and identification of dynamical systems.

The adopted fuzzy inference system is the zero order TSK (Takagi–Sugeno–Kang), with the r^{th} rule stated in a linguistic manner as follows:

$$\textit{If } \tilde{x} \textit{ is } \tilde{X}_r,\ \dot{\tilde{x}} \textit{ is } \dot{\tilde{X}}_r,\ \cdots,\ \textit{and } \tilde{x}^{(n-1)} \textit{ is } \tilde{X}_r^{(n-1)},\ \textit{then } \hat{d}_r = \hat{D}_r \quad ; \quad r = 1, 2, \cdots, N$$

where $\tilde{X}_r$, $\dot{\tilde{X}}_r$, $\cdots$, and $\tilde{X}_r^{(n-1)}$ are fuzzy sets, whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each one of the N fuzzy rules.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ an be computed by a weighted average:

$$\hat{d}(\tilde{\mathbf{x}}) = \frac{\sum_{r=1}^{N} w_r \cdot \hat{D}_r}{\sum_{r=1}^{N} w_r} \tag{4}$$

or, similarly,

$$\hat{d}(\tilde{\mathbf{x}}) = \hat{\mathbf{D}}^{\mathrm{T}} \mathbf{\Psi}(\tilde{\mathbf{x}}) \tag{5}$$

where, $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \dots, \hat{D}_N]$ is the vector containing the attributed values $\hat{D}_r$ to each rule r, $\mathbf{\Psi}(\tilde{\mathbf{x}}) = [\psi_1, \psi_2, \dots, \psi_N]$ is a vector with components $\psi_r(\tilde{\mathbf{x}}) = w_r / \sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule, which can be computed from the membership values with any fuzzy intersection operator (t-norm).

4 Fuzzy Feedback Linearization

Considering that fuzzy logic can perform universal approximation [7], we propose the adoption of a TSK fuzzy inference system within the feedback linearization controller to compensate for modeling inaccuracies and consequently enhance the trajectory tracking of uncertain nonlinear systems.

Therefore, the control law with the fuzzy compensation scheme can be stated as follows

$$u = b^{-1}[-f + x_d^{(n)} - k_0\tilde{x} - k_1\dot{\tilde{x}} - \cdots - k_{n-1}\tilde{x}^{(n-1)} - \hat{d}(\tilde{\mathbf{x}})] \tag{6}$$

and the related closed-loop system is:

$$\tilde{x}^{(n)} + k_{n-1}\tilde{x}^{(n-1)} + \ldots + k_1\dot{\tilde{x}} + k_0\tilde{x} = \tilde{d} \tag{7}$$

with $\tilde{d} = \hat{d} - d$.

Now, defining $\mathbf{k}^{\mathrm{T}}\tilde{\mathbf{x}} = k_{n-1}\tilde{x}^{(n-1)} + \ldots + k_1\dot{\tilde{x}} + k_0\tilde{x}$, where $\mathbf{k} = [c_0\lambda^n, c_1\lambda^{n-1}, \dots, c_{n-1}\lambda]$, λ is a strictly positive constant and c_i states for binomial coefficients, *i. e.*

$$c_i = \binom{n}{i} = \frac{n!}{(n-i)!\, i!}\,, \quad i = 0, 1, \dots, n-1 \tag{8}$$

the convergence of the closed-loop signals to a bounded region is assured.

Theorem 2. *Consider the uncertain nonlinear system (1) and Assumption 1, then the fuzzy feedback linearization controller defined by (5) and (6) ensures the exponential convergence of the tracking error vector to a closed region* $\Omega = \{\mathbf{x} \in \mathbb{R}^n \mid |\tilde{x}^{(i)}| \le \zeta_i \lambda^{i-n}\varepsilon, i = 0, 1, \dots, n-1\}$*, with* ζ_i *defined by (9).*

$$\zeta_i = \begin{cases} 1 & \textit{for} \quad i = 0 \\ 1 + \sum_{j=0}^{i-1} \binom{i}{j} \zeta_j & \textit{for} \quad i = 1, 2, \dots, n-1. \end{cases} \tag{9}$$

Proof: Considering the universal approximation feature of fuzzy logic [7], the output of the adopted inference system (5) can approximate the disturbance d to an arbitrary degree of accuracy, *i. e.* $|\hat{d}(\tilde{x}) - d| \le \varepsilon$ for an arbitrary $\varepsilon > 0$. Thus, from (7) one has

$$|\tilde{x}^{(n)} + k_{n-1}\tilde{x}^{(n-1)} + \ldots + k_1\dot{\tilde{x}} + k_0\tilde{x}| \le \varepsilon \tag{10}$$

From (8), inequality (10) may be rewritten as

$$-\varepsilon \le \tilde{x}^{(n)} + c_{n-1}\lambda\tilde{x}^{(n-1)} + \cdots + c_1\lambda^{n-1}\dot{\tilde{x}} + c_0\lambda^n\tilde{x} \le \varepsilon \tag{11}$$

Multiplying (11) by $e^{\lambda t}$ yields

$$-\varepsilon e^{\lambda t} \leq \frac{d^n}{dt^n}(\tilde{x}e^{\lambda t}) \leq \varepsilon e^{\lambda t} \tag{12}$$

Integrating (12) between 0 and t gives

$$-\frac{\varepsilon}{\lambda}e^{\lambda t} + \frac{\varepsilon}{\lambda} \leq \frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t}) - \left.\frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t})\right|_{t=0} \leq \frac{\varepsilon}{\lambda}e^{\lambda t} - \frac{\varepsilon}{\lambda} \tag{13}$$

or conveniently rewritten as

$$-\frac{\varepsilon}{\lambda}e^{\lambda t} - \left(\left|\left.\frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t})\right|_{t=0}\right| + \frac{\varepsilon}{\lambda}\right) \leq \frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t}) \leq \frac{\varepsilon}{\lambda}e^{\lambda t} + \left(\left|\left.\frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t})\right|_{t=0}\right| + \frac{\varepsilon}{\lambda}\right) \tag{14}$$

The same reasoning can be repeatedly applied until the n^{th} integral of (12) is reached:

$$\begin{aligned} -\frac{\varepsilon}{\lambda^n}e^{\lambda t} - \left(\left|\left.\frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t})\right|_{t=0}\right| + \frac{\varepsilon}{\lambda}\right)\frac{t^{n-1}}{(n-1)!} - \cdots + \\ - \left(|\tilde{x}(0)| + \frac{\varepsilon}{\lambda^n}\right) \leq \tilde{x}e^{\lambda t} \leq \frac{\varepsilon}{\lambda^n}e^{\lambda t} + \\ + \left(\left|\left.\frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t})\right|_{t=0}\right| + \frac{\varepsilon}{\lambda}\right)\frac{t^{n-1}}{(n-1)!} + \cdots + \left(|\tilde{x}(0)| + \frac{\varepsilon}{\lambda^n}\right) \end{aligned} \tag{15}$$

Furthermore, dividing (15) by $e^{\lambda t}$, it can be easily verified that, for $t \to \infty$,

$$-\frac{\varepsilon}{\lambda^n} \leq \tilde{x}(t) \leq \frac{\varepsilon}{\lambda^n} \tag{16}$$

Considering the $(n-1)^{\text{th}}$ integral of (12)

$$\begin{aligned} -\frac{\varepsilon}{\lambda^{n-1}}e^{\lambda t} - \left(\left|\left.\frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t})\right|_{t=0}\right| + \frac{\varepsilon}{\lambda}\right)\frac{t^{n-2}}{(n-2)!} - \cdots + \\ - \left(\left|\dot{\tilde{x}}(0)\right| + \frac{\varepsilon}{\lambda^{n-1}}\right) \leq \frac{d}{dt}(\tilde{x}e^{\lambda t}) \leq \frac{\varepsilon}{\lambda^{n-1}}e^{\lambda t} + \\ + \left(\left|\left.\frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t})\right|_{t=0}\right| + \frac{\varepsilon}{\lambda}\right)\frac{t^{n-2}}{(n-2)!} + \cdots + \left(\left|\dot{\tilde{x}}(0)\right| + \frac{\varepsilon}{\lambda^{n-1}}\right) \end{aligned} \tag{17}$$

and noting that $d(\tilde{x}e^{\lambda t})/dt = \dot{\tilde{x}}e^{\lambda t} + \tilde{x}\lambda e^{\lambda t}$, by imposing the bounds (16) to (17) and dividing again by $e^{\lambda t}$, it follows that, for $t \to \infty$,

$$-2\frac{\varepsilon}{\lambda^{n-1}} \leq \dot{\tilde{x}}(t) \leq 2\frac{\varepsilon}{\lambda^{n-1}} \tag{18}$$

Now, applying the bounds (16) and (18) to the $(n-2)^{\text{th}}$ integral of (12) and dividing once again by $e^{\lambda t}$, it follows that, for $t \to \infty$,

$$-6\frac{\varepsilon}{\lambda^{n-2}} \leq \ddot{\tilde{x}}(t) \leq 6\frac{\varepsilon}{\lambda^{n-2}} \tag{19}$$

The same procedure can be successively repeated until the bounds for $\tilde{x}^{(n-1)}$ are achieved:

$$-\left[1+\sum_{i=0}^{n-2}\binom{n-1}{i}\zeta_i\right]\frac{\varepsilon}{\lambda}\leq\tilde{x}^{(n-1)}\leq\left[1+\sum_{i=0}^{n-2}\binom{n-1}{i}\zeta_i\right]\frac{\varepsilon}{\lambda} \tag{20}$$

where the coefficients ζ_i $(i=0,1,\ldots,n-2)$ are related to the previously obtained bounds of each $\tilde{x}^{(i)}$ and can be summarized as in (9).

In this way, by inspection of the integrals of (12), as well as (16), (18), (19), (20) and the other omitted bounds, it follows that the tracking error exponentially converges to the n-dimensional box determined by the limits $|\tilde{x}^{(i)}|\leq\zeta_i\lambda^{i-n}\varepsilon, i=0,1,\ldots,n-1$, where ζ_i is defined by (9). □

Corollary 3. *It must be noted that the proposed control scheme provides a smaller tracking error when compared with the conventional feedback linearization controller. By setting the output of the fuzzy inference system to zero, $\hat{d}(\tilde{x})=0$, Theorem 2 implies that the resulting bounds are $|\tilde{x}^{(i)}|\leq\zeta_i\lambda^{i-n}\delta, i=0,1,\ldots,n-1$. Considering that $\varepsilon<\delta$, from the universal approximation feature of $\hat{d}$, it can be concluded that the tracking error obtained with the fuzzy feedback linearization controller is smaller than the associated with the conventional scheme.*

5 Illustrative Example

In order to illustrate the controller design methodology, consider a controlled Van der Pol oscillator

$$\ddot{x}-\mu(1-x^2)\dot{x}+x=\upsilon \tag{21}$$

with a dead-zone in the control input defined according to

$$\upsilon=\begin{cases}u+0.2 & \text{if} \quad u\leq-0.2\\ 0 & \text{if} \quad -0.2<u<0.2\\ u-0.2 & \text{if} \quad u\geq0.2\end{cases} \tag{22}$$

For control purposes, equation (22) can be rewritten as a combination of a linear and a saturation function [8, 12]:

$$\upsilon=u+d(u) \tag{23}$$

where $d(u)$ can be obtained from (22) and (23) as:

$$d(u)=\begin{cases}0.2 & \text{if} \quad u\leq-0.2\\ -u & \text{if} \quad -0.2<u<0.2\\ -0.2 & \text{if} \quad u\geq0.2\end{cases} \tag{24}$$

Based on (6) and considering $d(u)$ as uncertainty, a fuzzy feedback linearization controller can be chosen as follows

$$u=x-\mu(1-x^2)\dot{x}+\ddot{x}_d-2\lambda\dot{\tilde{x}}-\lambda^2\tilde{x}-\hat{d}(\tilde{x},\dot{\tilde{x}}) \tag{25}$$

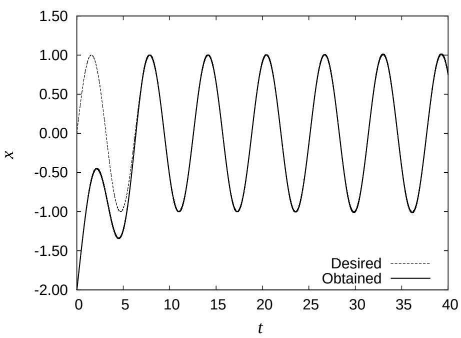
In order to evaluate the performance of the proposed control law (25), a numerical simulation was carried out. The simulation study was performed with an implementation in C, with sampling rates of 500 Hz for control system and 1 kHz for the Van der Pol oscillator, and the differential equations were numerically solved using the fourth order Runge-Kutta method. The chosen parameters for the Van der Pol oscillator and controller were $\mu=1$ and $\lambda=0.8$.

Regarding the fuzzy inference system, the number of fuzzy rules and the type of the membership functions, as well as how they are distributed over the input space, could be heuristically defined to accommodate designer's experience and experimental knowledge. The fuzzy rule base adopted in this work is presented in Table 1, where NB, NM, NS, ZO, PS, PM and PB represent, respectively, Negative–Big, Negative–Medium, Negative–Small, Zero, Positive–Small, Positive–Medium and Positive–Big. Triangular and trapezoidal (at the ends) membership functions are adopted for both $\tilde{X}_r$ and $\dot{\tilde{X}}_r$, with the central values defined respectively as $C_{\tilde{x}} = \{-20; -2; -0.2; 0.0; 0.2; 2; 20\} \times 10^{-2}$ and $C_{\dot{\tilde{x}}} = \{-16; -1.6; -0.16; 0.0; 0.16; 1.6; 16\} \times 10^{-2}$. The chosen fuzzy intersection operator was the minimum t-norm. It should be also emphasized that the input space could be partitioned and represented in many other ways, and that the system designer may test each one of them in order to improve the output value $\hat{d}$. With respect to the output of each rule, the following values were heuristically adopted for NB to PB: $\hat{D}_r = \{-20\,;\,-5\,;\,-2.5\,;\,0.0\,;\,2.5\,;\,5\,;\,20\}$.

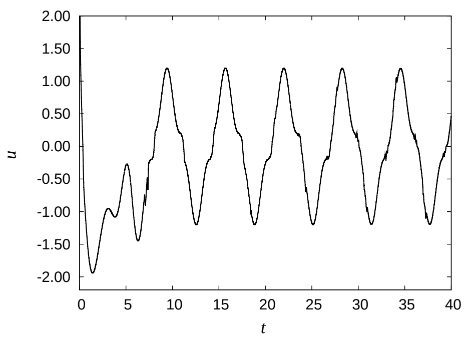
Table 1: Adopted fuzzy rule base.

$\tilde{x}$ / $\dot{\tilde{x}}$	NB	NM	NS	ZO	PS	PM	PB
NB	PB	PB	PB	PM	PM	PS	ZO
NM	PB	PB	PM	PM	PS	ZO	NS
NS	PB	PM	PM	PS	ZO	NS	NM
ZO	PM	PM	PS	ZO	NS	NM	NM
PS	PM	PS	ZO	NS	NM	NM	NB
PM	PS	ZO	NS	NM	NM	NB	NB
PB	ZO	NS	NM	NM	NB	NB	NB

In this way, considering that the initial state and initial desired state are not equal, $\tilde{\mathbf{x}}(0) = [-2.0, -0.4]$, Figures 1–3 show the obtained results for the tracking of $\mathbf{x}_d = [\sin t, \cos t]$.



(a) Tracking performance.



(b) Control action.

Figure 1: Trajectory tracking with $\mathbf{x}_d = [\sin t, \cos t]$.

As observed in Figure. 1(a), even in the presence of modeling imprecisions, the proposed control scheme allows the actuated Van der Pol oscillator to track the desired trajectory.

Now, in order to demonstrate the improved performance of the fuzzy feedback linearization controller, the tracking error associated with the last simulation is shown in Fig. 2. For comparison purposes, the tracking error obtained with conventional feedback linearization is also presented. It can be easily verified that the proposed controller provides a smaller tracking error when compared with the conventional one.

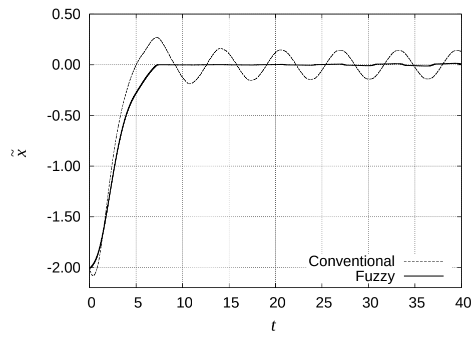
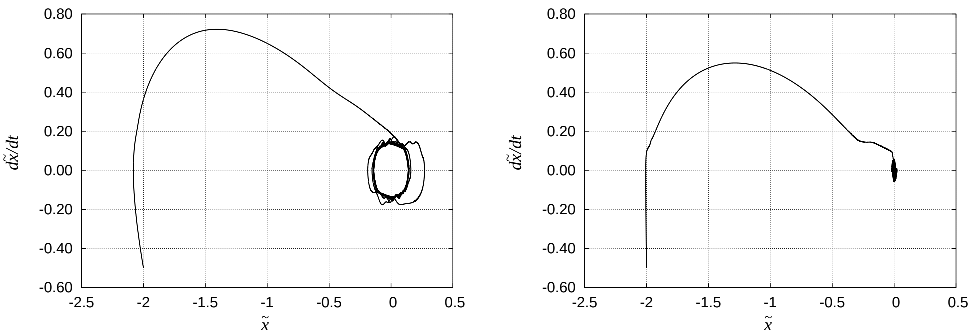


Figure 2: Tracking error with conventional and fuzzy feedback linearization.

The phase portraits of the tracking errors obtained with conventional as well as fuzzy feedback linearization are shown in Fig. 3. Note that the convergence region related to the proposed control scheme is much smaller than the associated with its uncompesated counterpart, which confirms Corollary 3.



(a) Conventional. (b) Fuzzy.

Figure 3: Phase portrait of the error with conventional and fuzzy feedback linearization.

6 Concluding Remarks

In this paper, a fuzzy feedback linearization controller is developed to deal with uncertain single-input–single-output nonlinear systems. To enhance the tracking performance, the feedback linearization controller is combined with a fuzzy inference system for uncertainty/disturbance compensation. The boundedness and convergence properties of the tracking error vector are analytically proven. To evaluate the control system performance, the proposed scheme is applied to the Van der Pol oscillator. By means of numerical simulations, the improved performance over the conventional feedback linearization controller is confirmed.

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Bibliography

[1] Boukezzoula, R.; Galichet, S.; Foulloy, L. (2007); Fuzzy feedback linearizing controller and its equivalence with the fuzzy nonlinear internal model control structure, *Int J Appl Math Comput Sci*, ISSN 1641-876X, 17(2):233-248.

[2] Chen. F.C. (1990); Back-propagation neural networks for nonlinear self-tuning adaptive control, *IEEE Control Syst Mag*, ISSN 0272-1708, 10(3):44-48.

[3] Couceiro, M.S.; Ferreira, N.M.F.; Machado, J.A.T. (2012); Hybrid adaptive control of a dragonfly model, *Commun Nonlinear Sci Numer Simul*, ISSN 1007-5704, 17(2):893-903.

[4] Deng, H.; Li, H.X.; Wu, Y.H. (2008); Feedback-linearization-based neural adaptive control for unknown nonaffine nonlinear discrete-time systems, *IEEE Trans Neural Netw*, ISSN 1045-9227, 19(9):1615-1625.

[5] Hojati, M.; Gazor, S. (2002); Hybrid adaptive fuzzy identification and control of nonlinear systems, *IEEE Trans Fuzzy Syst*, ISSN 1063-6706, 10(2):198-210.

[6] Kang, H.J.; Kwon, C.; Lee, H.; Park, M. (1998); Robust stability analysis and design method for the fuzzy feedback linearization regulator, *IEEE Trans Fuzzy Syst*, ISSN 1063-6706, 6(4):464-472.

[7] Kosko, B. (1994); Fuzzy systems as universal approximators, *IEEE Trans Comput*, 43(11):1329-1333.

[8] Lewis, F.L.; Tim, W.K.; Wang, L.Z.; Li, Z.X. (1999); Deadzone compensation in motion control systems using adaptive fuzzy logic control, *IEEE Trans Control Syst Technol*, ISSN 1063-6536, 7(6):731-742.

[9] Lu, Z.; Shieh, L.S.; Chen, G.; Coleman, N.P. (2006); Adaptive feedback linearization control of chaotic systems via recurrent high-order neural networks, *Inf Sci*, ISSN 0020-0255, 176(16):2337-2354.

[10] Pedro J.O.; Dahunsi, O.A. (2011); Neural network based feedback linearization control of a servo-hydraulic vehicle suspension system, *Int J Appl Math Comput Sci*, ISSN 1641-876X, 21(1):137-147.

[11] Slotine, J.J.E.; Li, W. (1991); *Applied Nonlinear Control*, Prentice Hall.

[12] Wang, X.S.; Su, C.Y.; Hong, H. (2004); Robust adaptive control of a class of nonlinear systems with unknown dead-zone, *Autom*, ISSN 0005-1098, 40(3):407-413.

[13] Yeşildirek, A.; Lewis, F.L. (1995); Feedback linearization using neural networks, *Autom*, ISSN 0005-1098, 31(11):1659-1664.